Translating Between Words and Math

(Also available in WeScheme)

Students learn to model verbal expressions with a visual tool known as "Circles of Evaluation".

Lesson Goals	Students will be able to... • Model an arithmetic expression using Circles of Evaluation. • Translate between verbal expressions, Circles of Evaluation, and mathematical expressions. • Recognize when expressions in words are ambiguous.
Student-facing Lesson Goals	• Let's analyze the structure of verbal expressions using Circles of Evaluation.
Materials	• PDF of all Handouts and Page • Lesson Slides • Printable Lesson Plan (a PDF of this web page)
Supplemental Materials	• Additional Printable Pages for Scaffolding and Practice
Preparation	• This activity involves a card sort. The easiest way to prepare for the card sort is to give each pair of students an envelope containing the three sets of cards. Keep each set (Expressions, Circles of Evaluation, and Verbal Expressions) together with rubber bands or paperclips. We recommend printing each set on a different color of paper.
Key Points For The Facilitator	• This lesson may challenge students' ideas of math as a subject that is entirely black and white. This theme - that there are oftentimes a variety of completely valid ways of seeing a seemingly simple problem - will emerge again.

Glossary

arithmetic expression :: a mathematical expression that consists of numbers and operators, which does not have an equal sign. When we apply the laws of arithmetic, arithmetic expressions can be simplified to a single numeric value.

circle of evaluation :: a diagram of the structure of a mathematical expression

equation :: a statement that two expressions are equal

function :: a relation from a set of inputs to a set of possible outputs, where each input is related to exactly one output

operator :: a symbol that manipulates two Numbers and produces a result

Circles of Evaluation *30 minutes*

Overview

Students match Circles of Evaluation to ***arithmetic expressions***, and then they consider how those arithmetic expressions in words map onto Circles of Evaluation.

Launch

Give each pair of students an envelope and explain that it contains three sets of cards, which you will print from [Card Sort (Arithmetic Expressions)](), [Card Sort (Circles of Evaluation)](), and [Card Sort (Verbal Expressions)]().

- Look through the first set of cards. What do you notice?
  - *Students should notice that the first set of cards includes* ***arithmetic expressions****. The expressions on these cards each include the number 15, the number 3, and an* ***operator****. Note that because these are expressions - not* ***equations*** *- they do not include an equal sign.*
- Now, look through the second set of cards. What do you notice?
  - *Students should observe that each card includes an oval-shaped diagram. They may also notice the position of the numbers and operator within that diagram.*

This second set of cards includes Circles of Evaluation. A ***Circle of Evaluation*** helps students visualize the structure of the mathematical expressions they encounter. For the time being, here's what students need to know about them:

- Every Circle must have one - and only one! - operator (or ***function***!), written at the top.
- The inputs of the operator are written left to right, in the middle of the Circle.

Share these two key concepts before explaining the following instructions.

- With your partner, match each Arithmetic Expression card with the corresponding Circle of Evaluation card. Do not sort the Verbal Expression cards yet.
- Create a separate pile for any cards that do not have a match.
  - *Cards with no match include:* $15 \times {}^{1}/_{3}$*, the Circle of Evaluation with the function between the operators rather than at the top of the Circle, and the Circle of Evaluation for* $3 + 15$*.*
- Lay the cards out on the table in front of you so you can clearly see both the Circle of Evaluation and the expression.
- Discuss any questions that arose.

Circulate as students sort their cards, ensuring that they are carefully analyzing each card so that they see the connection between the arithmetic expression and the Circle of Evaluation. This first phase of matching cards should be relatively quick - but it might raise questions on a few topics:

- **Symbols**. Circles of Evaluation utilize `*` to represent multiplication and `/` to represent division. (Why? Circles of Evaluation are a bridge representation - one which can eventually be used to help students learn to code! These are also the symbols used to type mathematical expressions into a search bar! *Although we use these symbols on all of our materials, you and your students can use whichever operator symbols are most comfortable.*)
- **Order of terms**. While $15 + 3$ and $3 + 15$ both evaluate to the same *answer*, they are not the same *expression*. Highlight this difference for students.
- **Position of the operator**. The operator always belongs at the top of the Circle, and not in between terms. Explain to students that this is a convention we must follow when working with Circles of Evaluation.

- Now, you’re going to receive a set of cards with verbal expressions - expressions written out in *words*.
- One at a time, take turns reading each card from the Verbal Expressions set out loud. After reading the card, place it with the corresponding Circle of Evaluation and Expression cards. Some sets will include just 3 cards, and others will have more.
- Explain to your partner how and why you placed each card.
- You and your partner must agree on each card’s placement before advancing to the next.

Again, circulate during the activity. As students match Verbal Expressions to their Arithmetic Expressions and Circles of Evaluation, additional challenges may arise. In particular:

- **Order matters.** When translating, encourage students to translate **precisely**. Emphasize that, although $15 + 3$ and $3 + 15$ evaluate to the same result, they are in fact structurally different expressions, as reflected by the verbal translation.
- **One-third of 15**. This card will likely motivate some discussion. Take advantage of opportunities to help students connect multiplication and division as inverse operations. Explain to students that another way to represent this expression is $1/3 \times 15$. Dividing by 3 produces the same outcome as multiplying by $1/3$.
- **Translating subtraction.** Students often translate "3 less than 15" into $3 - 15$, rather than $15 - 3$. Discuss the meaning of the word "less" in this context. Sometimes, asking, “What value is 3 less than 15?” can help students to make the connection. (The word "than" tends to force the numbers to appear in the *opposite* order in which the language arranges them.)

Reading expressions aloud can often help students think about the meaning of the words.

- Turn to Translating. Each row of the table represents a single arithmetic expression, written in three different ways. Fill in the empty spaces so that all three representations match.
- Next, complete Matching Words to Circles of Evaluation.

Ensure that students are confident creating and translating simple Circles, as the next section of the lesson introduces another layer of complexity.

Investigate

During the launch, students looked at the Circle of Evaluation for "15 increased by 3."

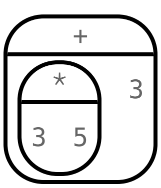
Let's say we want to replace 15 with 3×5. Now, our expression looks like this: $3 \times 5 + 3$.

Translating this mathematical expression into words requires students to see the underlying structure of the expression (e.g. - Multiply 3 by 5 first? Or add 5 and 3?). Then, they must access (possibly new/unfamiliar) vocabulary to describe what they see. Finally, they need to fit the right vocabulary onto the structure in the right way.

Fortunately, there is a simpler way…

Circles of Evaluation can contain other Circles of Evaluation.

The Circle of Evaluation for $3 \times 5 + 3$ looks like this:



Because Circles of Evaluation highlight the structure of any given expression, translating into words is a far less daunting task: the inner Circle clearly shows a product, which is being increased by 3 (as the outer Circle indicates).

Teacher Note: Your students do not need to know that multiplication precedes addition in the subsequent activities.

The following activities allow students an opportunity to explore nested Circles of Evaluation.

- First, practice Translating from Words to Circles.
- Then, translate in the *other* direction on Translating from Circles to Words.

- *Note: There are multiple correct translations! Invite students to share their responses and evaluate the clarity of each translation as a class.*
- When you're finished, complete Translation Table (1) and Translation Table (2) to practice moving between all three representations (the mathematical expression, the Circle of Evaluation, the verbal expression).
  - *Note: In Part 1, the same nested Circle is used in multiple expressions - but not all expressions! In Part 2, the structure of the Circles of Evaluation shift from expression to expression.*
- *Optional:* Try Matching Math to Words, where you will match mathematical expressions with their corresponding expressions in words. (If you get stuck, feel free to draw Circles to help you.)

Be sure to spend a moment going over students' solutions. Some translations into words are clearer than others; the subsequent section of this lesson will explore that notion in greater depth.

Synthesize

- We did lots of different translations between Circles of Evaluation, verbal expressions, and arithmetic expressions.
- Was there any type of translation that was more challenging for you?
- Is there more than one way to draw the Circle of Evaluation for $1 + 2$? If so, is one way more "correct" than the other?

The Ambiguity of Words *20 minutes*

Overview

Students diagram arithmetic expressions using Circles of Evaluations to consider how different mathematical interpretations can lead to different outcomes.

Launch

- Read this sentence: "Bruno told Gus that Mr. Schneider suspected that he had cheated on the science test."
- Who do you think is in trouble: Bruno or Gus?
    - *Discuss the two different possible interpretations of the sentence, which illustrate how even grammatically correct sentences in English can create confusion!*
- How could you rewrite this sentence to make it clearer?

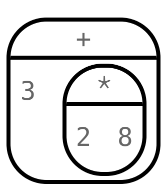
Math is precise, but that precision is difficult to preserve when we switch to words. Often, sentences can be *ambiguous*, meaning that there is more than one way to interpret them!

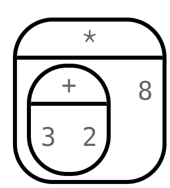
One reason that Circles of Evaluation are so powerful is that they eliminate the ambiguity we encounter when representing expressions with words. They also delineate expressions more clearly than traditional mathematical notation. In this lesson, we tackle verbal expressions that have *more than one* possible mathematical translation.

Investigate

Take a look at this expression: "the sum of three and two multiplied by eight"

Are we multiplying first and then adding (as represented by the Circle on the left), or adding first and then multiplying (as represented by the Circle on the right)?





In this case, there are multiple ways to translate the sentence. (Would inserting a comma after the word “two” provides clarity?)

- Complete The Ambiguity of Words, drawing two possible Circles for each verbal expression.
- What happens when you translate each Circle into a mathematical expression? Do the expressions produce the same result?
    - *The expressions are structurally different, and generally produce different results (with two noteworthy exceptions!)*
- Did you notice anything interesting about the last two expressions, compared to the others on the page?
    - *These expressions use only multiplication or only addition. As a result, the two expressions you wrote evaluated to the same outcome.*
- Now, try Rewriting Ambiguous Expressions.
- Did the two versions of the expressions produce the same results?
    - *No, each interpretation of the expression produces a different result.*

We don't want students to think *all* mathematical expressions in words are ambiguous, as that is simply not the case! Emphasize that only certain verbal structures create this confusion; some phrases are indeed clearer than others. The following activity emphasizes this idea.

- On Ambiguous or Clear?, identify the expressions that have two different numeric translations.
- When you encounter an expression that is ambiguous, rewrite it two times - once for each possible interpretation.
- When you encounter an expression that is clear, draw its Circle of Evaluation.

Synthesize

- Why are some expressions in words ambiguous and others are not?
- Do you think that expressions written in the language of math have ambiguity?
- Are Circles of Evaluation ever ambiguous?