Name: ____

Algebra

Fall 2025 Student Workbook - WeScheme Edition

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Table of Contents

Pioneers in Computing and Mathematics

The pioneers pictured below are featured in our Computing Needs All Voices lesson. To learn more about them and their contributions, visit https://bit.ly/bootstrap-pioneers.

We are in the process of expanding our collection of pioneers. If there's someone else whose work inspires you, please let us know at https://bit.ly/pioneer-suggestion.

Notice and Wonder

Write down what you Notice and Wonder from the What Most Schools Don't Teach video.
"Notices" should be statements, not questions. What stood out to you? What do you remember? "Wonders" are questions.

What do you Notice?	What do you Wonder?

Windows and Mirrors

1) Think about the stories you've just encountered. Identify something(s) from the film and/or posters that served as a mirror for you, connecting you with your own identity and experience of the world. Write about who or what you connected with and why.

2) Identify something(s) from the film or the posters that served as a window for you, giving you insight into other people's experiences or expanding your thinking in some way.

Reflection: Try Thinking About Ketchup

This reflection is designed to follow reading LA Times Perspective: A solution to tech's lingering diversity problem? Try thinking about ketchup

1) Think of a time when someone else had a strategy or idea that you would never have thought of, but was interesting to you and/or pushed your thinking to a new level.

2) Think of a time when you had an idea that felt "out of the box". Did you share your idea? Why or why not?

3) The author argues that tech companies with diverse teams have an advantage. Why?

4) What suggestions did the article offer for tech companies looking to diversify their teams?

5) What is one thing of interest to you in the author's bio?

6) Based on your experience of exceptions to mainstream assumptions, propose another pair of questions that could be used in place of "Where do you keep your ketchup?" and "What would you reach for instead?"

Perspective: A solution to tech’s lingering diversity problem? Try thinking about ketchup

By Dexter Thomas • Published March 16, 2016 6:24 PM PT in the Los Angeles Times

Diversity is a hot, and controversial, topic in Silicon Valley. But why do so many people care about it?

At first glance, the answer may seem simple: Improving minorities’ access to tech jobs is the right thing to do.

But when I moderated a panel Monday at SXSW on diversity in the tech industry, I was surprised none of the panelists talked much about what was “right.”

Instead, they talked about what was right for business.

Sarah Wagener, vice president of talent acquisition and diversity at Pandora, agreed during the panel that pushing to hire more diverse candidates is the “right thing” to do.

“But,” she said, “it’s been the ‘right thing to do’ for a long time, and we’re still having this conversation.”
If you’re trying to make the case at your company for diversifying your workforce, she said, your argument needs to be focused on “real business outcomes.”

In other words, recruiting people from underrepresented backgrounds should be understood not as an obligation that could lower the bar and weigh your company down, but as an opportunity that could raise the bar, and lift your company above the competition.

Instantly, Wagener’s statements reminded me of ketchup.

If you haven’t heard it yet, the “ketchup question” is a thought experiment that’s become something of a meme in some corners of the tech community thanks to a popular episode of the Reply All podcast. It starts as an innocent question:

Where do you keep your ketchup?

If you’re like most people in the United States, odds are that you keep your ketchup in the refrigerator. But depending on where you grew up, you might keep it in the cupboard.

Imagine that you reach for the ketchup bottle and find it empty. You need a substitute sauce, and grab whatever is nearby. If that bottle is in the refrigerator, you may opt for mayo. But if it’s in the cupboard, the seasoning closest at hand might be malt vinegar, or Tabasco, or salt and pepper.

Start-up culture is often centered around new ways of solving “problems” — ride-sharing apps such as Lyft and Uber solve the problem of getting around town without a car, for example. The “ketchup question” shows how a slight difference in perspective can lead a coworker toward a completely different solution that might never occur to you. That extra perspective could lead to a fresh new idea that could take your company to the top.

But without a diverse team? It’s gonna be mayo every time.

What do we do about it?

Most people aren’t chief executives of a major company, and may feel like they have no sway in the hiring process.
So I asked two of the panelists to give some suggestions that could be useful for employees of all levels, regardless of the industry in which they work.

Karla Monterroso, vice president of programs at Code 2040, an organization that works to place black and Latino students in engineering internships at tech companies, said that job listings could be an unexpected barrier to attracting diverse talent.
Using seemingly innocent words like “hacker” or “rockstar” in job listings could unintentionally give the impression to some women that the company would not be a hospitable place to work, said Monterroso. She recommended reading articles on the topic of bias and having informal conversations with coworkers.

More directly, she said, using these articles as “evidence” to suggest small changes in recruitment practices could be an easy first step in attracting new talent.

James Talbot, a software engineer at San Francisco web publishing startup Medium, was concerned with what happens after a new recruit is hired. He suggested using social media to follow people who have different perspectives than you, for 30 days. The key, he said, is to listen to what they have to say, simply exposing yourself to their conversations — not commenting or arguing with them.
This is important, he said, because even after a recruiter hires a person from an underrepresented community, adapting to the workplace environment can be another challenge. If people get into a job but have to deal with racist or sexist comments and insensitive treatment, they may simply leave – and take their unique perspectives and talent elsewhere.
People often say that the cause of the lack of diversity in many tech companies is the lack of an easy way to find available candidates.

“People always give excuses, saying the problem is the ‘pipeline,’” Talbot said.

“But who wants to be on a pipeline into a sewer?”

Dexter Thomas is from San Bernardino and is a PhD candidate in East Asian studies at Cornell University. He has taught media studies and Japanese and is writing a book about Japanese hip-hop. Thomas began working in new media as a student director of programming at KUCR-FM (88.3), independently producing podcasts as well as music and news programs. He has written for several outlets internationally on topics as diverse as Internet and youth culture, social justice and video games. He left The Times in 2016.

The Math Inside video games

- Video games are all about *change!* How fast is this character moving? How does the score change if the player collects a coin? Where on the screen should we draw a castle?
- We can break down a game into parts, and figure out which parts change and which ones stay the same. For example:
    - Computers use **coordinates** to position a character on the screen. These coordinates specify how far from the left (x-coordinate) and the bottom (y-coordinate) a character should be. Negative values can be used to "hide" a character, by positioning them somewhere off the screen.
    - When a character moves, those coordinates change by some amount. When the score goes up or down, it *also* changes by some amount.
- From the computer’s point of view, the whole game is just a bunch of numbers that are changing according to some equations. We might not be able to see those equations, but we can definitely see the effect they have when a character jumps on a mushroom, flies on a dragon, or mines for rocks!
- Modern video games are *incredibly* complex, costing millions of dollars and several years to make, and relying on hundreds of programmers and digital artists to build them. But building even a simple game can give us a good idea of how the complex ones work!

Notice and Wonder

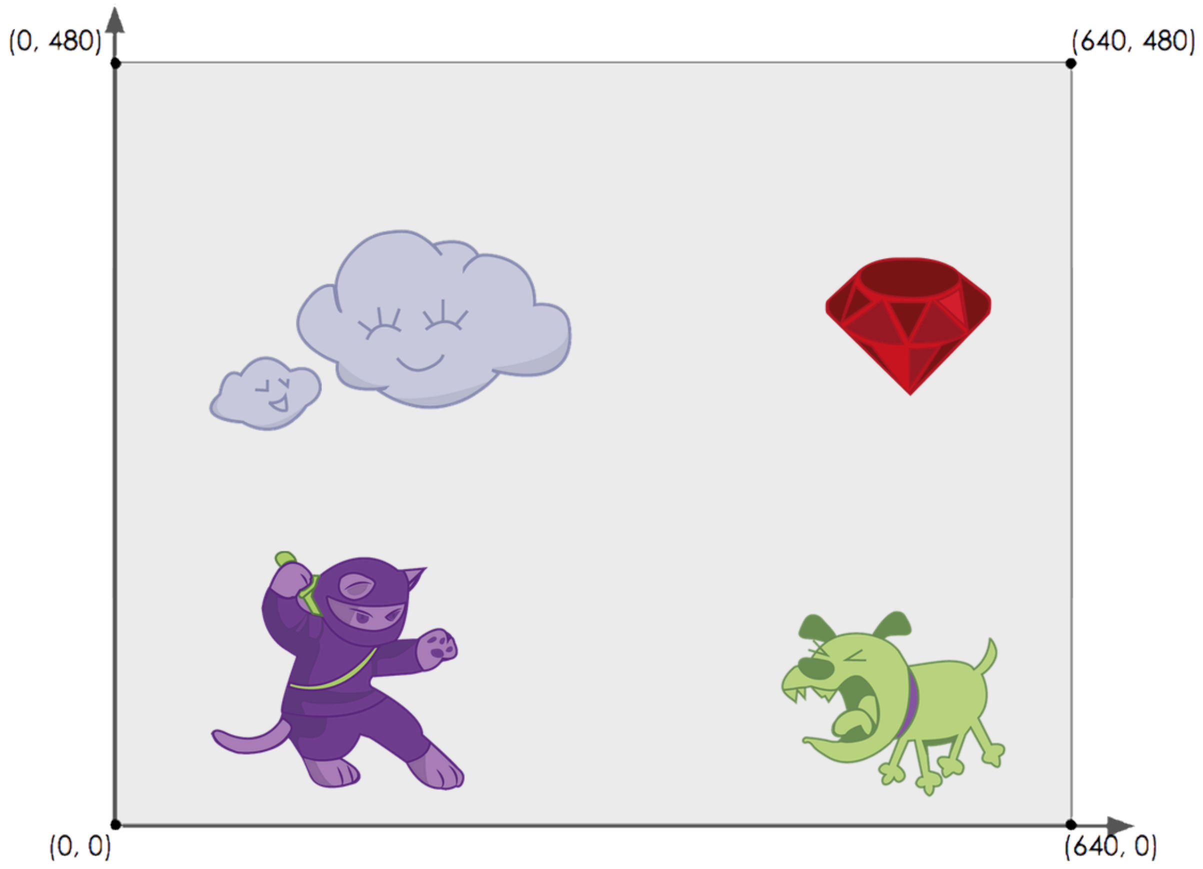
Write down what you Notice and Wonder about the Ninja Cat Game.
"Notices" should be statements, not questions. What stood out to you? What do you remember?

What do you Notice?	What do you Wonder?

Reverse Engineer a video game

This page is designed to be used with the Ninja Cat Game.

What is changing in the game? What variables is the program keeping track of? The first example is filled in for you.

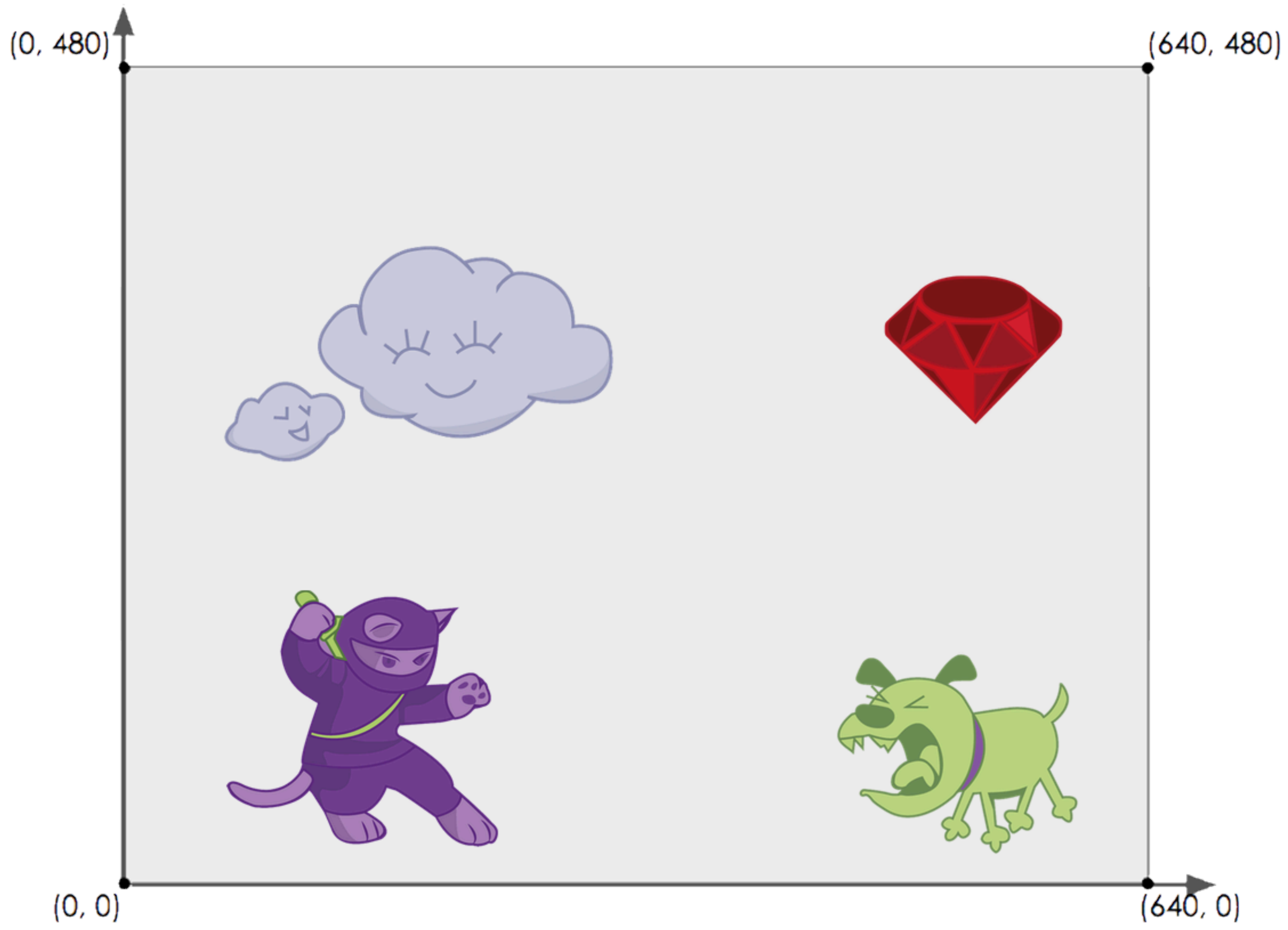


Thing in the Game	What Changes About It?	More Specifically... what variable(s) are being tracked?
Dog	*Position*	*x-coordinate*

Rubric: Video Game

	☐ **Wow!**	☐ **Getting There**	☐ **Needs Improvement**
Game Images	The images are appropriately sized and face the right direction. They make sense in the game and it is easy to discern which image is the danger / target / player.	The sizing of the images is slightly off and/or they face the wrong way. The images cause the game to feel a little confusing for the player.	The images take up way too much/little space in the game or are not on a transparent background. The game feels confusing and jumbled as a result.
Danger and Target Speed	The danger and target move at appropriate speeds for game play to be fun.	The speed of the danger and/or target are slightly too fast or too slow for the game to be fun to play	The speed of the danger and target are wrong, causing the game to be too difficult, too easy or very confusing.
Danger and Target Orientation	The danger and target move in appropriate directions for the game to be fun.	The direction of either the danger or target don't make sense.	The direction of the danger and target don't make sense.
Boundary Detection	Onscreen detection is appropriate, allowing the danger and target to fly across the screen and return smoothly.	The programmer needs to optimize onscreen detection to improve game play and/or there is some "glitching" of the danger and target near the edge of the screen.	The danger and target do not return when they go off screen.
Player Movement	The player moves in a variety of directions at an appropriate speed for game play to be fun.	The player's movement does not completely make sense. Hitting a random key produces an error.	The player does not move at all.
Collisions	The collisions happen at appropriate times.	The collisions happen slightly too early / late, when images are already overlapping or have not yet made contact.	The timing of the collisions is way off, causing the game to feel confusing and disorienting.
Code Quality	The programmer provides contracts and clear purpose statements for each and every function. There are examples provided for every valid keypress, and the code does not crash when an invalid key is pressed. There are no failed examples.	Occasionally, the programmer forgets a Contract or provides a confusing purpose statement. There is one failed example.	Coding seems rushed, with frequent missing contracts and purpose statements. There are multiple failed examples.

Estimating Coordinates



The coordinates for the PLAYER (NinjaCat) are: (____________________, ____________________)
x y

The coordinates for the DANGER (Dog) are: (____________________, ____________________)
x y

The coordinates for the TARGET (Ruby) are: (____________________, ____________________)
x y

Brainstorm Your Own Game

Created by: ______________________________

Background

Our game takes place: ______________________________
In space? The desert? A mall?

Player

The Player is a ______________________________
The Player moves only up and down.

Target

Your Player GAINS points when they hit The Target.

The Target is a ______________________________
The Target moves only to the left or right.

Danger

Your Player LOSES points when they hit The Danger.

The Danger is a ______________________________
The Danger moves only to the left or right.

Artwork/Sketches/Proof of Concept

Below is a **640x480 rectangle**, representing your game screen.

- Label the bottom-left corner (0,0).
- Label the other three corners with their corresponding coordinates.
- In the rectangle, sketch a picture of your game!

Images of Dog, Cat and Ruby

Cut out these images and use them with a number line on the board to facilitate class discussion about locating game characters with their coordinates.

Order of Operations

If you were to write instructions for getting ready for school, it would matter very much which instruction came first!

Imagine what might happen if someone tried to follow these steps:

1. Put on your sneakers.
2. Tie your sneakers.
3. Put on your socks.

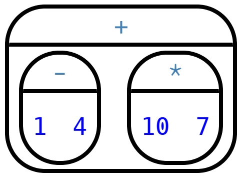
Sometimes we need multiple expressions in mathematics, and the order matters there, too!
Mathematicians didn't always agree on the **Order of Operations**, but at some point it became important to establish conventions that would allow them to work together.

To help us organize our math thinking into something we can trust, we can *diagram* an expression using the **Circles of Evaluation**.

For example, this expression:

$1 - 4 + 10 \times 7$

can be diagrammed as:



Order of Operations is important when programming, too!

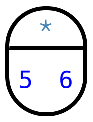
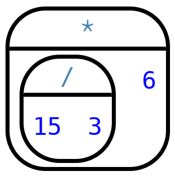
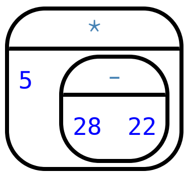
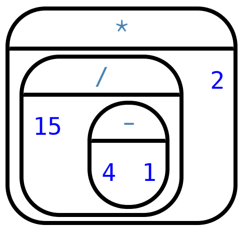
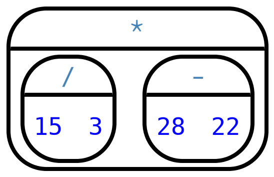
To convert a **Circle of Evaluation** into Code, we walk through the circle from outside-in, moving left-to-right.

1. Type an open parenthesis when we *start* a circle.
2. Once we're in a circle, we first write the **function** at the top, then write the inputs from left to right.
3. Type a close parenthesis when we *end* a circle.

So, the Circle of Evaluation above would be programmed as:
`(+ (- 1 4) (* 10 7))`

Circles of Evaluation - Notice and Wonder

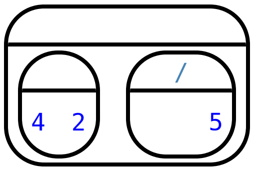
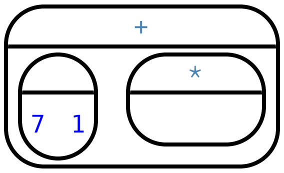
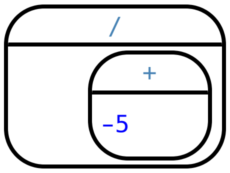
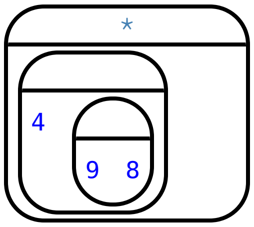
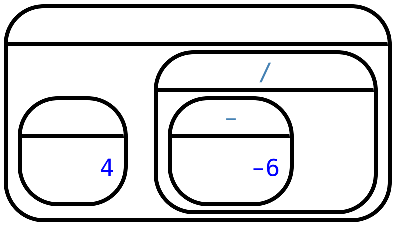
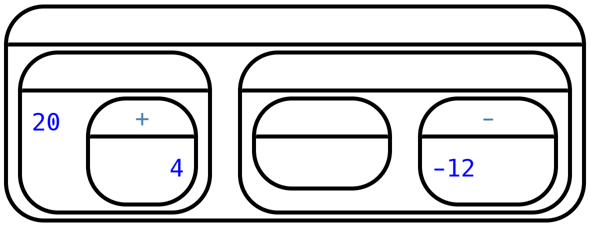
Let's take a look at a few ***Circles of Evaluation*** before we learn to draw them ourselves.

What do you Notice?	What do you Wonder?

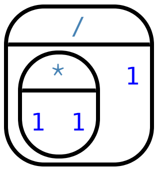
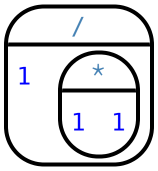
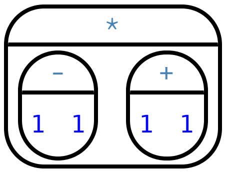
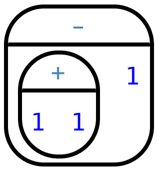
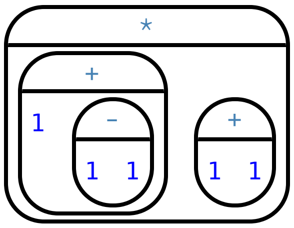
Complete the Circles of Evaluation

For each expression on the left, finish the Circle of Evaluation on the right by filling in the blanks.

	Arithmetic Expression	Circle of Evaluation
1	$4 + 2 - \frac{10}{5}$	
2	$7 - 1 + 5 \times 8$	
3	$\frac{-15}{-5 + 8}$	
4	$(4 + (9 - 8)) \times 5$	
5	$6 \times 4 + \frac{9 - -6}{5}$	
★	$\frac{20}{6 + 4} - \frac{5 \times 9}{-12 - 3}$	

Matching Expressions to Diagrams

Draw a line from each Circle of Evaluation on the left to the corresponding arithmetic expression on the right.

Circle of Evaluation			Arithmetic Expression
	**1**	**A**	1 ÷ (1 × 1)
	**2**	**B**	(1 + 1) - 1
	**3**	**C**	(1 × 1) ÷ 1
	**4**	**D**	(1 + (1 - 1)) × (1 + 1)
	**5**	**E**	(1 - 1) × (1 + 1)

Expressions -> Circles of Evaluation

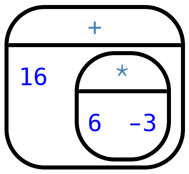
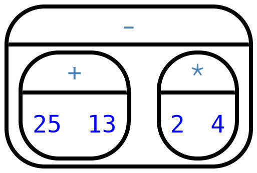
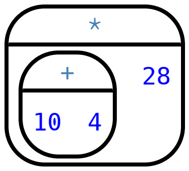
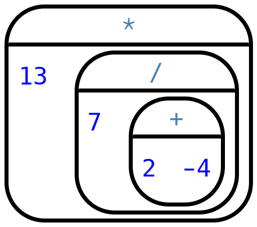
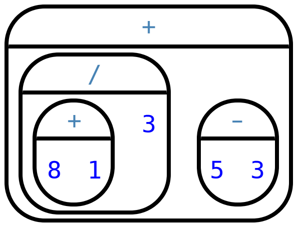
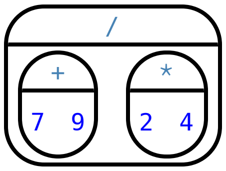
Translate each of the arithmetic expressions below into Circles of Evaluation.

	Arithmetic Expression	Circle of Evaluation
1	(6 ÷ 2) - (5 - 3)	
2	9 - (2 × 4)	
3	8 - (1 + (2 × 3))	
4	(1 + (4 × 7)) - 3	

★ Rewrite each of these expressions with one less pair of parentheses without changing its Order of Operations.

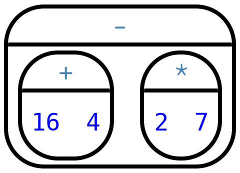
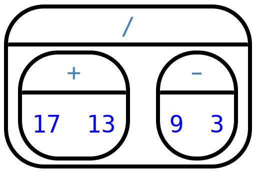
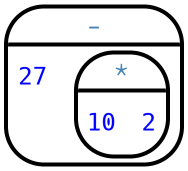
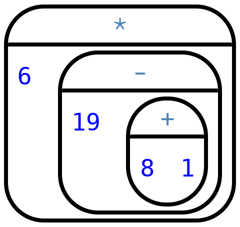
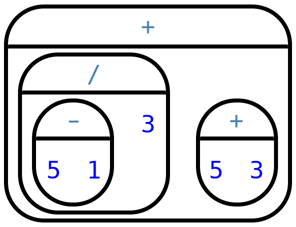
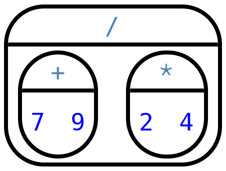
Complete the Code!

For each Circle of Evaluation on the left, finish the Code on the right by filling in the blanks.

	Circle of Evaluation	Code
1		`(+ __________ (* __________ -3))`
2		`( __________ (+ __________ 13) ( __________ __________ 4))`
3		`( __________ (+ __________ 4) __________ )`
4		`( __________ 13 ( __________ 7 ( __________ 2 -4)))`
5		`( __________ ( __________ ( __________ 8 1) 3) ( __________ 5 3))`
6		`(/ (+ __________ __________) (* __________ __________))`

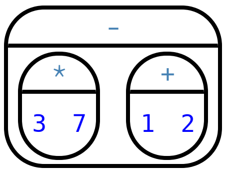
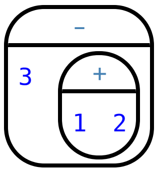
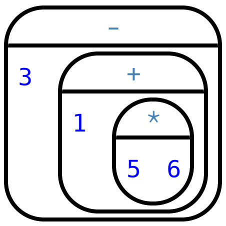
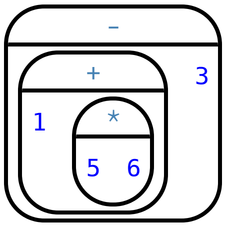
Complete the Code by adding Parentheses!

For each Circle of Evaluation on the left, finish the Code on the right by adding parentheses.

	Circle of Evaluation	Code
1		`- + 16 4 * 2 7`
2		`/ + 17 13 - 9 3`
3		`- 27 * 10 2`
4		`6 - 19 + 8 1`
5		`+ / - 5 1 3 + 5 3`
6		`/ + 7 9 * 2 4`

Expressions -> Circles of Evaluation -> Code 1

Complete the table by translating each of the arithmetic expressions below to code using the provided Circle of Evaluation.

	Arithmetic Expression	Circle of Evaluation	Code
1	$3 \times 7 - (1 + 2)$		
2	$3 - (1 + 2)$		
3	$3 - (1 + 5 \times 6)$		
4	$1 + 5 \times 6 - 3$		

Expressions -> Circles of Evaluation -> Code 2

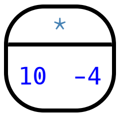
Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

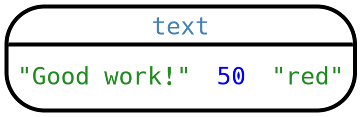
	Arithmetic Expression	Circle of Evaluation	Code
1	6 × 8 + (7 - 23)		
2	18 ÷ 2 + 24 × 4 - 2		
3	(22 - 7) ÷ (3 + 2)		
4	24 ÷ 4 × 2 - 6 + 20 × 2		

Notice and Wonder - More than +, - , ÷, ×

Part A

Here are two Circles of Evaluation . One of them is familiar, but the other is very different from what we've been working with.





1) **Focus on the Circles of Evaluation.** What do you Notice is different about the one on the right?

2) What do you Wonder about the Circle of Evaluation on the Right?

3) Can you figure out the Name for the function in the second Circle of Evaluation?

4) What do you think this expression will evaluate to? ____________________

Part B

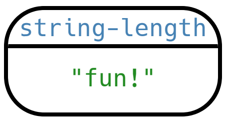
5) Convert this Circle of Evaluation to Code:

6) Test the code out in WeScheme!

7) What does the `50` mean to the computer? *Try replacing it with different values, and see what you get.*

8) What does the `"red"` mean to the computer? *Try replacing it with different values, and see what you get.*

Here is another Circle of Evaluation to explore.



9) Convert this Circle of Evaluation to code: ____________________

10) What do you think this expression will evaluate to? ____________________

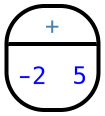
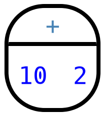
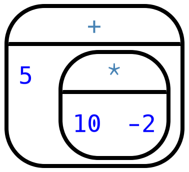
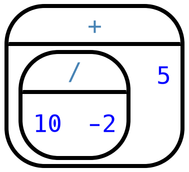
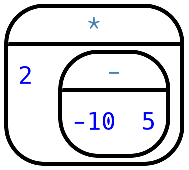
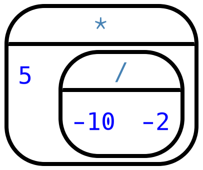
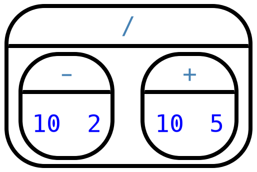
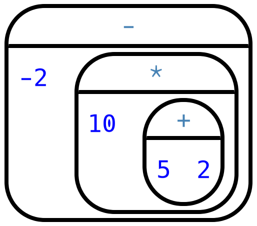
Expressions -> Circles of Evaluation -> Code - Challenge

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code. *Hint: Two useful functions are* `sqr` *and* `sqrt`.

	Arithmetic Expression	Circle of Evaluation	Code
1	$45 - 9 \times (3 + (2 - 4)) - 7$		
2	$50 \div 5 \times 2 - ((3 + 4) \times 2 - 5)$		
3	$\frac{16 + 3^2}{\sqrt{49} - 2}$		

Matching Circles of Evaluation & Code

Cut out the cards in the table below, mix them up, and try to match the Circle of Evaluation with the Arithmetic Expression.

1 	A - 2 + 5	2 	B 10 + 2
3 	C 5 + (10 × - 2)	4 	D (10 ÷ - 2) + 5
5 	E 2 × (- 10 - 5)	6 	F 5 × (- 10 ÷ - 2)
7 	G (10 - 2) ÷ (10 + 5)	8 	H - 2 - (10 × (5 + 2))

Drawing the Structure 1

For each arithmetic expression on the left, draw its Circle of Evaluation on the right.

	Arithmetic Expression	Circle of Evaluation
1	4 - (6 - 17)	
2	25 + 14 - 12	
3	1 + 15 × 5	
4	15 ÷ (10 + 4 × 2)	

(optional)

Drawing the Structure 2

For each arithmetic expression on the left, draw its Circle of Evaluation on the right.

	Arithmetic Expression	Circle of Evaluation
1	6 + 17 - - 2	
2	(2 + 17) × (12 - 8)	
3	23 × 14 × (3 + 20)	
4	5 - 17 + 14 × 5	

(optional)

Drawing the Structure 3

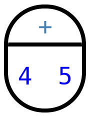
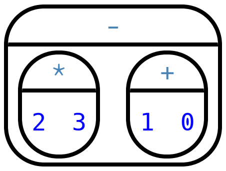
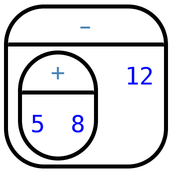
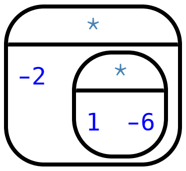
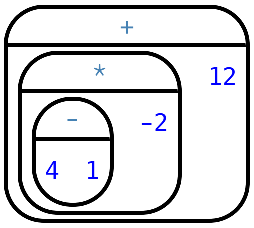
For each expression on the left, draw its Circle of Evaluation on the right.

	Arithmetic Expression	Circle of Evaluation
1	$9 \times (17 + 2)$	
2	$(2 + 17) \times (12 - 8)$	
3	$19 - (12 + 11)$	
4	$\frac{7}{7 \times (9 + 8)}$	

(optional)

Circles of Evaluation -> Mathematical Expressions

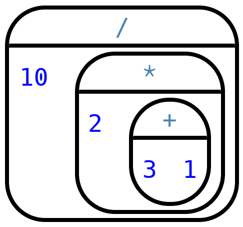
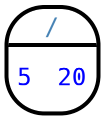
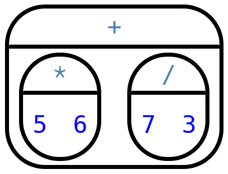
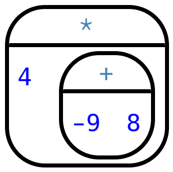
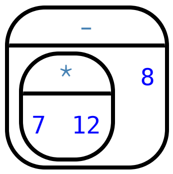
For each Circle of Evaluation on left, write the arithmetic expression on the right.

	Circle of Evaluation	Arithmetic Expression
1		+
2		+
3		+
4		+
5		+

(optional)

Circles of Evaluation -> Mathematical Expressions 2

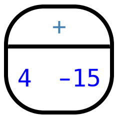
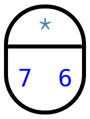
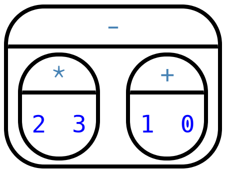
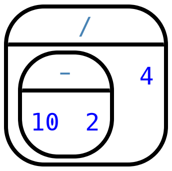
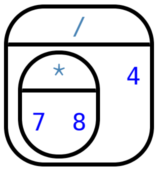
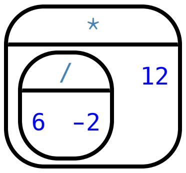
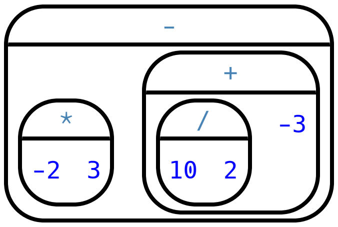
For each Circle of Evaluation on left, write the arithmetic expression on the right

	Circle of Evaluation	Arithmetic Expression
1		+
2		+
3		+
4		+
5		+

(optional)

Evaluating Circles of Evaluation

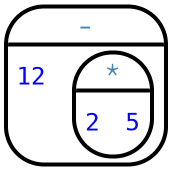
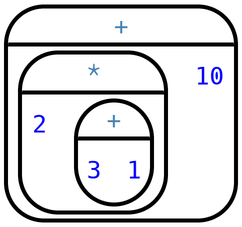
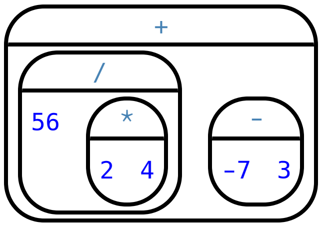
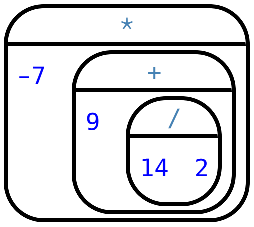
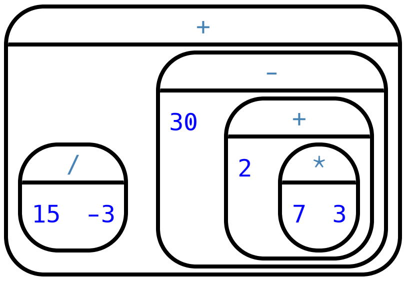
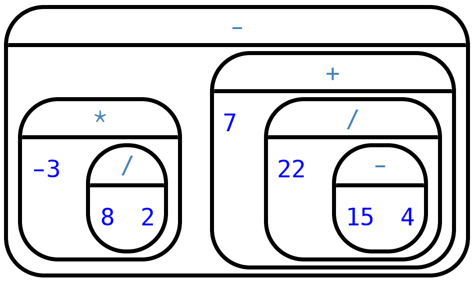
Write each Circle of Evaluation as an arithmetic expression and evaluate it.

	Circle of Evaluation	Arithmetic Expression	Answer
1			
2			
3			
4			
5			
6			
7			

(optional)

Evaluating Circles of Evaluation 2

Write each Circle of Evaluation as an arithmetic expression and evaluate it.

	Circle of Evaluation	Arithmetic Expression	Answer
1			
2			
3			
4			
5			
6			

Why isn't this expression Commutative?

You may have heard that "addition is commutative, so $a + b$ can always be written as $b + a$."
We know, for example, $1 + 2$ can be transformed to $2 + 1$.

Suppose another student tells you that $1 + 2 \times 3$ can be rewritten as $2 + 1 \times 3$.
This is obviously wrong, but *why* isn't that how the commutative property works? *Take a moment to think: What's the problem?*

1) Draw the Circles of Evaluation to figure it out!

$1 + 2 \times 3$	$2 + 1 \times 3$

2) What do these Circles of Evaluation show us about why we can't use the commutative property to rewrite $1 + 2 \times 3$ as $2 + 1 \times 3$?

3) Draw the Circles of Evaluation to decide whether or not these expressions will evaluate to the same thing.

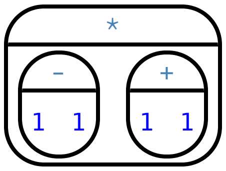
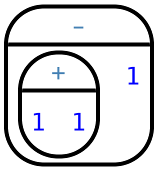
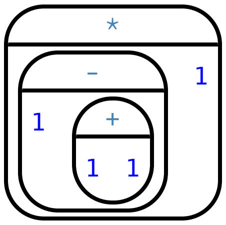
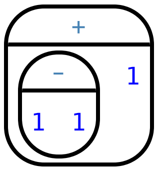
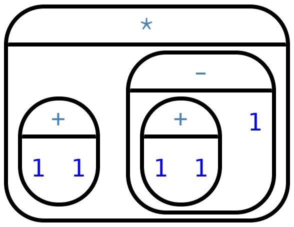
$5 + 21 \times 36$	$21 \times 36 + 5$

4) Will $5 + 21 \times 36$ and $21 \times 36 + 5$ evaluate to the same thing? How do you know from looking at the Circles of Evaluation?

(optional)

Matching Circles of Evaluation to Code

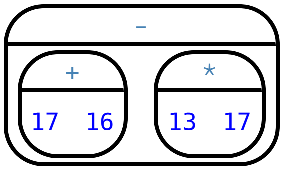
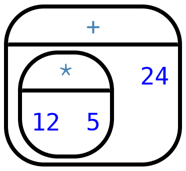
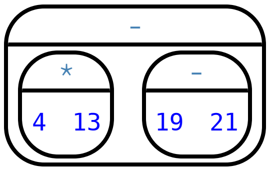
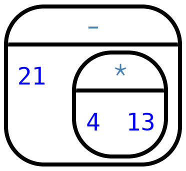
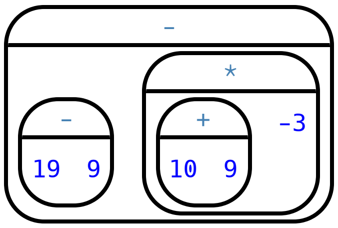
Draw a line from each Circle of Evaluation on the left to the corresponding Code on the right.

Circle of Evaluation			Code
	1	A	`(* (- 1 (+ 1 1)) 1)`
	2	B	`(* (- 1 1) (+ 1 1))`
	3	C	`(* (+ 1 1) (- (+ 1 1) 1))`
	4	D	`(- (+ 1 1) 1)`
	5	E	`(+ (- 1 1) 1)`

(optional)

Circles of Evaluation -> Code

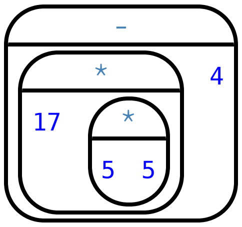
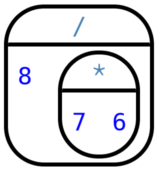
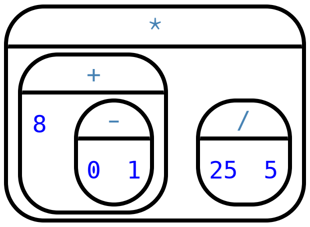
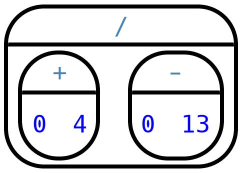
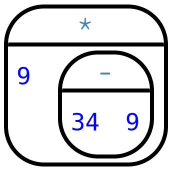
For each Circle of Evaluation on the left-hand side, write the code for the Circle on the right-hand side

	Circle of Evaluation	Code
1		
2		
3		
4		
5		

(optional)

Circles of Evaluation -> Code 2

For each Circle of Evaluation on the left-hand side, write the code for the Circle on the right-hand side

	Circle of Evaluation	Code
1		
2		
3		
4		
5		

(optional)

Expressions -> Circles of Evaluation -> Code 3

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	9 ÷ 3 + 7 - 8 ÷ 4		
2	6 × (5 + 3) - 2		
3	3 - (1 + 5 × 6)		
4	15 ÷ 3 + (2 + 1)		

(optional)

Expressions -> Circles of Evaluation -> Code 4

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	15 - 9 ÷ (2 + 1)		
2	(9 + 6) × 7 + 8 ÷ 2		
3	7 - (8 × 3 + 2)		
4	5 + 8 ÷ 2 × 4		

(optional)

Expressions -> Circles of Evaluation -> Code 5

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	6 + (5 - 3) ÷ 2		
2	- 15 ÷ 3 × (2 + 1)		
3	8 - 6 ÷ (- 2 + - 1) × - 4		
4	10 ÷ - 5 × 3 - - 7		

Expressions -> Circles of Evaluation -> Code 6

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.

	Arithmetic Expression	Circle of Evaluation	Code
1	7 × - 4 + - 10 ÷ 2		
2	- 5 ÷ 5 × 4 - 8		
3	9 × 3 + - 6 - 8 × 4		
4	6 + (- 5 + 3) ÷ 2		

(optional)

Expressions -> Circles of Evaluation -> Code - w/Square Roots

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code.
HINT: The function name is `sqrt`.

	Arithmetic Expression	Circle of Evaluation	Code
1	$\sqrt{9}$		
2	$\sqrt{5+1}$		
3	$\sqrt{4}+1$		
4	$3 \times \sqrt{3}+\sqrt{7}$		

Expressions -> Circles of Evaluation -> Code - Challenge 2

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code. *Hint: Two useful functions are* `sqr` *and* `sqrt`.

	Arithmetic Expression	Circle of Evaluation	Code
1	$8 - (9 + 2 \times (4 - 1))$		
2	$2 \times 4^2 + 8 \div 4 \times 2$		
3	$(10 - (3 + 4)) \times \frac{7 - \sqrt{4}}{5 \times (2 + 4)} + 7$		

(optional)

Expressions -> Circles of Evaluation -> Code - Challenge 3

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code. *Hint: Two useful functions are* `sqr` *and* `sqrt`.

	Arithmetic Expression	Circle of Evaluation	Code
1	$27 - 5 \times (4^2 - 16) + \sqrt{9}$		
2	$3 \times 4^2 - 2 \times \sqrt{25 - 4^2}$		
3	$5^2 \times (8 - (3 + 2)) - \frac{\sqrt{100}}{2}$		

(optional)

Expressions -> Circles of Evaluation -> Code - Challenge 4

Translate each of the arithmetic expressions below into Circles of Evaluation, then translate them to Code. *Hint: Two useful functions are* `sqr` *and* `sqrt`.

	Arithmetic Expression	Circle of Evaluation	Code
1	$45 \div 3^2 + 8 \times -2 - \sqrt{16}$		
2	$11 + (5 - 3)^2 \div 5 - 6 \times 2$		
3	$2^3 + \frac{8^2 + 4^2}{9 - 5} \times 2 \times (9 - 4 \times 2)$		

(optional)

Introduction to Programming in a Nutshell

The **Editor** is a software program we use to write Code. Our Editor allows us to experiment with Code on the right-hand side, in the **Interactions Area**. For Code that we want to *keep*, we can put it on the left-hand side in the **Definitions Area**. Clicking the "Run" button causes the computer to re-read everything in the Definitions Area and erase anything that was typed into the Interactions Area.

Data Types

Programming languages involve different ***data types***, such as Numbers, Strings, Booleans, and even Images.

- Numbers are values like `1`, `0.4`, `1/3`, and `-8261.003`.
    - Numbers are *usually* used for quantitative data and other values are *usually* used as categorical data.
- Strings are values like `"Emma"`, `"Rosanna"`, `"Jen and Ed"`, or even `"08/28/1980"`.
    - All strings *must* be surrounded by quotation marks.
- Booleans are either `true` or `false`.

All values evaluate to themselves. The program `42` will evaluate to `42`, the String `"Hello"` will evaluate to `"Hello"`, and the Boolean `false` will evaluate to `false`.

Operators

Operators (like `+`, `-`, `*`, `<`, etc.) are treated the same way as functions: after all, they have inputs and outputs and obey the same rules!

Applying Functions

Functions (and operators!) work much the way they do in math. Every function has a name, takes some inputs, and produces some output. The function name is written first, followed by a list of ***arguments***.

- In math this could look like $f(5)$ or $g(10, 4)$.
- In WeScheme, these examples would be written as `(f 5)` and `(g 10 4)`.
- Applying the operator $+$ to the inputs 1 and 2 would look like `(+ 1 2)`.
- Applying a function to make images would look like `(star 50 "solid" "red")`.
- There are many other functions in WeScheme, for example `sqr`, `sqrt`, `triangle`, `square`, `string-repeat`, etc.

Functions have ***contracts***, which help explain how a function should be used. Every Contract has three parts:

- The *Name* of the function - literally, what it's called.
- The *Domain* of the function - what *type(s) of value(s)* the function consumes, and in what order.
- The *Range* of the function - what *type of value* the function produces.

Strings and Numbers

*Make sure you've loaded WeScheme, clicked "Run", and are working in the **Interactions Area** on the right. Hit Enter/return to evaluate expressions you test out.*

Strings

String values are always in quotes.

- Try typing your name *(in quotes!)*.
- Try typing a sentence like "I'm excited to learn to code!" *(in quotes!)*.
- Try typing your name with the opening quote, but *without the closing quote.* Read the error message!
- Now try typing your name *without any quotes.* Read the error message!

1) Explain what you understand about how strings work in this programming language. ______________________

____

Numbers

2) Try typing `42` into the Interactions Area and hitting "Enter". Is `42` the same as `"42"` ? Why or why not?

____

3) What is the largest number the editor can handle?

____

4) Try typing `0.5` . Then try typing `.5` . Then try clicking on the answer. Experiment with other decimals.

Explain what you understand about how decimals work in this programming language. ______________________

____

5) What happens if you try a fraction like `1/3` ? ______________________

____

6) Try writing **negative** integers, fractions and decimals. What do you learn? ______________________

____

____

Booleans

Boolean-producing expressions are yes-or-no questions, and will always evaluate to either `true` ("yes") or `false` ("no").
What will the expressions below evaluate to? *Write down your prediction, then type the code into the Interactions Area to see what it returns.*

	Prediction	Result		Prediction	Result
1) `(<= 3 4)`			2) `(string>? "a" "b")`		
3) `(= 3 2)`			4) `(string<? "a" "b")`		
5) `(< 2 4)`			6) `(string=? "a" "b")`		
7) `(>= 5 5)`			8) `(string<>? "a" "a")`		
9) `(>= 4 6)`			10) `(string>=? "a" "a")`		
11) `(<> 3 3)`			12) `(string<>? "a" "b")`		
13) `(<> 4 3)`			14) `(string>=? "a" "b")`		

15) In your own words, describe what `<` does. ______________________________

16) In your own words, describe what `>=` does. ______________________________

17) In your own words, describe what `<>` does. ______________________________

	Prediction:	Result:
18) `(string=? "a tree" "trees")`		
19) `(string=? "tree" "tree")`		
20) `(string-contains? "catnap" "cat")`		
21) `(string-contains? "cat" "catnap")`		

22) In your own words, describe what `string-contains` does. Can you generate another expression using `string-contains` that returns true?

____

★ There are infinite string values ("a", "aa", "aaa"...) and infinite number values out there (...-2,-1,0,-1,2...). But how many different *Boolean* values are there? ______________________________

Applying Functions

Open WeScheme and click "Run". We will be working in the Interactions Area on the right.

Test out these two expressions and record what you learn below:

- `(regular-polygon 40 6 "solid" "green")`
- `(regular-polygon 80 5 "outline" "dark-green")`

1) You've seen data types like Numbers, Strings, and Booleans. What data type did the `regular-polygon` function produce? ____________

2) How would you describe what a regular polygon is? ____

____

3) The `regular-polygon` function takes in four pieces of information (called arguments). Record what you know about them below.

	Data Type	Information it Contains
Argument 1		
Argument 2		
Argument 3		
Argument 4		

There are many other functions available to us in Pyret. We can describe them using ***contracts***. The Contract for `regular-polygon` is:

```
; regular-polygon :: Number, Number, String, String -> Image
```

- Each Contract begins with the function name: *in this case* `regular-polygon`
- Lists the data types required to satisfy its Domain: *in this case* Number, Number, String, String
- And then declares the data type of the Range it will return: *in this case* Image

Contracts can also be written with more detail, by annotating the Domain with *variable names*:

```
; regular-polygon :: ( Number , Number , String , String ) -> Image
                       size     number-of-sides  fill-style  color
```

4) We know that a square is a regular polygon because ____

____

5) What code would you write to make a big, blue square using the `regular-polygon` function?

________________ (____________ , ____________ , ____________ , ____________)
function-name size :: Number number-of-sides :: Number fill-style :: String color :: String

6) Pyret also has a `square` function whose contract is:

```
; square :: ( Number , String , String ) -> Image
              size     fill-style  color
```

What code would you write to make a big blue square using the `square` function?

________________ (____________ , ____________ , ____________)
function-name size :: Number fill-style :: String color :: String

7) Why does `square` need fewer arguments to make a square than `regular-polygon`? ______________________

____

★ Where else have you heard the word ***contract*** used before?

Practicing Contracts: Domain & Range

Note: The contracts on this page are not defined in WeScheme and cannot be tested in the editor.

is-beach-weather

Consider the following Contract:
`; is-beach-weather :: Number, String -> Boolean`

1) What is the **Name** of this function? ______________________

2) How many arguments are in this function's **Domain**? ______________________

3) What is the **Type** of this function's **first argument**? ______________________

4) What is the **Type** of this function's **second argument**? ______________________

5) What is the **Range** of this function? ______________________

6) Circle the expression below that shows the correct application of this function, based on its Contract.

A. `(is-beach-weather 70 90)`
B. `(is-beach-weather 80 100 "cloudy")`
C. `(is-beach-weather "sunny" 90)`
D. `(is-beach-weather 90 "stormy weather")`

cylinder

Consider the following Contract:
`; cylinder :: Number, Number, String -> Image`

7) What is the **Name** of this function? ______________________

8) How many arguments are in this function's **Domain**? ______________________

9) What is the **Type** of this function's **first argument**? ______________________

10) What is the **Type** of this function's **second argument**? ______________________

11) What is the **Type** of this function's **third argument**? ______________________

12) What is the **Range** of this function? ______________________

13) Circle the expression below that shows the correct application of this function, based on its Contract.

A. `(cylinder "red" 10 60)`
B. `(cylinder 30 "green")`
C. `(cylinder 10 25 "blue")`
D. `(cylinder 14 "orange" 25)`

Matching Expressions and Contracts

Match the Contract (left) with the expression that uses it correctly (right).
Note: The contracts on this page are not defined in Pyret and cannot be tested in the editor.

Contract			Expression
`; make-id :: String, Number -> Image`	1	A	`(make-id "Savannah" "Lopez" 32)`
`; make-id :: String, Number, String -> Image`	2	B	`(make-id "Pilar" 17)`
`; make-id :: String -> Image`	3	C	`(make-id "Akemi" 39 "red")`
`; make-id :: String, String -> Image`	4	D	`(make-id "Raïssa" "McCracken")`
`; make-id :: String, String, Number -> Image`	5	E	`(make-id "von Einsiedel")`

Contract			Expression
`; is-capital :: String, String -> Boolean`	6	A	`(show-pop "Juneau" "AK" 31848)`
`; is-capital :: String, String, String -> Boolean`	7	B	`(show-pop "San Juan" 395426)`
`; show-pop :: String, Number -> Image`	8	C	`(is-capital "Accra" "Ghana")`
`; show-pop :: String, String, Number -> Image`	9	D	`(show-pop 3751351 "Oklahoma")`
`; show-pop :: Number, String -> Number`	10	E	`(is-capital "Albany" "NY" "USA")`

Contracts for Image-Producing Functions

Log into WeScheme and click "Run". Experiment with each of the functions listed below, trying to find an expression that will build. Record the contract and example code for each function you are able to successfully build!

Name		Domain		Range
`; triangle`	`::`	`Number, String, Sting`	`->`	`Image`
`(triangle 80 "solid" "green")`				
`; star`	`::`		`->`	
`; circle`	`::`		`->`	
`; rectangle`	`::`		`->`	
`; text`	`::`		`->`	
`; square`	`::`		`->`	
`; ellipse`	`::`		`->`	
`; regular-polygon`	`::`		`->`	
`; rhombus`	`::`		`->`	
`; right-triangle`	`::`		`->`	
`; isosceles-triangle`	`::`		`->`	
`; radial-star`	`::`		`->`	
`; star-polygon`	`::`		`->`	
`; triangle/sas`	`::`		`->`	
`; triangle/asa`	`::`		`->`	

Catching Bugs when Making Triangles

Learning about a Function through Error Messages

1) Type `triangle` into the Interactions Area of WeScheme and hit "Enter". What do you learn? ______________________________

2) We know that all functions will need an open parenthesis and at least one input! Type `(triangle 80)` in the Interactions Area and hit Enter/return. Read the error message. What hint does it give us about how to use this function?

____

3) Using the hint from the error message, experiment until you can make a triangle. What is the contract for `triangle`?

____

What Kind of Error is it?

- **syntax errors** - the computer cannot make sense of the code because of unclosed strings, missing commas or parentheses, etc.
- **contract errors** - the function isn't given what it needs (the wrong type or number of arguments are used)

4) In your own words, the difference between **syntax errors** and **contract errors** is: ______________________________

____

Finding Mistakes with Error Messages

The following lines of code are all BUGGY! Read the code and the error messages below. See if you can find the mistake WITHOUT typing it into WeScheme.

5) `(triangle 20 "solid")`

```
triangle: expects 3 arguments, but given 2: 20 solid at: line 1, column 0, in <interactions>
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

6) `(triangle "solid" "red" 20)`

```
triangle: expects a non-negative number as 1st argument, but given: solid; other arguments were:
red 20 at: line 1, column 0, in <interactions>
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

7) `(triangle 20 40 "solid" "red")`

```
triangle: expects 3 arguments, but given 4: 20 40 solid red at: line 1, column 0, in
<interactions>
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

8) `(triangle 20 solid "red")`

```
solid: this variable is not defined at: line 1, column 0, in <interactions>
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

★ `(triangle 20 "striped" "red")`

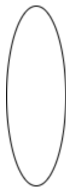
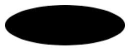
```
triangle: expects a style ("solid" / "outline") or an opacity value [0-255]) as 2nd argument,
but given: "striped"; other arguments were: 20 "red" at: line 1, column 0, in <interactions>
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

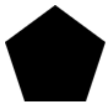
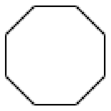
Using Contracts

For questions 1,2,4,5,8 & 9, use the contracts provided to find expressions that will generate images similar to the ones pictured. Test your code in WeScheme before recording it.

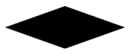
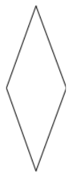
```
; ellipse :: ( Number , Number , String , String ) -> Image
               width    height   fill-style color
```

1)		
2)		
3)	Write an expression using `ellipse` to produce a circle.	

```
; regular-polygon :: ( Number , Number , String , String ) -> Image
                       side-length number-of-sides fill-style color
```

4)		
5)		
6)	Use `regular-polygon` to write an expression for a square!	
7)	How would you describe a **regular polygon** to a friend?	

```
; rhombus :: ( Number , Number , String , String ) -> Image
               size     top-angle fill-style color
```

8)		
9)		
10)	Write an expression to generate a `rhombus` that is a square!	

Triangle Contracts

Respond to the questions. Go to WeScheme to test your code.

1) What kind of triangle does the `triangle` function produce? ____________________________
There are lots of other kinds of triangles! And WeScheme has lots of other functions that make triangles!

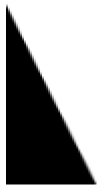
```
; triangle :: (Number, String, String) -> Image
               size    fill-style color
; right-triangle :: (Number, Number, String, String) -> Image
                     base    height  fill-style color
; isosceles-triangle :: (Number, Number, String, String) -> Image
                         leg     angle   fill-style color
```

2) Why do you think `triangle` only needs one number, while `right-triangle` and `isosceles-triangle` need two numbers?

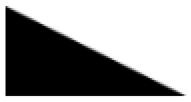
____

____

3) Write `right-triangle` expressions for the images below using `100` as one argument for each.

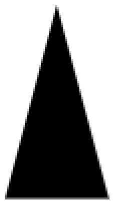


____

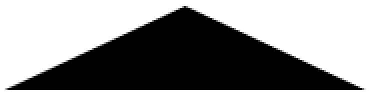


____

4) Write `isosceles-triangle` expressions for the images below using `100` as one argument for each.

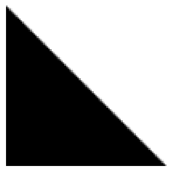


____



____

5) Write 2 expressions that would build **right-isosceles** triangles. Use `right-triangle` for one expression and `isosceles-triangle` for the other expression.



____

____

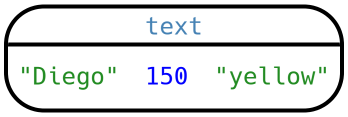
6) Which do you like better? Why? ________________________________

Composing with Circles of Evaluation

Notice and Wonder

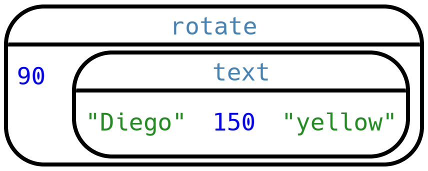
Suppose we want to see the `text` *"Diego" written vertically in yellow letters of size 150. Let's use Circles of Evaluation to look at the structure:*

We can start by generating the Diego image.



→

And then use the `rotate` function to rotate it 90 degrees.



```
(text "Diego" 150 "yellow")
```

```
(rotate 90 (text "Diego" 150 "yellow"))
```

1) What do you Notice? ____________________

____

2) What do you Wonder? ____________________

____

Let's Rotate an Image of Your Name!

Suppose you wanted the computer to show your name in your favorite color and rotate it so that it's diagonal...

Write your name (any size), in your favorite color

3) Draw the circle of evaluation:

5) Convert the Circle of Evaluation to code:

`rotate` **the image so that it's diagonal**

4) Draw the circle of evaluation:

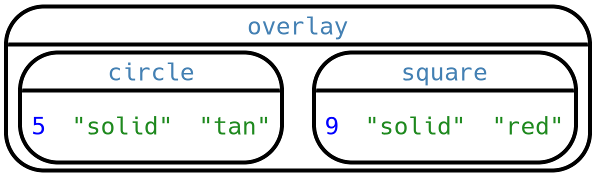
6) Convert the Circle of Evaluation to code:

Circle of Evaluation to Code (Scaffolded)

Complete the Code by Filling in the Blanks!

Finish the Code by filling in the blanks.

1)

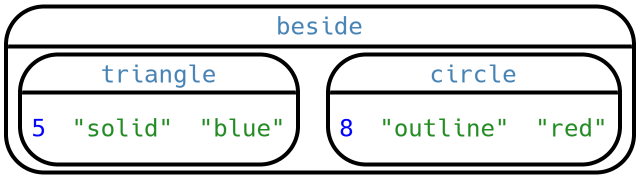


```
(overlay (circle ____ "solid" ____________________) (____________________ 9 ____________________ "red"))
```

Complete the Code by adding Parentheses

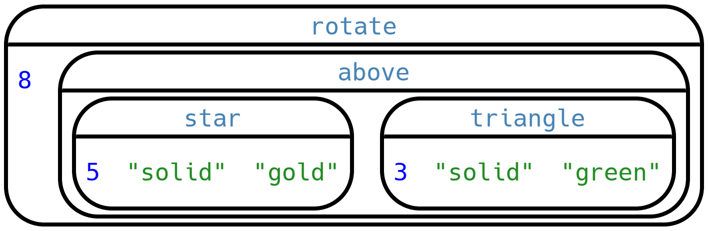
For each Circle of Evaluation, finish the Code by adding parentheses.

2)



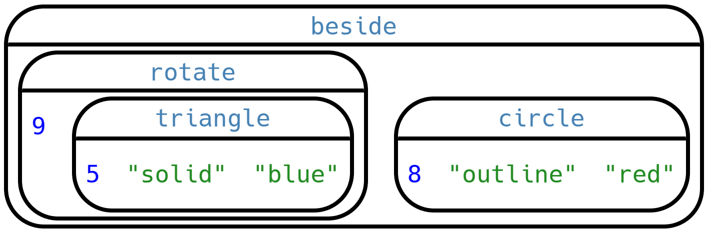
```
beside    triangle    5    "solid"    "blue"    circle    8    "outline"  "red"
```

3)



```
rotate    8    above    star    5    "solid"    "gold"    triangle    3    "solid"    "green"
```

4)



```
beside    rotate    9    triangle    5    "solid"    "blue"    circle    8    "outline"    "red"
```

Frayer Model: Domain and Range

My Definition

Facts and Characteristics

Domain

Examples

Non-Examples

My Definition

Facts and Characteristics

Range

Examples

Non-Examples

(optional)

Frayer Model: Function and Variable

My Definition

Facts and Characteristics

Function

Examples

Non-Examples

My Definition

Facts and Characteristics

Variable

Examples

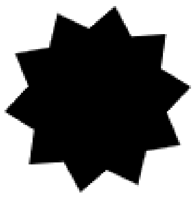
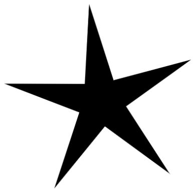
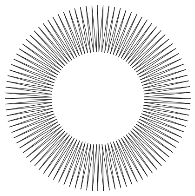
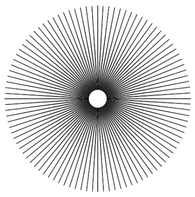
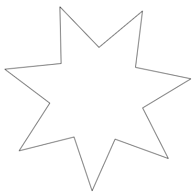
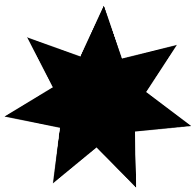
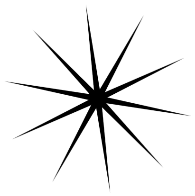
Non-Examples

(optional)

Radial Star

```
; radial-star :: (____Number____, ____Number____, ____Number____, ____String____, ____String____) -> Image
                      points          outer-radius    inner-radius     fill-style         color
```

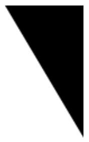
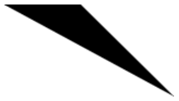
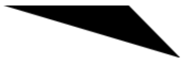
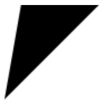
Using the Contract above, match the images on the left to the expressions on the right. Test the code at WeScheme.

	1	A	`(radial-star 5 200 50 "solid" "black")`
	2	B	`(radial-star 7 200 100 "solid" "black")`
	3	C	`(radial-star 7 200 100 "outline" "black")`
	4	D	`(radial-star 10 200 150 "solid" "black")`
	5	E	`(radial-star 10 200 20 "solid" "black")`
	6	F	`(radial-star 100 200 20 "outline" "black")`
	7	G	`(radial-star 100 200 100 "outline" "black")`

(optional)

Triangle Contracts (SAS & ASA)

Type each expression (left) below into the WeScheme and match it to the image it creates (right).

Expression			Image
`(triangle-sas 120 45 70 "solid" "black")`	1	A	
`(triangle-sas 120 90 70 "solid" "black")`	2	B	
`(triangle-sas 120 135 70 "solid" "black")`	3	C	
`(triangle-sas 70 135 120 "solid" "black")`	4	D	

Contracts

Think about how you would describe each `triangle-sas` argument to someone who'd never used the function before.

5) Annotate the Contract below using descriptive variable names.

`triangle-sas` :: (___Number___, ___Number___, ___Number___, ___String___, ___String___) `-> Image`

If you have a printed workbook, add examples of each of the triangle functions we've explored to your contracts pages.

★ If you have time, experiment with the `triangle-asa` function.

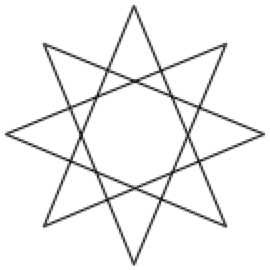
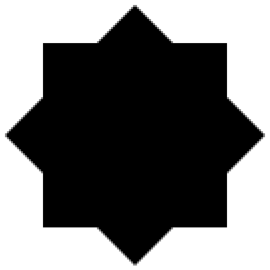
```
; triangle-asa :: (__Number__, __Number__, __Number__, __String__, __String__) -> Image
                  top-left-angle  left-side  bottom-angle  fill-style   color
```

★ Why did these two functions need to take in one more Number than `right-triangle` did?

Star Polygon

```
; star-polygon :: (___Number___, ___Number___, ___Number___, ___String___, ___String___) -> Image
                    side-length   points-on-polygon  points-to-skip-for-star  fill-style  color
```

1. Using the Contract above, write expressions to create images like those pictured below.
2. Go to WeScheme to test your code.
3. Then write expressions to generate two more star polygons of your choosing.
   Sketch them and record your working code.

1		______________________________
2		______________________________
3		______________________________
4		______________________________

(optional)

Function Composition — Green Star

1) Draw a Circle of Evaluation and write the Code for a **solid, green star, size 50**. *Then go to WeScheme to test your code.*

Circle of Evaluation:

Code: ______________________________

Using the star described above as the **original**, draw the Circles of Evaluation and write the Code for each exercise below. Test your code in the editor.

2) A solid, green star, that is triple the size of the original (using `scale`)	3) A solid, green star, that is half the size of the original (using `scale`)
4) A solid, green star of size 50 that has been rotated 45 degrees counter-clockwise	5) A solid, green star that is 3 times the size of the original **and** has been rotated 45 degrees

Function Composition — Your Name

You'll be investigating these functions with your partner:

```
; text :: String, Number, String -> Image          ; frame :: Image -> Image
; flip-horizontal :: Image -> Image                ; above :: Image, Image -> Image
; flip-vertical :: Image -> Image                  ; beside :: Image, Image -> Image
```

1) In the editor, write the code to make an image of your name in big letters in a color of your choosing using `text`. Then draw the Circle of Evaluation and write the Code that will create the image.

Circle of Evaluation for an "image of your name":

Code for an "image of your name": ______________________________

Using the "image of your name" described above as the **original**, draw the Circles of Evaluation and write the Code for each exercise below. Test your ideas in the editor to make sure they work.

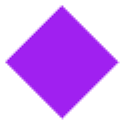
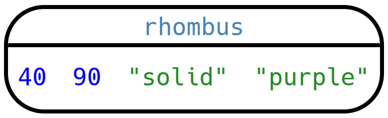
2) The framed "image of your name".	3) The "image of your name" flipped vertically.
4) The "image of your name" above a vertical reflection of the "image of your name"	5) The "image of your name" flipped horizontally beside "the image of your name".

Function Composition — scale-xy

You'll be investigating these two functions with your partner:

```
; scale/xy :: ( Number , Number , Image ) -> Image
              x-scale-factor y-scale-factor img-to-scale
```

```
; overlay :: ( Image , Image ) -> Image
               top     bottom
```

The Image:	Circle of Evaluation:	Code:
		`(rhombus 40 90 "solid" "purple")`

Starting with the image described above, write Circles of Evaluation and Code for each exercise below. *Be sure to test your code!*

1) A purple rhombus that is stretched 4 times as wide.	2) A purple rhombus that is stretched 4 times as tall

3) The tall rhombus from #1 overlayed on the wide rhombus (#2).

★ Overlay a red rhombus onto the last image you made in #3.

More than one way to Compose an Image!

What image will each of the four expressions below evaluate to?
If you're not sure, go to WeScheme and type them into the Interactions Area and see if you can figure out how the code constructs its image.

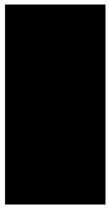
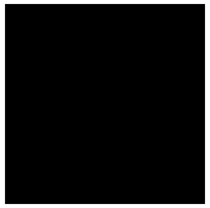
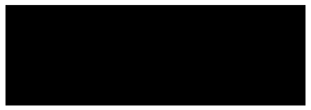
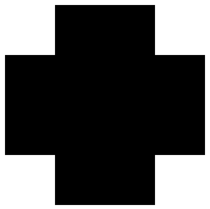
```
(beside (rectangle 200 100 "solid" "black")(square 100 "solid" "black"))

(scale/xy 1 2(square 100 "solid" "black"))

(scale 2 (rectangle 100 100 "solid" "black"))

(above
    (rectangle 100 50 "solid" "black")
    (above
        (rectangle 200 100 "solid" "black")
        (rectangle 100 50 "solid" "black")))
```

For each image below, identify 2 expressions that could be used to compose it. *The bank of expressions at the top of the page includes one possible option for each image.*

1		
2		
3		
★		

Function Cards

Print and cut these out, for use with the unplugged "function composition" activity.

`; double :: Number -> Number`
`; consumes a number, and multiplies that number by 2`
	`; half :: Number -> Number`
	`; consumes a number, and produces a number that is half the input`

`; add5 :: Number -> Number`
`; consumes a number, adds five, and produces the result`
	`; sub10 :: Number -> Number`
	`; consumes a number, subtracts ten, and produces the result`

`; sqr :: Number -> Number`
`; consumes a number, squares it, and produces the result`
	`; neg :: Number -> Number`
	`; consumes a number, multiplies it by -1, and produces the result`

`; add1 :: Number -> Number`
`; consumes a number, adds one, and produces the result`
	`; f :: Number -> Number`
	`; consumes a number, subtracts seven, and produces the result`

`; g :: Number -> Number`
`; consumes a number, adds six, and produces the result`
	`; h :: Number -> Number`
	`; consumes a number, subtracts one, and produces the result`

Defining Values in a Nutshell

In math, we use values, expressions and definitions.

- **Values** include things like: -98.1 $\quad {}^{2}/_{3}$ $\quad 42$
- **Expressions** include things like: 1×3 $\quad \sqrt{16}$ $\quad 5 - 2$
  - These evaluate to results, and typing any of them in as code produces some answer.
- **Definitions** are different from values and expressions, because *they do not produce results*. Instead, they simply create names for values, so that those names can be re-used to make the Math simpler and more efficient.
  - Definitions always have both a name and an expression.
  - The name goes on the left and is defined by an equals sign to be the result of a value-producing expression on the right:
    $x = 4$
    $y = 9 + x$
  - The above examples tells us:
    "x is defined to be 4."
    "y is defined to be 13."
  - **Important: there is no "answer" to a definition**, and typing in a definition as code will produce no result.
  - Notice that *once a value has been defined, it can be used in subsequent definitions*. In the example above…
    The definition of `y` refers to `x`.
    The definition of `x`, on the other hand, *cannot* refer to `y`, because it comes before `y` is defined.

In WeScheme, these definitions are written a little differently, making it clear that we're talking about definitions:

- Try typing these definitions into the Definitions Area on the left, clicking "Run", and then *using* them in the Interactions Area on the right.
  - `(define x 4)`
  - `(define y (+ 9 x))`

Just like in math, definitions in our programming language can only refer to previously-defined values.

- Here are a few more value definitions. Feel free to type them in, and make sure you understand them.
  - `(define x (+ 5 1))`
  - `(define y (* x 7))`
  - `(define food "Pizza!")`
  - `(define dot (circle y "solid" "red"))`

Defining Values - Explore

Open the Defining Values Starter File and click "Run".

1) What do you Notice?

2) What do you Wonder?

For each of the expressions listed below, write your *prediction* for what you expect WeScheme to produce? Once you have completed your predictions, test them out one at a time in the Interactions Area.

	Prediction	Result		Prediction	Result
3) `x`			4) `(+ x 5)`		
5) `(- y 9)`			6) `(* x y)`		
7) `z`			8) `t`		
9) `gold-star`			10) `my-name`		
11) `swamp`			12) `c`		

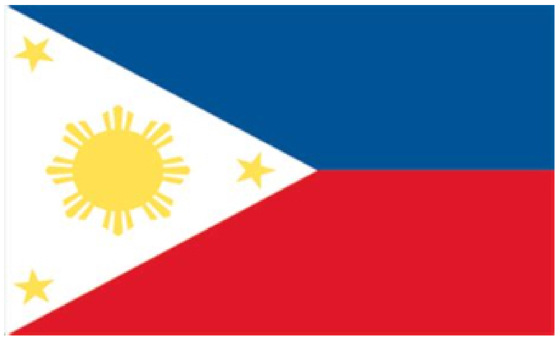
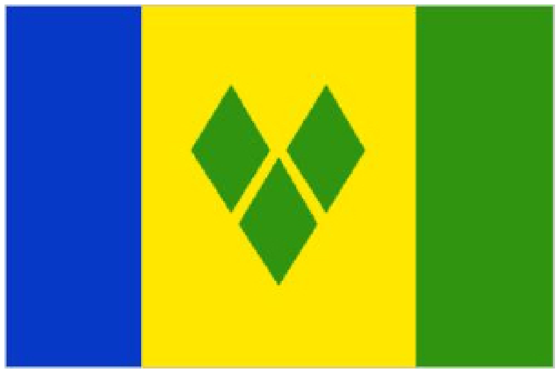
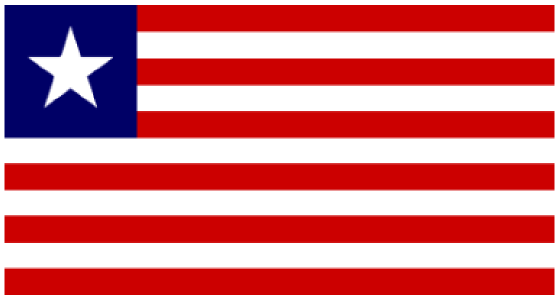
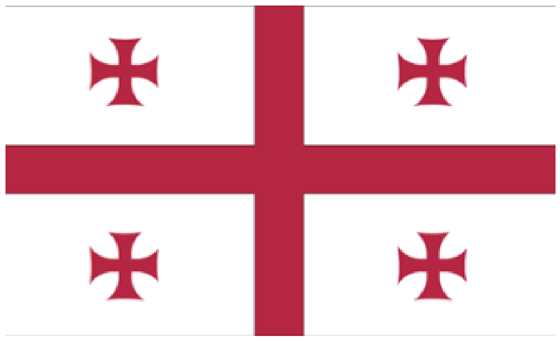
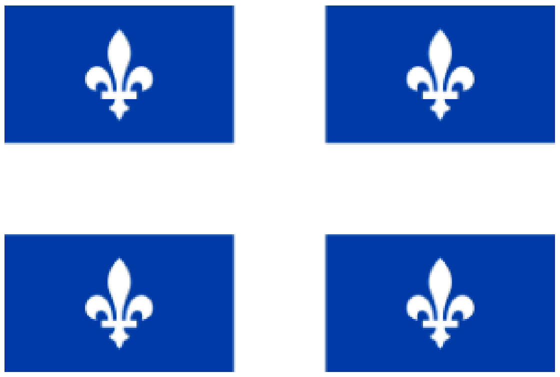
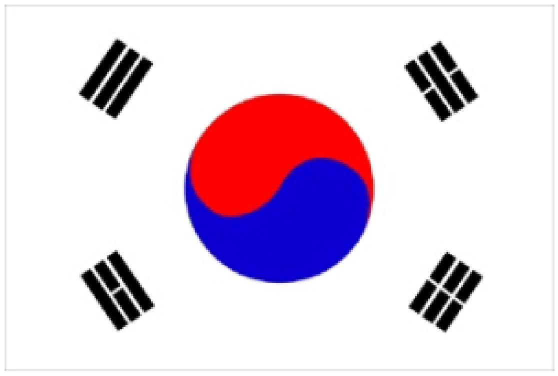
13) In the code, find the definitions of `exampleA`, `exampleB`, and `exampleC`. These all define the same shape, but their definitions are split across several lines. Suppose you *had* to split your code across multiple lines like this. Which one of these is the easiest to read, and why?

14) Define at least 2 more variables in the Definitions Area, click "Run" and test them out. Once you know they're working, record the code you used below.

15) What have you learned about defining values?

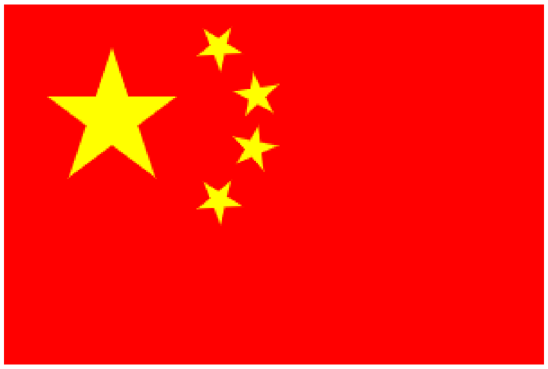
Which Value(s) Would it Make Sense to Define?

For each of the images below, identify which element(s) you would want to define before writing code to compose the image.
Hint: what gets repeated?

Philippines	St. Vincent & the Grenadines
	
1) ________________________________	2) ________________________________
Liberia	**Republic of Georgia**
	
3) ________________________________	4) ________________________________
Quebec	**South Korea**
	
5) ________________________________	6) ________________________________

Chinese Flag

The image value on the left called `china` is defined by the code on the right.



```
(define china
  (translate
     (rotate 40 (star 15 "solid" "yellow"))
     120 175
     (translate
       (rotate 80 (star 15 "solid" "yellow"))
       140 150
       (translate
          (rotate 60 (star 15 "solid" "yellow"))
          140 120
          (translate
             (rotate 40 (star 15 "solid" "yellow"))
             120 90
             (translate
                (scale 3 (star 15 "solid" "yellow"))
                60 140
                (rectangle 300 200 "solid"
"red")))))))
```

1) What image do you see repeated in the flag?

____

2) **Highlight or underline** every place in the code ➡️ that you see the repeated expression for that image.

3) Write the code to **define a value** for the repeated expression.

____

4) Open the Flag of China Starter File, **save a copy** and click "Run". **Simplify the code**, replacing the repeated expressions with the value you defined. Do you still get the same image when you click "Run"? If not, check your work.

5) Change the color of all the stars to black, then change their size to 20. Would this have been easier with the original code? Why or why not?

____

6) Here is the same code shown above, but all crammed into one line.

```
(define china (translate (rotate 40 (star 15 "solid" "yellow")) 120 175 (translate (rotate 80 (star
15 "solid" "yellow")) 140 150 (translate (rotate 60 (star 15 "solid" "yellow")) 140 120 (translate
(rotate 40 (star 15 "solid" "yellow")) 120 90 (translate (scale 3 (star 15 "solid" "yellow")) 60 140
(rectangle 300 200 "solid" "red")))))))
```

Is it easier or harder to read, when everything is all on one line? ______________________________

7) Professional programmers *indent* their code, by breaking long lines into shorter, more readable lines of code. In the indented code at the top of the page, notice that each `translate` is followed by several lines of code that all line up with each other, and that the lines under the *next* `translate` are shifted farther and farther to the right. What do you think is going on?

____

____

____

★ This file uses a function we haven't seen before! *Hint: Focus on the last instance of the function.* What is its name? ____________.

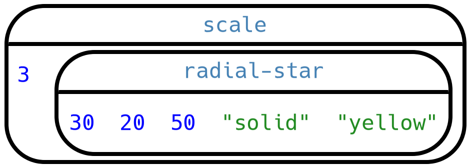
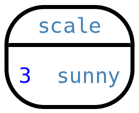
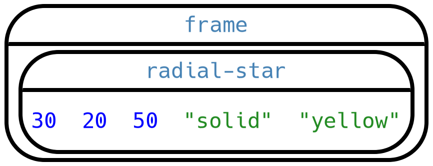
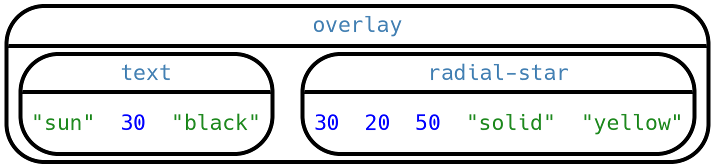
How many inputs are in its domain? ________. What are the types of those inputs? ______________________________

Why Define Values?

Take a close look at the Original Circle of Evaluation & Code and how it got simplified.

1) Write the code that must have been used to define the value of `sunny`. ______________________________

2) Complete the table using the first row as an example.

Original Circle of Evaluation & Code		Use the ***defined value*** **`sunny`** to simplify!
	→	
`(scale 3 (radial-star 30 20 50 "solid" "yellow"))`	→	Code: `(scale 3 sunny)`
Second Circle of Evaluation & Code		Use the ***defined value*** **`sunny`** to simplify!
	→	
`(frame (radial-star 30 20 50 "solid" "yellow"))`	→	Code:
Third Circle of Evaluation & Code		Use the ***defined value*** **`sunny`** to simplify!
	→	
`(overlay (text "sun" 30 "black") (radial-star 30 20 50 "solid" "yellow"))`	→	Code:

3) Define `sunny` in the Definitions Area using the code you recorded at the top of the page.

4) Test your code in the editor and make sure it produces what you would expect it to.

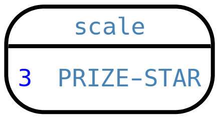
Writing Code using Defined Values

1) On the line below, **write the Code** to define `PRIZE-STAR` as the pink outline of a size 65 star.

Using the `PRIZE-STAR` definition from above, draw the Circle of Evaluation and write the Code for each of the exercises.
Be sure to test out your code in WeScheme before moving onto the next item. One Circle of Evaluation has been done for you.

2) The outline of a pink star that is three times the size of the original (using `scale`)

Circle of Evaluation:



Code:

3) The outline of a pink star that is half the size of the original (using `scale`)

Circle of Evaluation:

Code:

4) The outline of a pink star that is rotated 45 degrees
(It should be the same size as the original.)

Circle of Evaluation:

Code:

5) The outline of a pink star that is three times as big as the original **and** has been rotated 45 degrees

Circle of Evaluation:

Code:

6) How does defining values help you as a programmer?

Surface Area of a Rectangular Prism - Explore

1) What do you picture in your mind when you hear *rectangular prism*?

2) What do you picture in your mind when you hear *surface area*?

Open the Surface Area of a Rectangular Prism Starter File and click "Run".
Type `prism` into the Interactions Area (on the right) and hit "enter" to see an image of a rectangular prism.

3) How many faces does this prism have? ______

Defining Faces

Find PART 1 in the Definitions Area of the starter file (on the left). You will see a definition for `front` and `back`.

4) How did the author know to use width and height as the dimensions for `front`? ______

5) Why are `front` and `back` defined to be the same thing? ______

6) Using these definitions as a model, add definitions for the other faces of this prism to the Definitions Area (on the left).

Completing the List

Find PART 2 in the starter file. You'll see `(list front back)` ... so far the list only includes `front` and `back`.

7) Complete the faces list, then type `(print-imgs faces)` into the Interactions Area. What do you see?

Printing Your Paper Model

We're going to print the faces following directions in PART 3 and build a paper model of a rectangular prism.
Before you print and build your prism, you can change the length, width, and height of your prism at the top of the starter file. Be sure that all 3 dimensions are different, and that they are all small enough to fit on a sheet of paper. If you change them, record your new dimensions here.

LENGTH: ______ WIDTH: ______ HEIGHT: ______

10) Calculate the surface area of your prism, by adding the area of each face. ______ Show your work below.

Code for Calculating the Surface Area of a Prism

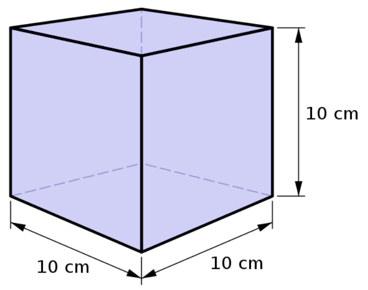
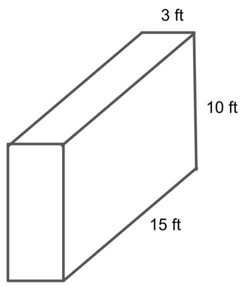
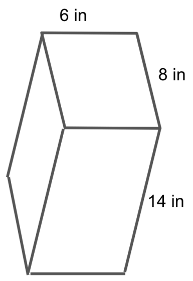
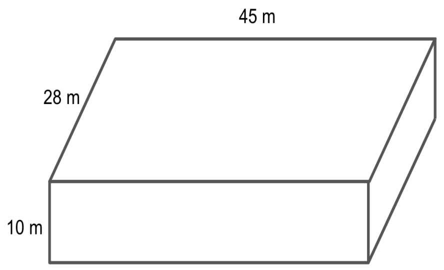
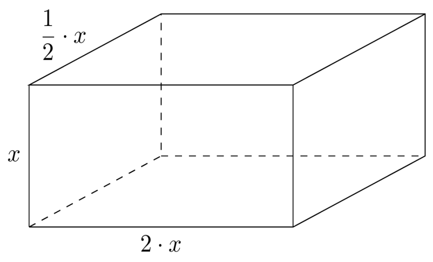
Follow the directions in PART 4 of the starter file to write code to calculate the surface area.

11) How many definitions did you write? ______

12) How does the surface area that the computer returns compare to the surface area you calculated by hand?

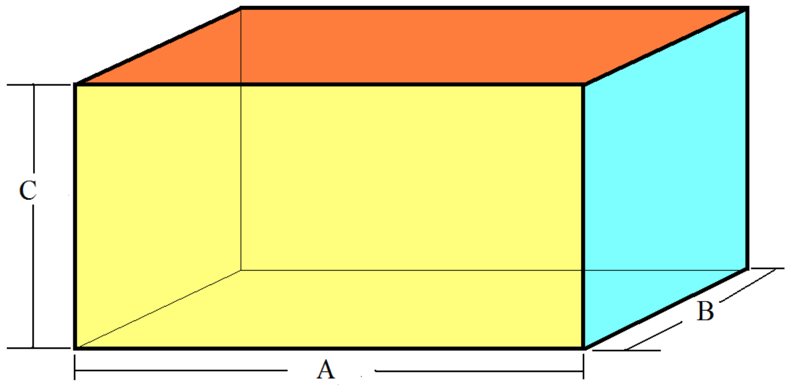
Surface Area of a Prism - Practice

Find the Surface Area of each rectangular prism below. Show your work in the right-hand column, and write your final answer in the blank.

1		Surface Area: ________________
2		Surface Area: ________________
3		Surface Area: ________________
4		Surface Area: ________________
5		Surface Area: ________________

Surface Area of a Prism - More than One Way

Students in Mr. Grattan's class were asked to write code that would calculate the surface area of this rectangular prism. Help them convert their strategies into algebraic expressions and code, and double check that each strategy works.



1) Della says, "Just find the area of the top, bottom, left, right, front and back and add them all together!" **Will it work?** __________

- Algebraic Expression: $AB + AB + BC + BC + AC + AC = 2AB + 2BC + 2AC$
- Code: __________

2) Orion says, "Just find the area of the front, top and right faces, add them together, and double the sum." **Will it work?** __________

- Algebraic Expression: __________
- Code: __________

3) Jules says, "Double the area of the front, double the area of the top, double the area of the side. Then add them up." **Will it work?** __________

- Algebraic Expression: __________
- Code: __________

4) Tate says, "Just multiply the length times the width times the height and double their product." **Will it work?** __________

- Algebraic Expression: __________
- Code: __________

5) Can you think of one other way to find the surface area of the prism?

- Description: __________
- Algebraic Expression: __________
- Code: __________

6) Whose strategy do you like best? __________

Why? __________

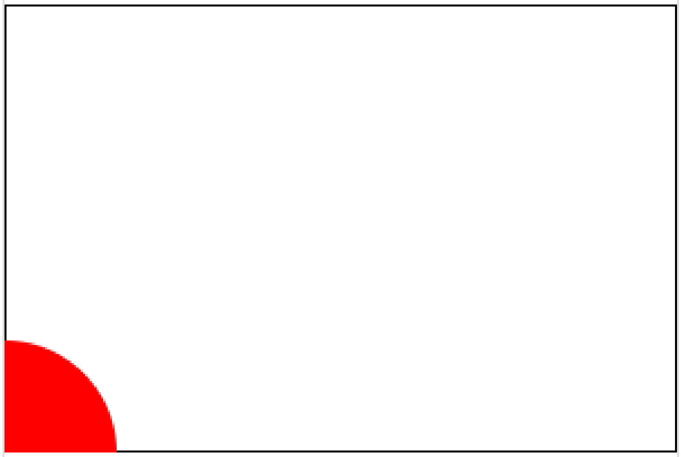
Making Sense of Coordinates

```
(define dot (circle 50 "solid" "red"))
(define background (rectangle 300 200 "outline" "black"))
```

Think of the background image as a sheet of graph paper with the origin (0,0) in the bottom left corner. The width of the rectangle is 300 and the height is 200. The numbers in `translate` specify a point on that graph paper, where the center of the top image (in this case `dot`) should be placed.

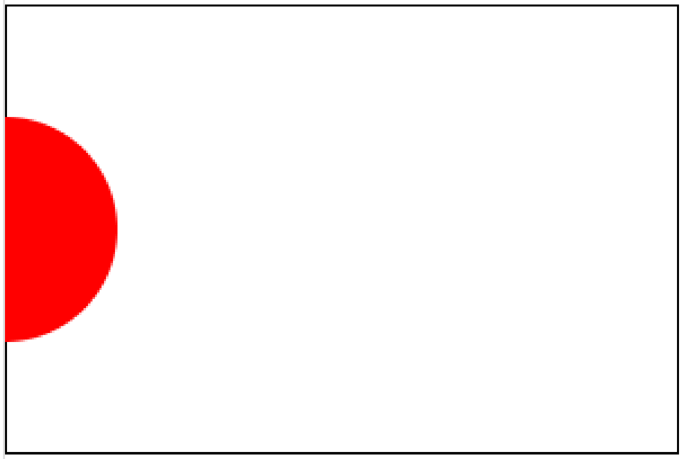
What coordinates would you expect were used to place the `dot` for each of the following images?

1)



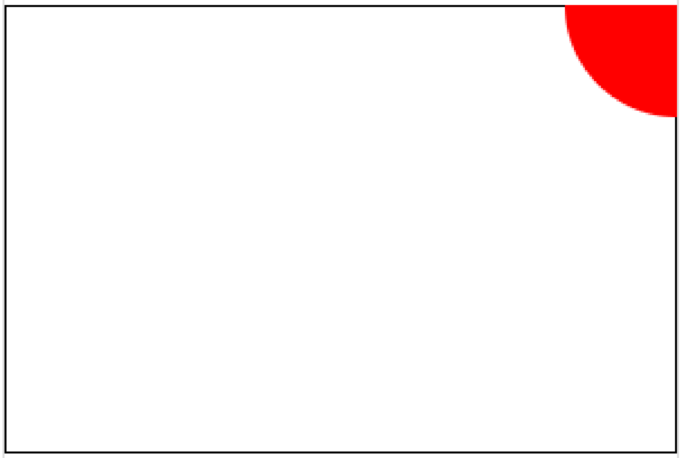
(translate dot ________ ________ background)

2)



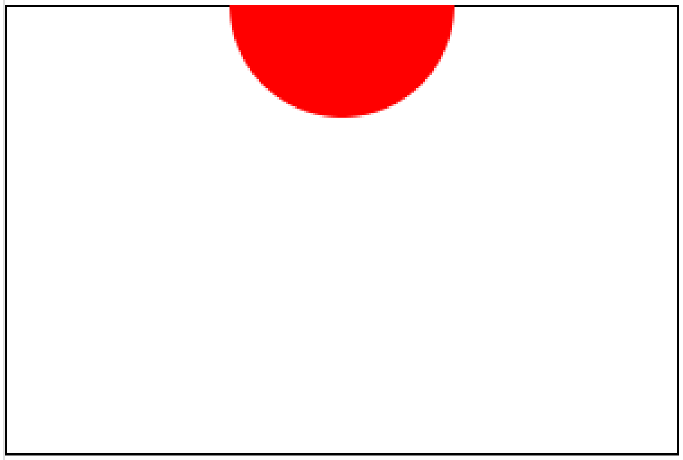
(translate dot ________ ________ background)

3)



(translate dot ________ ________ background)

4)



(translate dot ________ ________ background)

Investigating translate

Japan

For this section of the page, you will refer to the Flags Starter File.

1) Each language has its own symbol for commenting code so that programmers can leave notes that won't be read by the computer. In WeScheme, we use the semicolon (`;`). What color are comments in WeScheme? ______

2) Type `japan-flag` into the Interactions Area. What do you get back? ______

3) Type `japan` into the Interactions Area and compare the image to `japan-flag`.

- How are they alike? ______
- How are they different? ______

4) `japan` is composed using `dot` and `background`. Type each of those variables into the Interactions Area. What do you get back?

- `dot`: ______
- `background`: ______

5) These images are combined using the `translate` function. What is its contract? ______

6) Fix the `japan` code so that it matches the `japan-flag` image. What did you need to change? ______

7) How can you prove that you have placed the `dot` in exactly the right location? ______

The Netherlands

For this section of the page, you will refer to the Flags of Netherlands, France & Mauritius Starter File.

8) What was the programmer thinking when she coded the height of the red stripe as `(/ 200 3)`? ______

9) The center of the blue stripe is placed at (`150`, `(/ 200 6)`). How did the programmer know to use 150 as the x-coordinate? ______

10) What was the programmer thinking when she coded the y-coordinate as `(/ 200 6)`? ______

11) Explain the thinking behind coding the red stripe's y-coordinate as `(* 5 (/ 200 6))`. ______

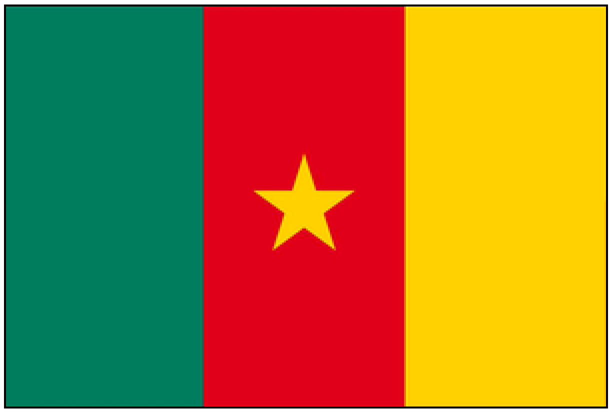
12) What advantages are there to representing height / length / width as fractions (as we see in this code) rather than using a computed value?

______

Decomposing Flags

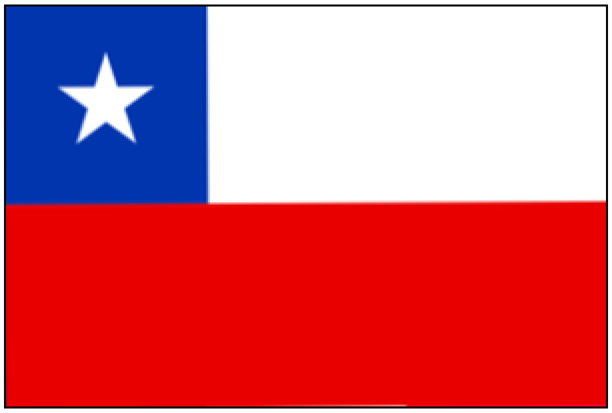
Each of the flags below is shown with their width and height. Identify the shapes that make up each flag. Use the flag's dimensions to estimate the dimensions of the different shapes. Then estimate the x and y coordinates for the point at which the center of each shape should be located on the flag. *Hint: The bottom left corner of each flag is at (0,0) and the top right corner is given by the flags dimensions.*

1) **Cameroon** (450 x 300)



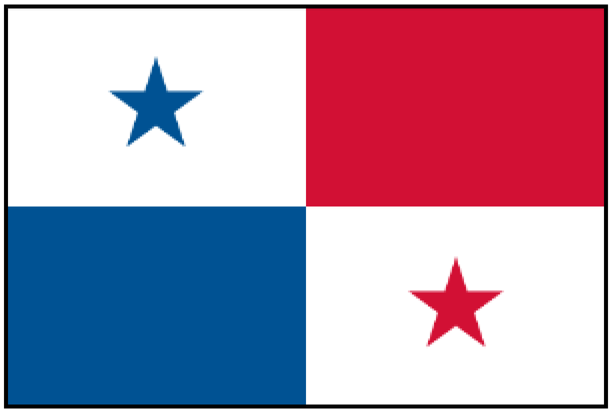
shape:	color:	width:	height:	x	y

2) **Chile** (420 x 280)



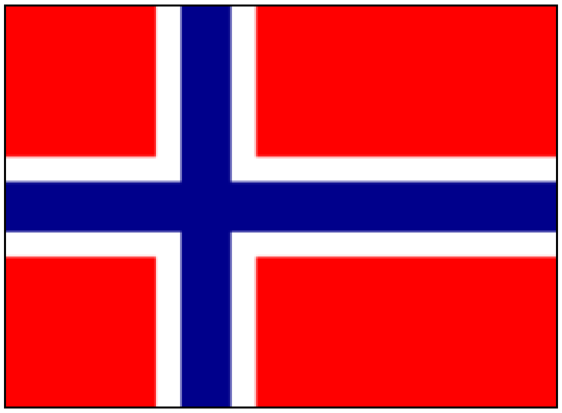
shape:	color:	width:	height:	x	y

3) **Panama** (300 x 200)



shape:	color:	width:	height:	x	y

4) **Norway** (330 x 240)



shape:	color:	width:	height:	x	y

(optional)

Coding and Designing the Alaskan Flag

Open the Flag of Alaska Starter File. Click run and type `alaska` *to see an image of the flag of Alaska.*

Exploring the Code

1) How many images are defined in the code? ______________________

2) How many images are placed using `translate` in order to generate the flag? ______________________

3) Why do your answers to these questions differ? ______________________

____

4) The code for the flag could have been written without defining any images. What are some reasons why defining images makes the code easier to work with?

____

____

The Story of the Flag of Alaska

Benny Benson holding the flag of Alaska that he designed

The Alaska state flag is based on a design created in 1926 for a Territory-wide contest for schoolchildren. The thirteen-year-old seventh-grade designer was Benny Benson from the Aleutian Islands. *(At the time, Alaska was not yet a state; it had been a US Territory since the land was purchased from Russia in 1867.)*

On the design submission, Benny had written the following explanation:

"The blue field is for the Alaska sky and the forget-me-not, an Alaska flower.
The North Star is for the future of the state of Alaska, the most northerly in the Union.
The dipper is for the Great Bear — symbolizing strength."

Benny's flag was officially adopted by the legislature in 1927.

Alaska was officially recognized as a state on January 3, 1959.

5) How old was Benny when Alaska achieved statehood?

____

6) Think of someone you know who is old enough to remember 1959. (Your teacher is not old enough!). Find a time this week to visit or call and ask them if they remember anything about when Alaska became a state! Record what you learn below.

____

____

(optional)

Notice and Wonder

As you investigate the Blank Game Starter File with your partner, record what you Notice, and then what you Wonder.
Remember, "Notices" are statements, not questions.

What do you Notice?	What do you Wonder?

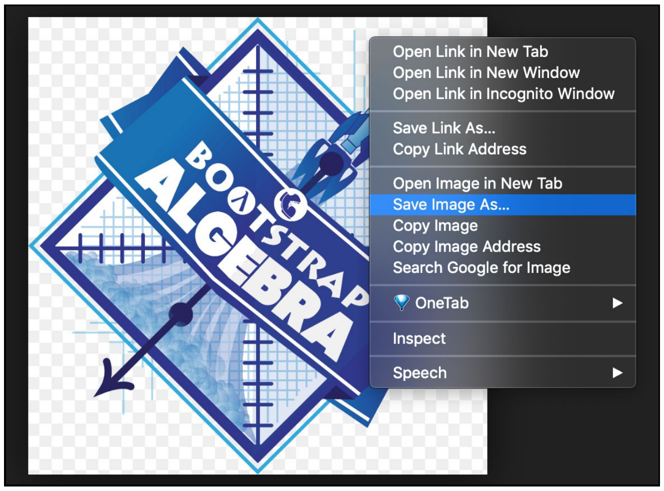
Quick Guide to Saving Images to Google Drive

Windows/MacOS:

1. Find the image you'd like to save. If using Google Image Search or a similar search engine, click once on the image to expand it.

2. Right-click (or 2-finger click on trackpad) on the expanded image.

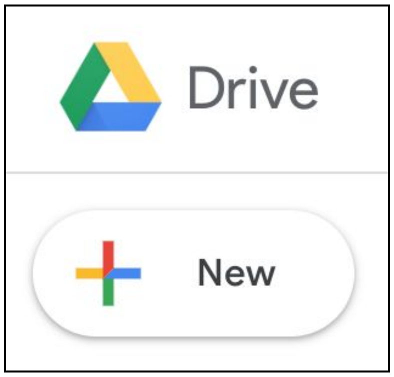
3. Select "Save Image As" (or "Save Picture As").



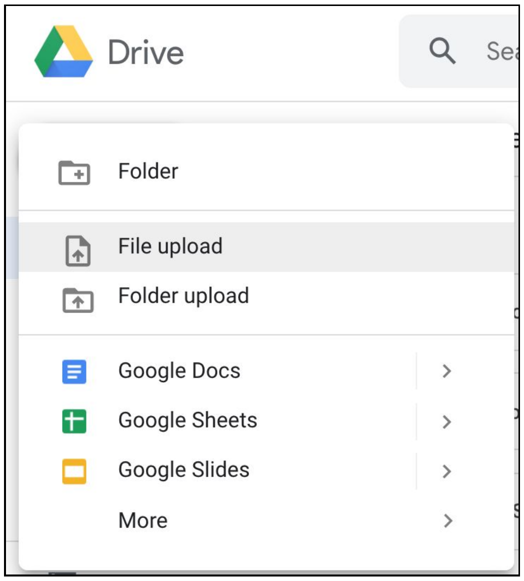
4. Name the file and select a location on your computer to save it to. (If saving several images, you can make a folder to make uploading faster.)

5. Open Google Drive (drive.google.com) and sign in if needed.

6. Click the "New" button near the top left.

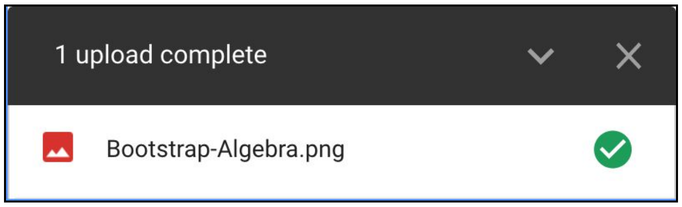


7. Select "File upload" (or "Folder upload" if you have a folder of images to upload).



8. Select the file (or folder) you want and click "Open".

9. Wait for the upload to finish (a green checkmark will appear).

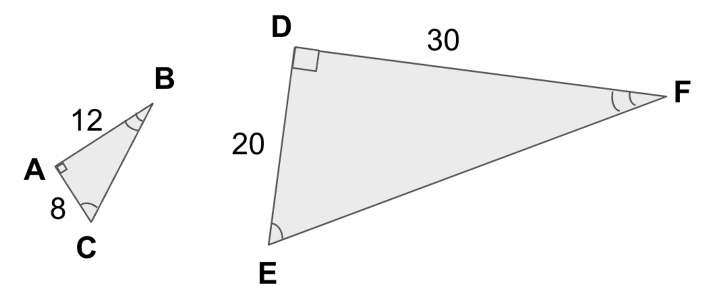


10. In WeScheme (www.wescheme.org), click the Images button and, if prompted, select the Google account you're using. (If the "Images" button isn't there, you'll need to sign in to WeScheme.)

    Select your image and you'll see the code for your image (using the `bitmap/url` function) appear!

Scaling Practice

```
; scale :: (Number, Image) -> Image
```
scale-factor image-producing-expression



The class was given an assignment to generate triangle DEF by scaling triangle `ABC`.

- Jourdan wrote: `(scale 1.5 ABC)`
- Roux wrote: `(scale (/ 30 12) ABC)`
- Zuni wrote: `(scale (/ 8 20) ABC)`
- Cedric wrote: `(scale (/ 30 20) ABC)`
- Josie wrote: `(scale 2.5 ABC)`
- Celine wrote: `(scale (/ 20 8) ABC)`

1) Whose expressions will work? ______________________________

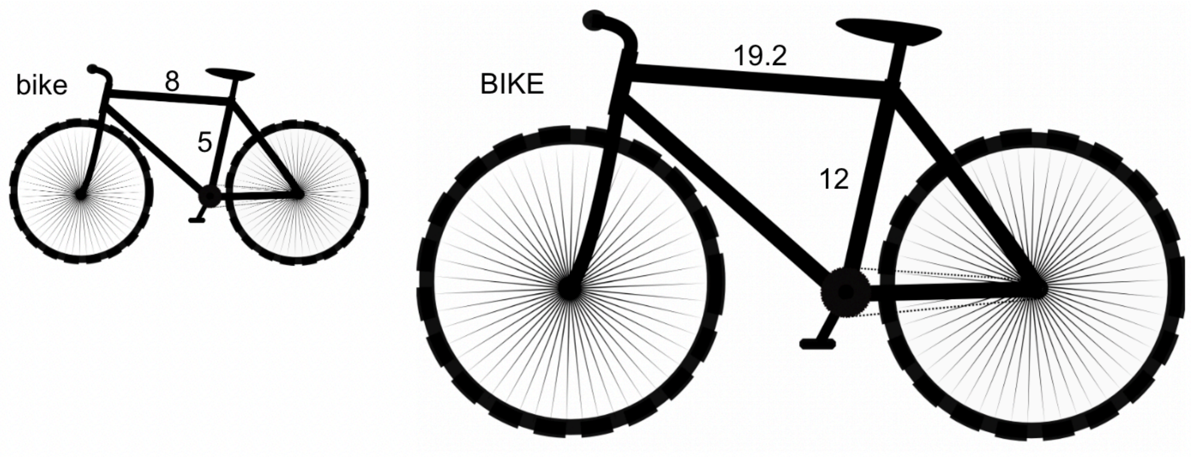
2) How do you know? ______________________________

____

____

3) Which one would you use and why? ______________________________

____



4) Write at least two expressions for generating the image titled BIKE by scaling `bike`.

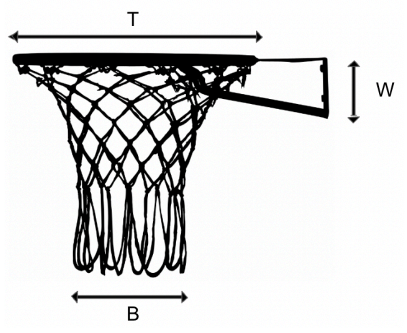
____

____

(optional)

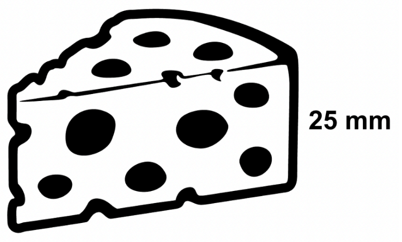
Scaling Practice (2)

Part 1: Complete the table below by filling in the missing fields for the original image and the three transformations.



Description	Original	Double-size	Triple-size	______________
expression	`hoop`	`(scale 2 hoop)`		`(scale 0.5 hoop)`
percent of original	100%		300%	50%
length of T		36	54	9
length of B	6			3
length of W		4		1

Part 2: Raffi wants to use this `cheese` image in his game. In thinking through what size he wants it to be, he comes up with the list of transformations described below. Help him to translate his ideas into code by matching each description to a `scale` expression.



Desired Resizing			Expression
New height of 75 mm	1	A	`(scale 1.5 cheese)`
60% as tall	2	B	`(scale 3 cheese)`
New height of 30 mm	3	C	`(scale 2 cheese)`
One and a half times as tall	4	D	`(scale 1.2 cheese)`
New height of 5 mm	5	E	`(scale 0.2 cheese)`
200% of the original size	6	F	`(scale 0.6 cheese)`
3/4 as tall	7	G	`(scale 0.75 cheese)`
New height of 12.5 mm	8	H	`(scale 0.05 cheese)`
5% as tall	9	I	`(scale 0.5 cheese)`

(optional)

Defining Functions in a Nutshell

Functions can be viewed in *multiple representations*.

Contract and Purpose

You already know one of them: ***Contracts***, which specify the Name, Domain, and Range of a function. Contracts are a way of thinking of functions as a *mapping* between one set of data and another. For example, a mapping from Numbers to Strings:

```
; f :: Number -> String
```

Examples

The goal of the ***Examples*** step is to *find the pattern* that represents what the function does.

Examples are essentially input-output tables, showing what the functions does with a list of specific inputs. *In our programming language, we write the table columns as code.*

How f is used	What f does
f(1)	$1 + 2$
f(2)	$2 + 2$
f(3)	$3 + 2$
f(4)	$4 + 2$

```
(EXAMPLE (f 1) (+ 1 2))
(EXAMPLE (f 2) (+ 2 2))
(EXAMPLE (f 3) (+ 3 2))
(EXAMPLE (f 4) (+ 4 2))
```

Definition

The final step in the Design Recipe is to *generalize the pattern* we see in our examples by writing a formal **function definition**. To do this we replace the inputs with ***variables*** that can work with any input.
In the example below, the definition for the examples above is written in both math and code:

$$f(x) = x + 2$$

```
(define (f x) (+ x 2))
```

Look for connections between these three representations!

- The function name is always the same, whether looking at the Contract, Examples, or Definition.
- The number of inputs in the Examples is always the same as the number of types in the Domain, which is always the same as the number of variables in the Definition.
- The "what the function does" pattern in the Examples is almost the same in the Definition, but with specific inputs replaced by variables.

The Great gt domain debate!

Kermit: The domain of `gt` is `Number, String, String`.
Oscar: The domain of `gt` is `Number`.
Ernie: I'm not sure who's right!
In order to make a triangle, we need a size, a color and a fill style…
but all we had to tell our actor was `(gt 20)`…and they returned `(triangle 20 "solid" "green")`.
Please help us!

1) What is the correct domain for gt?

2) What could you tell Ernie to help him understand how you know?

Let's Define Some New Functions!

1) **Let's define a function `rs` to generate solid red squares of whatever size we give them!**
If I say `(rs 5)`, what would our actor need to say?

____

Let's write a few more examples:

`(rs ______)` → ____

`(rs ______)` → ____

`(rs ______)` → ____

What changes in these examples? Name your variable(s): ______________________________
Let's define our function using the variable:

`(define (rs __________) ________________________________________________)`

2) **Let's define a function `bigc` to generate big solid circles of size 100 in whatever color we give them!**
If I say `(bigc "orange")`, what would our actor need to say?

____

Let's write a few more examples:

`(bigc __________)` → ____

`(bigc __________)` → ____

`(bigc __________)` → ____

What changes in these examples? Name your variable(s): ______________________________
Let's define our function using the variable:

`(define (bigc __________) ________________________________________________)`

3) **Let's define a function `ps` to build a pink star of size 50, with the input determining whether it's solid or outline!**
If I say `(ps "outline")`, what would our actor need to say?

____

Write examples for all other possible inputs:

`(ps __________)` → ____

`(ps __________)` → ____

What changes in these examples? Name your variable(s): ______________________________
Let's define our function using the variable:

`(define (ps __________) ________________________________________________)`

4) Add these new function definitions to your gt Starter File and test them out!

Let's Define Some More New Functions!

1) **Let's define a function `sun` to write SUNSHINE in whatever color and size we give it!**
If I say `(sun 5 "blue")`, what would our actor need to say?

____

Let's write a few more examples:

`(sun ______ ________ )` → ________________________________

`(sun ______ ________ )` → ________________________________

`(sun ______ ________ )` → ________________________________

What changes in these examples? Name your variable(s): ________________________
Let's define our function using the variable(s):

`(define (sun ________ ________ ) ________________________________ )`

2) **Let's define a function `me` to generate your name in whatever size and color we give it!**
If I say `(me 18 "gold")`, what would our actor need to say?

____

Let's write a few more examples:

`(me ______ ________ )` → ________________________________

`(me ______ ________ )` → ________________________________

`(me ______ ________ )` → ________________________________

What changes in these examples? Name your variable(s): ________________________
Let's define our function using the variable(s):

`(define (me ________ ________ ) ________________________________ )`

3) **Let's define a function `gr` to build a solid, green rectangle of whatever height and width we give it!**
If I say `(gr 10 80)`, what would our actor need to say?

____

Let's write a few more examples:

`(gr ______ ______ )` → `(rectangle ______ ______ "solid" "green")`

`(gr ______ ______ )` → `(rectangle ______ ______ "solid" "green")`

`(gr ______ ______ )` → `(rectangle ______ ______ "solid" "green")`

What changes in these examples? Name your variable(s): ________________________
Let's define our function using the variable(s):

`(define (gr ________ ________ ) ________________________________ )`

4) Add these new function definitions to your gt Starter File and test them out!

Describe and Define Your Own Functions!

1) **Let's define a function** ________________ **to generate...**

____

If I say ________________, what would our actor need to say? ______________________________________

Let's write a few more examples:

(________ ______________) → (________ ______________________________)

(________ ______________) → (________ ______________________________)

(________ ______________) → (________ ______________________________)

What changes in these examples? Name your variable(s): ______________________________

Let's define our function using the variable.

`(define (` ________ ________ `)` ______________________________________ `)`

2) **Let's define a function** ________________ **to generate...**

____

If I say ________________, what would our actor need to say? ______________________________________

Let's write a few more examples:

(________ ______________) → (________ ______________________________)

(________ ______________) → (________ ______________________________)

(________ ______________) → (________ ______________________________)

What changes in these examples? Name your variable(s): ______________________________

Let's define our function using the variable.

`(define (` ________ ________ `)` ______________________________________ `)`

3) **Let's define a function** ________________ **to generate...**

____

If I say ________________, what would our actor need to say? ______________________________________
Let's write a few more examples:

(________ ______________) → (________ ______________________________)

(________ ______________) → (________ ______________________________)

(________ ______________) → (________ ______________________________)

What changes in these examples? Name your variable(s): ______________________________

Let's define our function using the variable.

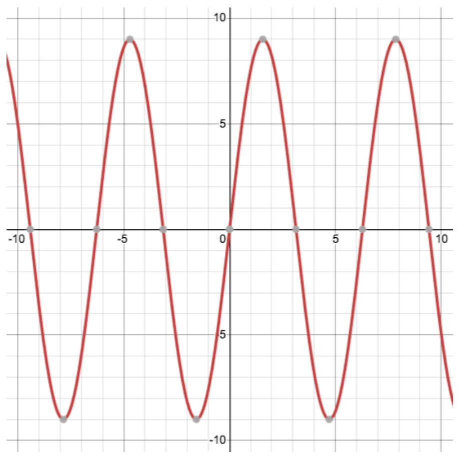
`(define (` ________ ________ `)` ______________________________________ `)`

4) Add your new function definitions to your gt Starter File and test them out!

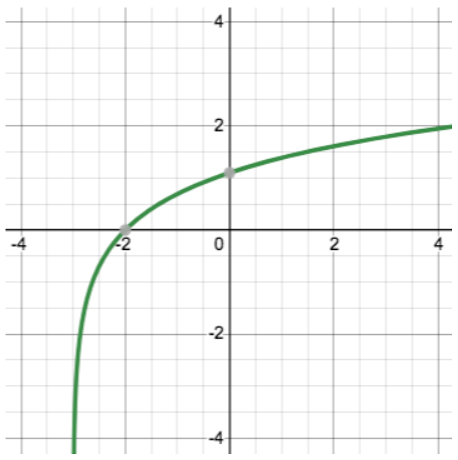
Identifying Functions from Graphs

Decide whether each graph below is a function. If it's not, prove it by drawing a vertical line that crosses the graph at more than one point.

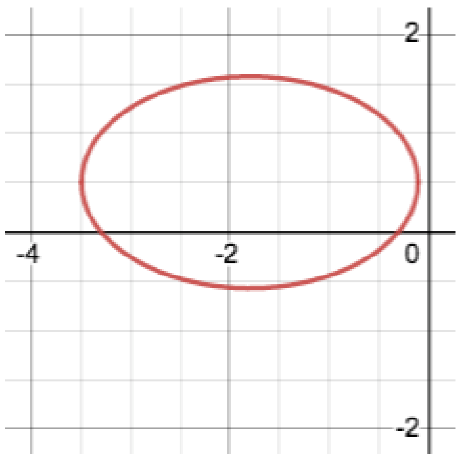
1) **Function** or **Not a Function**?



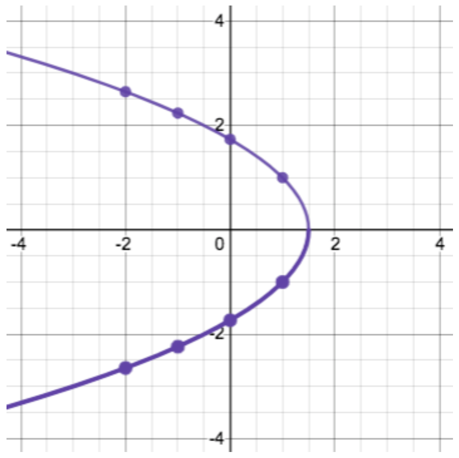
2) **Function** or **Not a Function**?



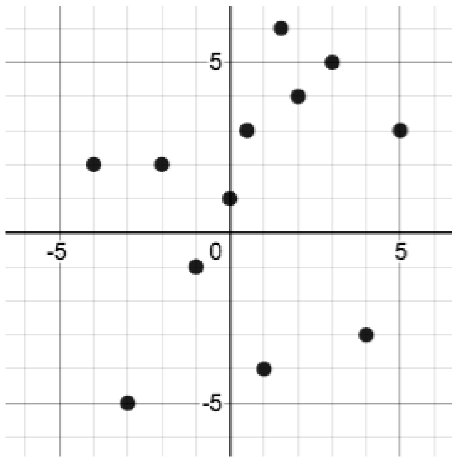
3) **Function** or **Not a Function**?



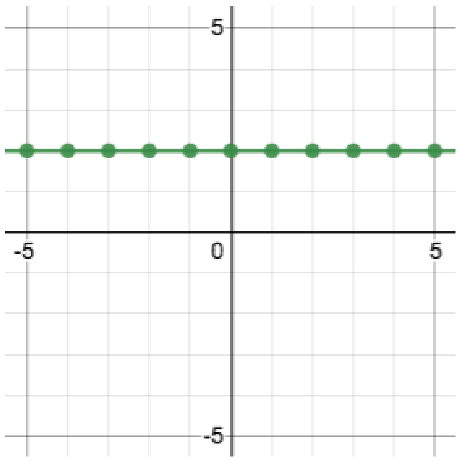
4) **Function** or **Not a Function**?



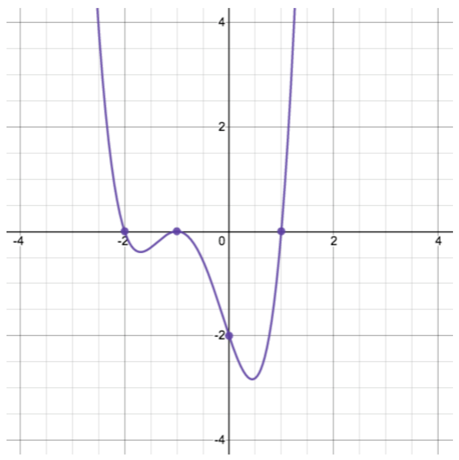
5) **Function** or **Not a Function**?



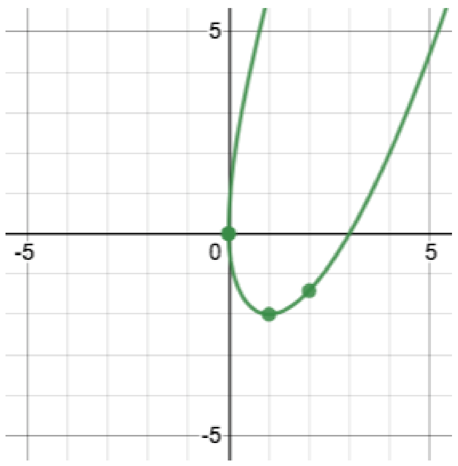
6) **Function** or **Not a Function**?



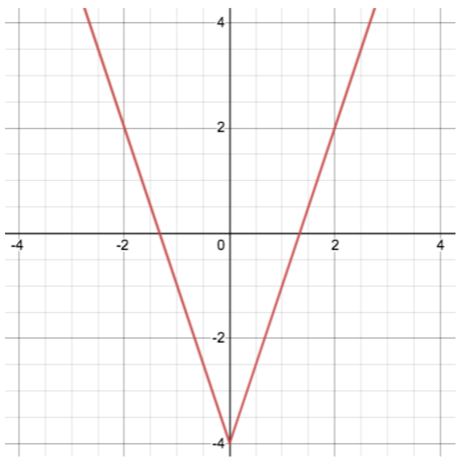
7) **Function** or **Not a Function**?



8) **Function** or **Not a Function**?



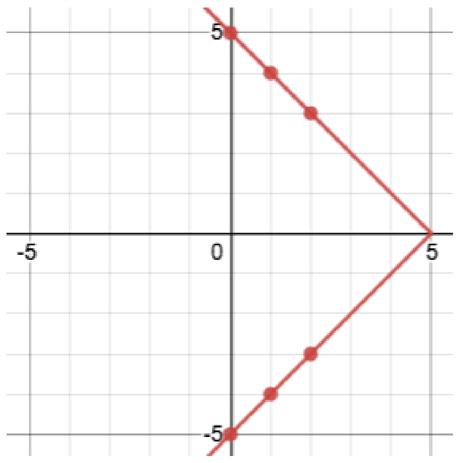
9) **Function** or **Not a Function**?



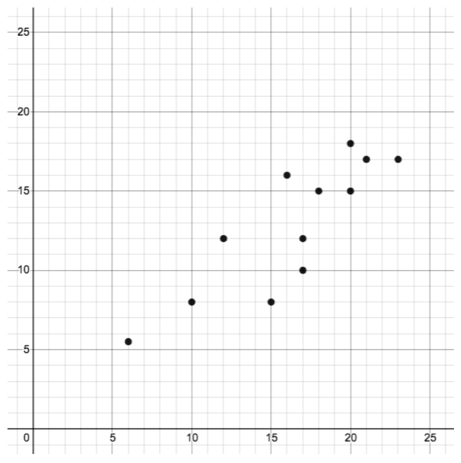
Identifying Functions from Graphs (2)

Decide whether each graph below is a function. If it's not, prove it by drawing a vertical line that crosses the graph at more than one point.

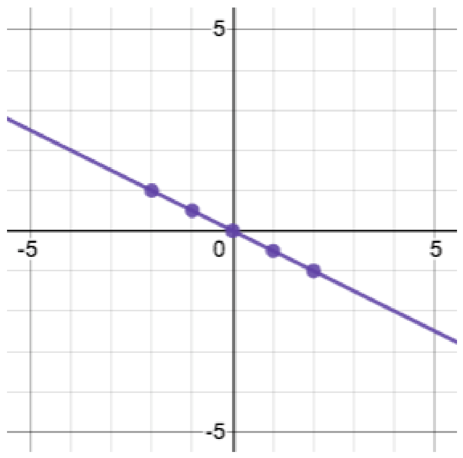
1) **Function** or **Not a Function**?



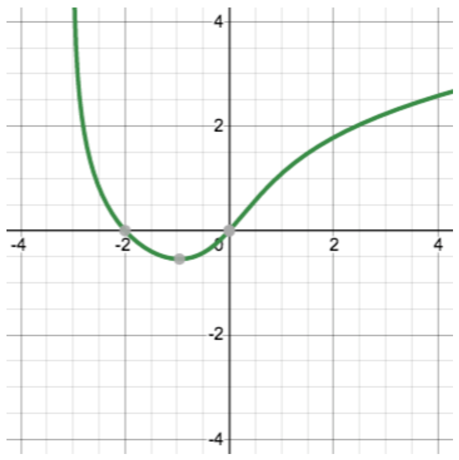
2) **Function** or **Not a Function**?



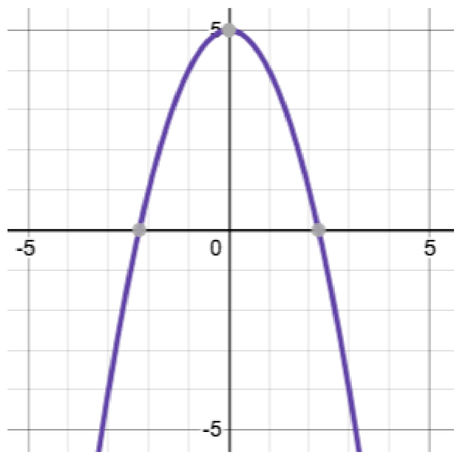
3) **Function** or **Not a Function**?



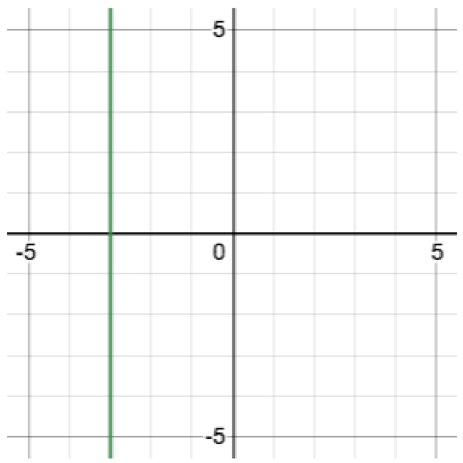
4) **Function** or **Not a Function**?



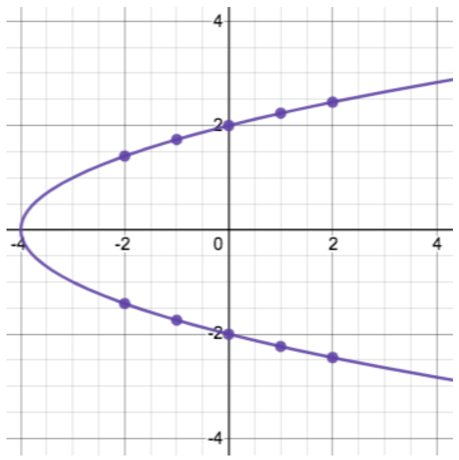
5) **Function** or **Not a Function**?



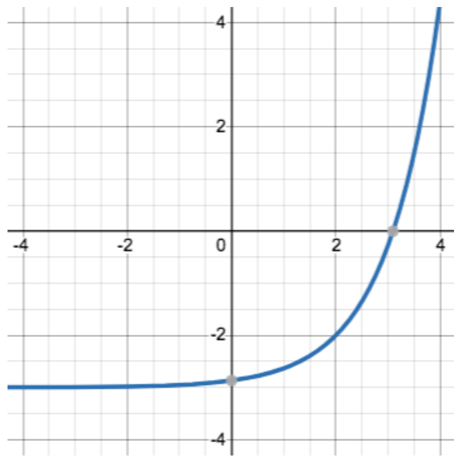
6) **Function** or **Not a Function**?



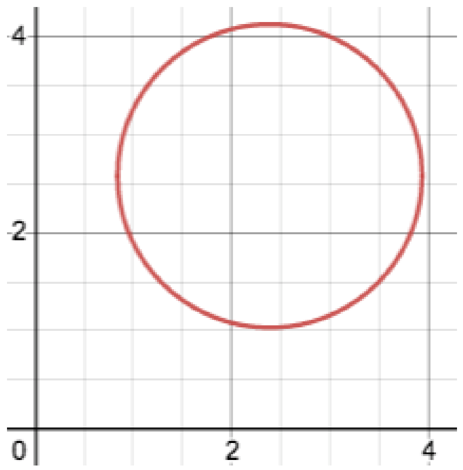
7) **Function** or **Not a Function**?



8) **Function** or **Not a Function**?



9) **Function** or **Not a Function**?



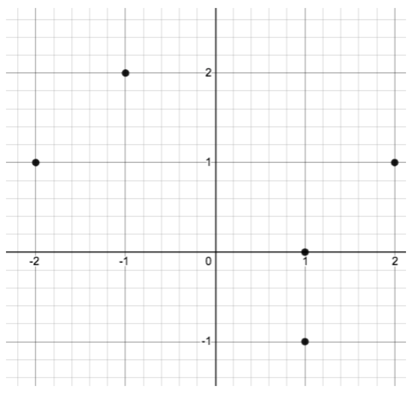
Notice and Wonder - Functions

Write down what you Notice and Wonder about the graphs you've just seen. At a later point you will *also* use this page to record what you Notice and Wonder about the tables you'll see. *Remember: "Notices" should be statements, not questions!*

What do you Notice?	What do you Wonder?

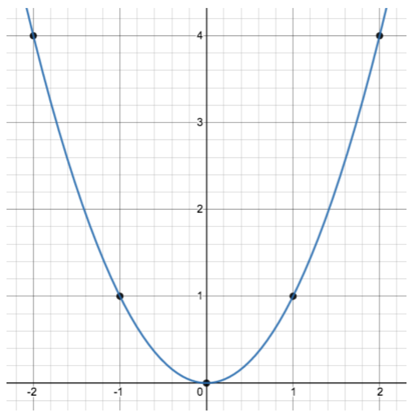
How Tables Fail the Vertical Line Test

1) Each of the graphs below is also represented by a table. Use the vertical line test to determine whether or not each graph represents a function.



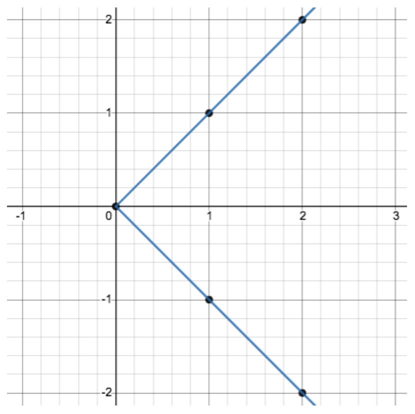
Function or **Not a Function**?

x	-2	-1	1	1	2
y	1	2	0	-1	1



Function or **Not a Function**?

x	-2	-1	0	1	2
y	4	1	0	1	4



Function or **Not a Function**?

x	2	1	0	1	2
y	2	1	0	-1	-2

2) For each graph that failed the vertical line test, label the offending points with their coordinates.

3) Find the same coordinates in the table below the graph and circle or highlight them.

4) What do the tables of the non-functions have in common? What could you look for in other tables to identify whether or not they could represent a function?

____

____

____

5) Use the process you just described to determine whether each table below could represent a function. Circle or highlight the points that would end up on the same vertical line.

x	y
0	-2
1	-2
2	-2
3	-2
4	-2

Function or **Not**?

x	y
0	-2
1	1
2	4
3	7
3	10

Function or **Not**?

x	y
0	3
1	4
-1	5
2	6
-2	7

Function or **Not**?

x	y
1	0
0	1
1	2
2	3
3	4

Function or **Not**?

Identifying Functions from Tables

Decide whether or not each table below could represent a function. If not, circle what you see that tells you it's not a function.
In a function, there is exactly one y-value (or output) for each x-value (or input). If a table has more than one y-value (or output) for the same x-value (or input), it can't represent a function.

1) **Function** or **Not**?

x	y
0	3
1	2
2	5
3	6
4	5

2) **Function** or **Not**?

ind	dep
5	3
1	4
-3	5
3	6
2	7

3) **Function** or **Not**?

input	output
0	2
5	2
2	2
6	2
3	2

4) **Function** or **Not**?

x	y
1	0
1	1
1	2
1	3
1	4

5) **Function** or **Not**?

tickets	$
2	0
1	2
2	4
3	6
4	8

6) **Function** or **Not**?

input	output
-4	-2
-3	-1
-2	0
-1	1
0	2

7) **Function** or **Not**?

ind	dep
10	9
3	2
9	8
17	16
3	5

8) **Function** or **Not**?

C	F
-40	-40
0	32
10	50
37	98.6
100	212

9) **Function** or **Not**?

input	output
0	7
-1	2
4	3
8	6
-5	-8

10) **Function** or **Not**?

$	games
10	5
11	25
12	45
13	65
14	85

11) **Function** or **Not**?

x	y
8	10
6	5
4	0
6	-5
8	-10

12) **Function** or **Not**?

miles	minutes
0	0
1	2
2	4
3	6
4	8

Identifying Functions from Tables & Graphs

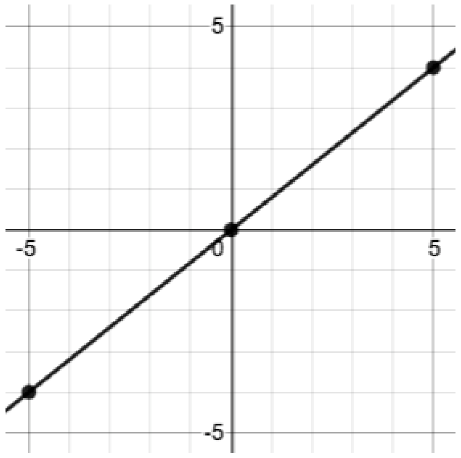
Decide whether or not each table or graph below could represent a function. If not, circle what tells you it's not a function.

In a function, there's exactly one y-value for each x-value. Any table or graph with more than one y-value for the same x-value, can't represent a function.

1) **Function** or **Not a Function**?

x	y
-2	5
0	2
2	4
4	7
6	8

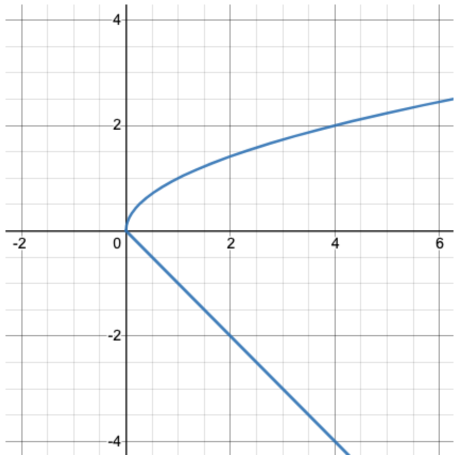
2) **Function** or **Not a Function**?



3) **Function** or **Not a Function**?

x	y
0	7
1	2
1	3
2	6
3	-8

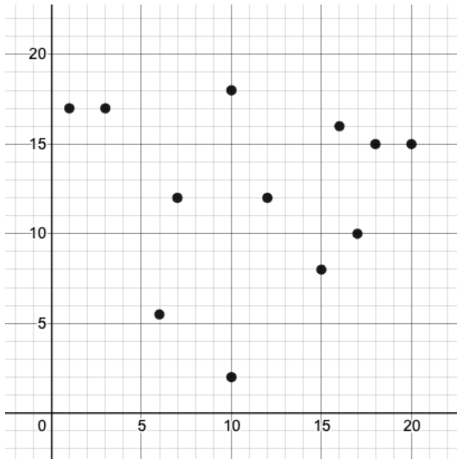
4) **Function** or **Not a Function**?



5) **Function** or **Not a Function**?

x	y
-1.5	-2
-1	-1
-0.5	0
0	1
0.5	2

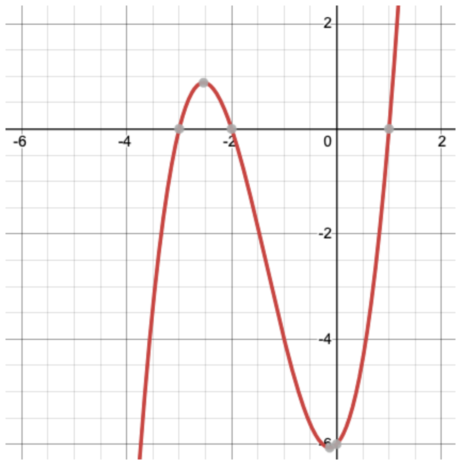
6) **Function** or **Not a Function**?



7) **Function** or **Not a Function**?

x	y
-1	1.5
0	1.5
1	1.5
2	1.5
3	1.5

8) **Function** or **Not a Function**?



9) **Function** or **Not a Function**?

x	y
8	1
5	2
4	3
5	4
8	5

Matching Examples and Definitions (Math)

Match each of the function definitions on the left with the corresponding table on the right.
It may help to circle or highlight what's changing in the $f(x)$ column of the table!

Function Definitions **Example Tables**

$f(x) = x - 2$ **1** **A**

x	$f(x)$
1	2×1
2	2×2
3	2×3

$f(x) = 2x$ **2** **B**

x	$f(x)$
15	$15 - 2$
25	$25 - 2$
35	$35 - 2$

$f(x) = 2x + 1$ **3** **C**

x	$f(x)$
10	$2 + 10$
15	$2 + 15$
20	$2 + 20$

$f(x) = 1 - 2x$ **4** **D**

x	$f(x)$
0	$1 - 2(0)$
1	$1 - 2(1)$
2	$1 - 2(2)$

$f(x) = 2 + x$ **5** **E**

x	$f(x)$
10	$2(10) + 1$
20	$2(20) + 1$
30	$2(30) + 1$

Function Notation - Substitution

Part 1

Complete each **row** of the table below, substituting the given value into the expression and evaluating.

	Function Definition	Expression	Substitution	Evaluates to
1)	$f(x) = x + 2$	$f(3)$	$3 + 2$	5
2)	$g(x) = x - 1$	$g(6)$		
3)	$h(x) = 3x$	$h(4)$		
4)	$k(x) = 2x - 1$	$k(5)$		

Part 2

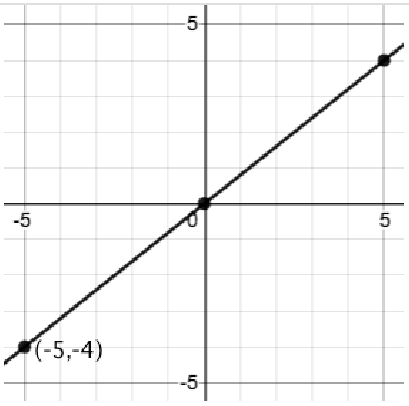
Each **column** below includes four different functions. Beneath each of them are a collection of different expressions for you to evaluate.

5) $m(x) = -2x + 3$	**6)** $n(x) = -x + 7$	**7)** $v(x) = 10x - 8$	**8)** $w(x) = x^2$
$m(3) = -2(3) + 3$	$n(5) =$	$v(7) =$	$w(-2) =$
-3			
$m(-4) =$	$n(-2) =$	$v(0) =$	$w(10) =$
$m(0) =$	$n(3.5) =$	$v(-10) =$	$w(0) =$
$m(0.5) =$	$n(0) =$	$v(2.5) =$	$w(1.5) =$

What do you Notice?	What do you Wonder?

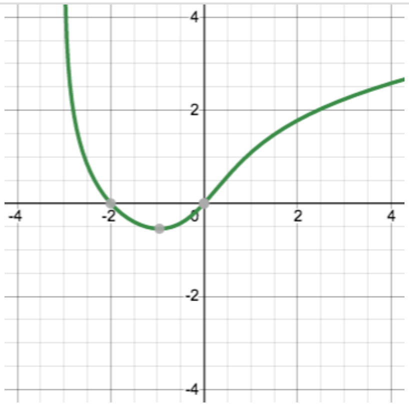
Function Notation - Graphs

For each graph, find the values described by the expressions below. *The first one has been done for you.*



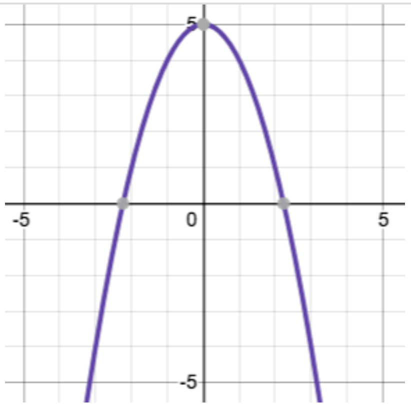
1) $f(-5) =$ -4

2) $f(5) =$ ________



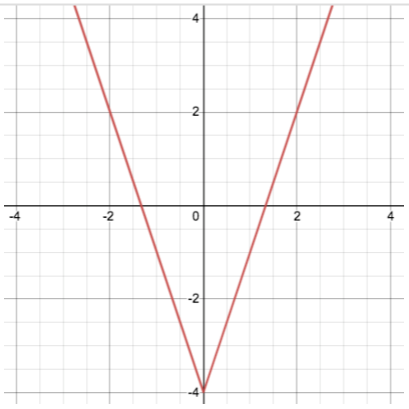
3) $g(-2) =$ ________

4) $g(0) =$ ________



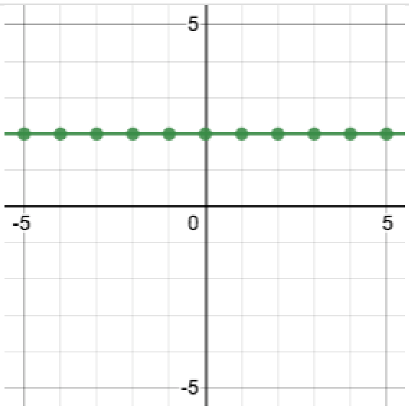
5) $h(0) =$ ________

6) $h(1) =$ ________



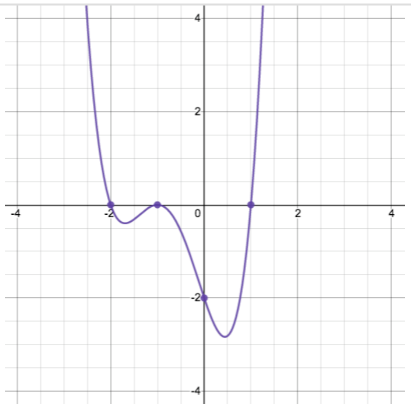
7) $j(-2) =$ ________

8) $j(0) =$ ________



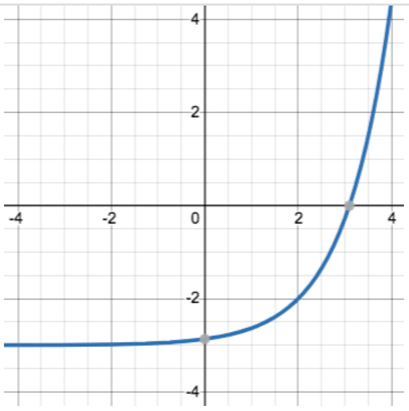
9) $k(3) =$ ________

10) $k(-2.5) =$ ________



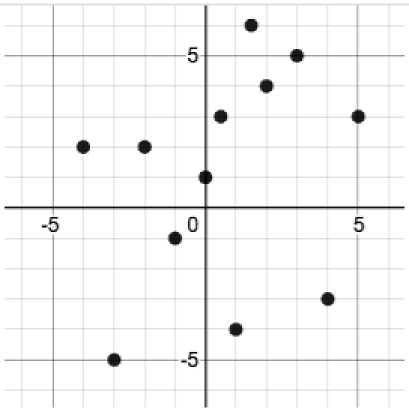
11) $m(0) =$ ________

12) $m(1) =$ ________



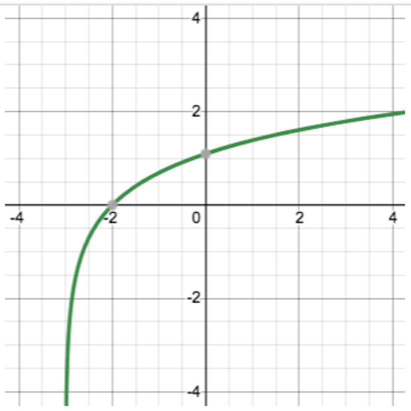
13) $n(2) =$ ________

14) $n(-\infty) \approx$ ________



15) $v(5) =$ ________

16) $v(2) =$ ________



17) $w(-2) =$ ________

18) $w(0) =$ ________

Function Notation - Tables

Find the values described by the expressions below each table.

Note: Not all of the relationships here are actually functions, which means that not all of these expressions can be evaluated!

x	$f(x)$
0	0
1	2
2	4
3	6
4	8

x	$g(x)$
5	3
1	4
-3	5
3	6
2	7

x	$h(x)$
0	2
5	2
2	2
6	2
3	2

x	$y(x)$
1	0
1	1
1	2
1	3
1	4

1) $f(3) =$ 6

2) $f(4) =$ ______

3) $g(1) =$ ______

4) $g(3) =$ ______

5) $h(0) =$ ______

6) $h(3) =$ ______

7) $y(1) =$ ______

8) $y(8) =$ ______

a	$b(a)$
-4	-2
-3	-1
-2	0
-1	1
0	2

c	$d(c)$
0	3
1	2
2	5
3	6
4	5

n	$m(n)$
0	0
-1	-1
-2	-2
-3	-3
-4	-4

q	$p(q)$
2	0
1	2
2	4
3	6
4	8

9) $b(-1) =$ ______

10) $b(0) =$ ______

11) $d(2) =$ ______

12) $d(4) =$ ______

13) $m(0) =$ ______

14) $m(-3) =$ ______

15) $p(1) =$ ______

16) $p(2) =$ ______

s	$r(s)$
0	7
-1	2
4	3
8	6
-5	-8

w	$v(w)$
10	5
11	25
12	45
13	65
14	85

y	$z(y)$
8	10
6	5
4	0
5	-5
8	-10

time	*l(time)*
10	9
3	2
9	8
17	16
5	5

17) $r(-1) =$ ______

18) $r(8) =$ ______

19) $v(11) =$ ______

20) $v(14) =$ ______

21) $z(6) =$ ______

22) $z(2) =$ ______

23) $l(10) =$ ______

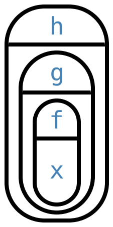
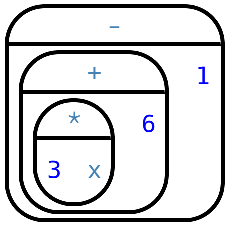
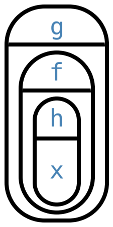
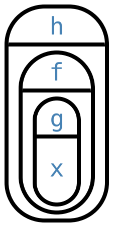
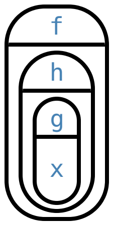
24) $l(3) =$ ______

Diagramming Function Composition

`f :: Number -> Number`
`Consumes a number, multiplies by 3 to produce the result`
	`g :: Number -> Number`
	`Consumes a number, adds six to produce the result`
		`h :: Number -> Number`
		`Consumes a number, subtracts one to produce the result`

$f(x) = 3x$	$g(x) = x + 6$	$h(x) = x - 1$

For each function composition diagrammed below, translate it into the equivalent Circle of Evaluation for Order of Operations. Then write expressions for *both* versions of the Circles of Evaluation, and evaluate them for $x = 4$. The first one has been completed for you.

	Function Composition	Order of Operations	Translate & Evaluate	
1			Composition:	`(h (g (f x)))`
			Operations:	`(- (+ (* 3 x) 6) 1)`
			Evaluate for x = 4	$h(g(f(4))) = ((3 \times 4) + 6) - 1 = 17$
2			Composition:	
			Operations:	
			Evaluate for x = 4	
3			Composition:	
			Operations:	
			Evaluate for x = 4	
4			Composition:	
			Operations:	
			Evaluate for x = 4	

Function Notation Challenge

$f(x) = 2x - 3$	$g(x) = 3x + 2$	$h(x) = x^2$	$k(x) = 2^x$

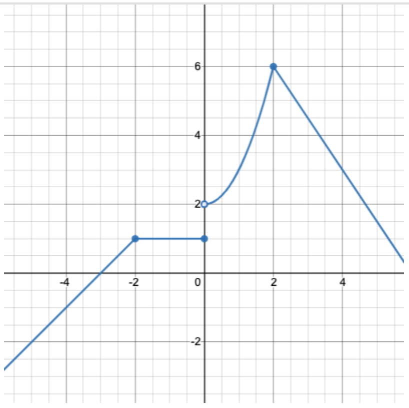
Evaluate each expression below using the function definitions above.

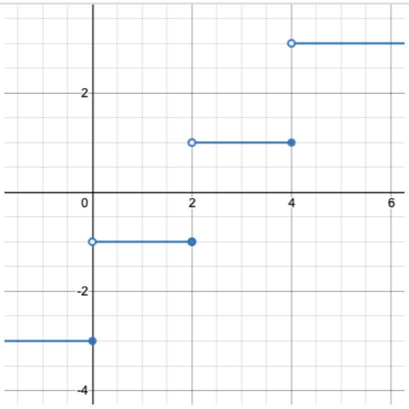
1) $f(4) =$	2) $f(4) - 3 =$
3) $f(4 - 3) =$	4) $g(4) + h(4) =$
5) $3 - f(5) =$	6) $h(3) - k(3) =$
7) $f(-5) =$	8) $g(\frac{1}{3}) =$
9) $5 \times g(4) =$	10) $h(4) + f(6) - 5 =$
11) $h(2) - 5 =$	12) $h(2 - 5) =$
13) $k(4 - 1) =$	14) $k(4) - 1 =$

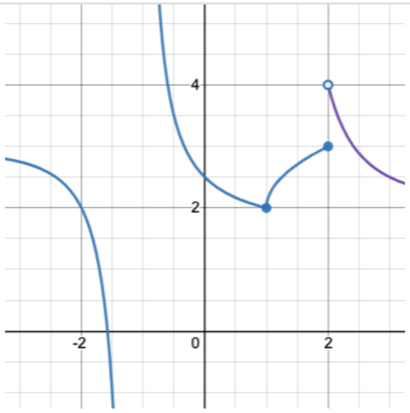
(optional)

Function Notation - Piecewise Graphs

Find the values described by the expressions below each graph. *The first one has been done for you.*







1) $f(0) =$ ___1___

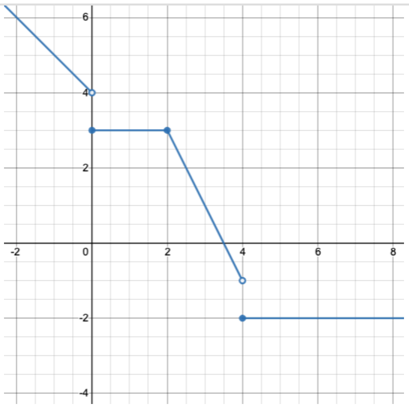
2) $f(-2) =$ __________

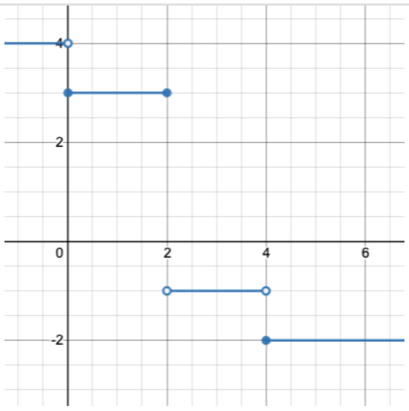
3) $g(1) =$ __________

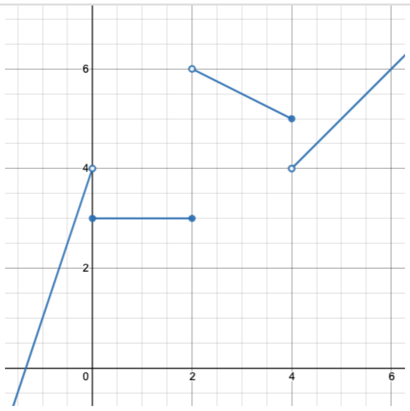
4) $g(4) =$ __________

5) $h(1) =$ __________

6) $h(2) =$ __________







7) $j(0) =$ __________

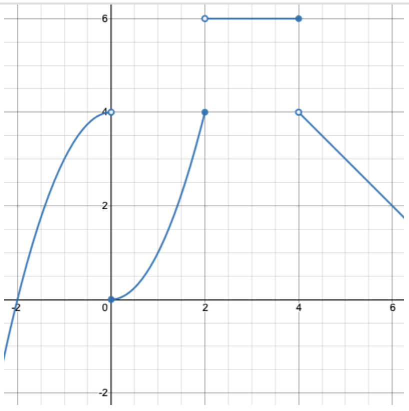
8) $j(4) =$ __________

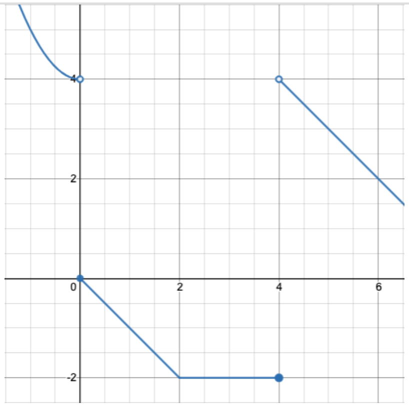
9) $k(1) =$ __________

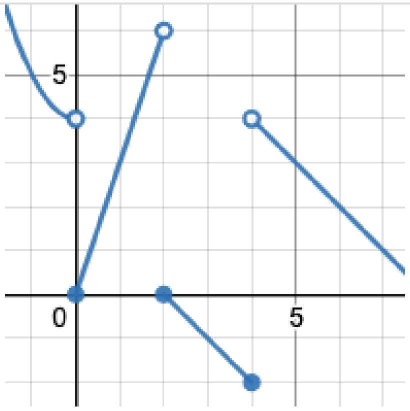
10) $k(3) =$ __________

11) $m(2) =$ __________

12) $m(4) =$ __________







13) $n(2) =$ __________

14) $n(5) =$ __________

15) $v(3) =$ __________

16) $v(4) =$ __________

17) $w(1) =$ __________

18) $w(4) =$ __________

(optional)

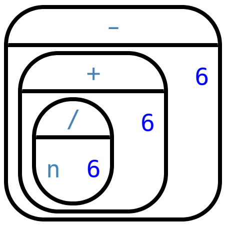
Function Composition: Matching

`g :: Number -> Number` `Consumes a number, multiplies by 6 to produce the result`	`h :: Number -> Number` `Consumes a number, subtracts 6 to produce the result`	`j :: Number -> Number` `Consumes a number, adds 6 to produce the result`	`k :: Number -> Number` `Consumes a number, divides by 6 to produce the result`
$g(n) = n \times 6$	$h(n) = n - 6$	$j(n) = n + 6$	$k(n) = n \div 6$

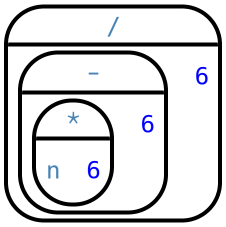
Draw a line from each expression on the left to the corresponding Circle of Evaluation on the right.

Function Notation **Circle of Evaluation**

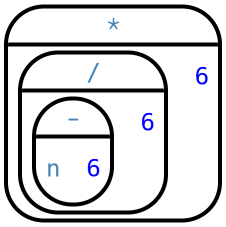
$g(h(j(n)))$ **1** **A**



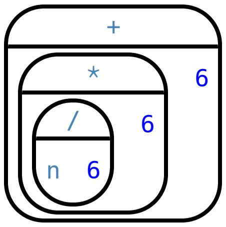
$h(j(k(n)))$ **2** **B**



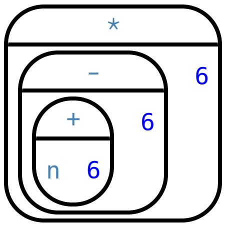
$g(k(h(n)))$ **3** **C**



$k(h(g(n)))$ **4** **D**



$j(g(k(n)))$ **5** **E**



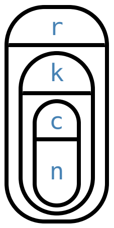
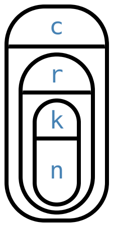
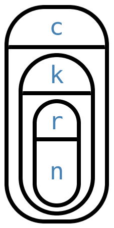
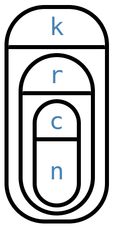
(optional)

Diagramming Function Composition (2)

`m :: Number -> Number`
`Consumes a number. divides by 2 to produce the result`
	`r :: Number -> Number`
	`Consumes a number. subtracts 5 to produce the result`
		`w :: Number -> Number`
		`Consumes a number. adds 4 to produce the result`

$k(n) = n \div 2$	$r(n) = n - 5$	$c(n) = n + 4$

For each function composition diagrammed below, translate it into the equivalent Circle of Evaluation for Order of Operations. Then write expressions for *both* versions of the Circles of Evaluation, and evaluate them for $n = 7$.

	Function Composition	Order of Operations	Translate & Evaluate	
1			Composition:	
			Operations:	
			Evaluate for n = 7	
2			Composition:	
			Operations:	
			Evaluate for n = 7	
3			Composition:	
			Operations:	
			Evaluate for n = 7	
4			Composition:	
			Operations:	
			Evaluate for n = 7	

(optional)

Matching Examples and Contracts

Match each set of examples (left) with the Contract that best describes it (right).

Examples | **Contract**

```
(EXAMPLE (f 5) (/ 5 2))
(EXAMPLE (f 9) (/ 9 2))
(EXAMPLE (f 24) (/ 24 2))
```

1 — A `; f :: Number -> Number`

```
(EXAMPLE (f 1)
  (rectangle 1 1 "outline" "red"))
(EXAMPLE (f 6)
  (rectangle 6 6 "outline" "red"))
```

2 — B `; f :: String -> Image`

```
(EXAMPLE (f "pink" 5)
  (star 5 "solid" "pink"))
(EXAMPLE (f "blue" 8)
  (star 8 "solid" "blue"))
```

3 — C `; f :: Number -> Image`

```
(EXAMPLE (f "Hi!") (text "Hi!" 50 "red"))
(EXAMPLE (f "Ciao!") (text "Ciao!" 50 "red"))
```

4 — D `; f :: Number, String -> Image`

```
(EXAMPLE (f 5 "outline")
  (star 5 "outline" "yellow"))
(EXAMPLE (f 5 "solid")
  (star 5 "solid" "yellow"))
```

5 — E `; f :: String, Number -> Image`

Matching Examples and Function Definitions

(1) Find the variables in `gt` and label them with the word "size".

```
(EXAMPLE (gt 20) (triangle 20 "solid" "green"))
(EXAMPLE (gt 50) (triangle 50 "solid" "green"))
(define (gt size) (gt size "solid" "green"))
```

(2) Highlight and label the variables in the example lists below.

(3) Then, using `gt` as a model, match the examples to their corresponding function definitions.

Examples			Definition
`(EXAMPLE (f "solid") (circle 8 "solid" "red"))` `(EXAMPLE (f "outline") (circle 8 "outline" "red"))`	**1**	**A**	`(define (f s) (star s "outline" "red"))`
`(EXAMPLE (f 2) (+ 2 2))` `(EXAMPLE (f 4) (+ 4 4))` `(EXAMPLE (f 5) (+ 5 5))`	**2**	**B**	`(define (f num) (+ num num))`
`(EXAMPLE (f "red") (circle 7 "solid" "red"))` `(EXAMPLE (f "teal") (circle 7 "solid" "teal"))`	**3**	**C**	`(define (f c) (star 9 "solid" c))`
`(EXAMPLE (f "red") (star 9 "solid" "red"))` `(EXAMPLE (f "grey") (star 9 "solid" "grey"))` `(EXAMPLE (f "pink") (star 9 "solid" "pink"))`	**4**	**D**	`(define (f s) (circle 8 s "red"))`
`(EXAMPLE (f 3) (star 3 "outline" "red"))` `(EXAMPLE (f 8) (star 8 "outline" "red"))`	**5**	**E**	`(define (f c) (circle 7 "solid" c))`

Creating Contracts From Examples

Write the contracts used to create each of the following collections of examples. The first one has been done for you.

1) `; big-triangle :: Number, String -> Image`

```
(EXAMPLE (big-triangle 100 "red")
  (triangle 100 "solid" "red"))
(EXAMPLE (big-triangle 200 "orange")
  (triangle 200 "solid" "orange"))
```

2) ______________________________

```
(EXAMPLE (purple-square 15)
  (rectangle 15 15 "outline" "purple"))
(EXAMPLE (purple-square 6)
  (rectangle 6 6 "outline" "purple"))
```

3) ______________________________

```
(EXAMPLE (sum 5 8) (+ 5 8))
(EXAMPLE (sum 9 6) (+ 9 6))
(EXAMPLE (sum 120 11) (+ 120 11))
```

4) ______________________________

```
(EXAMPLE (banner "Game Today!")
  (text "Game Today!" 50 "red"))
(EXAMPLE (banner "Go Team!")
  (text "Go Team!" 50 "red"))
(EXAMPLE (banner "Exit")
  (text "Exit" 50 "red"))
```

5) ______________________________

```
(EXAMPLE (twinkle "outline" "red")
  (star 5 "outline" "red"))
(EXAMPLE (twinkle "solid" "pink")
  (star 5 "solid" "pink"))
(EXAMPLE (twinkle "outline" "grey")
  (star 5 "outline" "grey"))
```

6) ______________________________

```
(EXAMPLE (half 5) (/ 5 2))
(EXAMPLE (half 8) (/ 8 2))
(EXAMPLE (half 900) (/ 900 2))
```

7) ______________________________

```
(EXAMPLE (Spanish 5) "cinco")
(EXAMPLE (Spanish 30) "treinta")
(EXAMPLE (Spanish 12) "doce")
```

Contracts, Examples & Definitions - bc

We've already found the Contract for gt, made Examples, and described the pattern with a Definition. Let's review the process.
Directions: Define a function called `gt`, which makes solid green triangles of whatever size we want.

Contract and Purpose Statement

Every contract has three parts…

; gt (function name) : *Number* (Domain) -> *Image* (Range)

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (*gt* (function name) *10* (input(s)) *)* *(triangle 10 "solid" "green")* (what the function produces))

(EXAMPLE (*gt* (function name) *20* (input(s)) *)* *(triangle 20 "solid" "green")* (what the function produces))

Definition

Write the definition, giving variable names to all your input values…

(define (*gt* (function name) *size* (variable(s)))

(triangle size "solid" "green") (what the function does with those variable(s)))

Now, let's apply the same steps to think through a new problem!
Directions: Define a function called `bc`, which makes solid blue circles of whatever radius we want.

Contract and Purpose Statement

Every contract has three parts…

; ________ (function name) : ________ (Domain) -> ________ (Range)

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (________ (function name) ________ (input(s)) *)* ________ (what the function produces))

(EXAMPLE (________ (function name) ________ (input(s)) *)* ________ (what the function produces))

Definition

Write the definition, giving variable names to all your input values…

(define (________ (function name) ________ (variable(s)))

________ (what the function does with those variable(s)))

Contracts, Examples & Definitions - Stars

Directions: Define a function called `sticker`, which consumes a color and draws a solid 50px star of the given color.

Contract and Purpose Statement

Every contract has three parts…

; ______________________ : ____ -> ______________

function name — Domain — Range

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________________________________ *)* ______________________________________)

function name — input(s) — what the function produces

(EXAMPLE (______________________________________ *)* ______________________________________)

function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)

function name — variable(s)

____)

what the function does with those variable(s)

Directions: Define a function called `gold-star`, which takes in a radius and draws a solid gold star of that given size.

Contract and Purpose Statement

Every contract has three parts…

; ______________________ : ____ -> ______________

function name — Domain — Range

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________________________________ *)* ______________________________________)

function name — input(s) — what the function produces

(EXAMPLE (______________________________________ *)* ______________________________________)

function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)

function name — variable(s)

____)

what the function does with those variable(s)

Contracts, Examples & Definitions - Name

Directions: Define a function called `name-color`, which makes an image of your name at size 50 in whatever color is given.

Contract and Purpose Statement

Every contract has three parts…

; ____________ : ________________________ -> ____________

function name | Domain | Range

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*________________________*)* ________________________)

function name | input(s) | what the function produces

*(EXAMPLE (*________________________*)* ________________________)

function name | input(s) | what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____________________________________)

function name | variable(s)

____)

what the function does with those variable(s)

Directions: Define a function called `name-size`, which makes an image of your name in your favorite color (be sure to specify your name and favorite color!) in whatever size is given.

Contract and Purpose Statement

Every contract has three parts…

; ____________ : ________________________ -> ____________

function name | Domain | Range

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*________________________*)* ________________________)

function name | input(s) | what the function produces

*(EXAMPLE (*________________________*)* ________________________)

function name | input(s) | what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____________________________________)

function name | variable(s)

____)

what the function does with those variable(s)

Do the Examples Have the Same Contracts?

For each pair of Examples below, decide whether the two examples have the same Contract. If they do, fill in the Contract in the space provided. If not, write a few words explaining how you know their contracts aren't the same.

1) ____

```
(EXAMPLE (mystery 30) (* 30 50))
(EXAMPLE (mystery 10)
  (text "Welcome!" 10 "darkgreen"))
```

2) ____

```
(EXAMPLE (mystery 30 40) (- 40 (* 2 30)))
(EXAMPLE (mystery 10 15) (- 15 (* 2 10)))
```

3) ____

```
(EXAMPLE (mystery "New York")
  (text "New York" 20 "red"))
(EXAMPLE (mystery 20)
  (text "New York" 20 "red"))
```

4) ____

```
(EXAMPLE (mystery "green" 32)
  (circle 32 "outline" "green"))
(EXAMPLE (mystery 18 "green")
  (circle 18 "outline" "green"))
```

5) ____

```
(EXAMPLE (mystery 6 9 10) (/ 6 (+ 9 10)))
(EXAMPLE (mystery 3 7) (/ 3 (+ 7 10)))
```

6) ____

```
(EXAMPLE (mystery "red" "blue")
  (text "blue" 25 "red"))
(EXAMPLE (mystery "purple" "Go Team!")
  (text "Go Team!" 25 "purple"))
```

Do the Examples Have the Same Contracts? (2)

For each pair of Examples below, decide whether the two examples have the same Contract. If they do, fill in the Contract in the space provided. If not, write a few words explaining how you know their contracts aren't the same.

1) ____

```
(EXAMPLE (mystery (triangle 70 "solid" "green"))
  (triangle 140 "solid" "green"))
(EXAMPLE (mystery (circle 100 "solid" "blue"))
  (circle 200 "solid" "blue"))
```

2) ____

```
(EXAMPLE (mystery "red")
  (triangle 140 "solid" "red"))
(EXAMPLE (mystery "blue" "circle")
  (circle 140 "solid" "blue"))
```

3) ____

```
(EXAMPLE (mystery "" 4 5) ( 4 5))
(EXAMPLE (mystery "sqrt" 25) (sqrt 25))
```

4) ____

```
(EXAMPLE (mystery "circle" 4) (* pi (sqr 4)))
(EXAMPLE (mystery "square" 5) (sqr 5))
```

5) ____

```
(EXAMPLE (mystery "dog") 3)
(EXAMPLE (mystery "cat") "kitten")
```

6) ____

```
(EXAMPLE (mystery "dog") 3)
(EXAMPLE (mystery "kitten") 6)
```

Matching Examples and Contracts (2)

Match each Example on the left with its Contract on the right. NOTE: Multiple examples may match to the same Contract!

Contract			Examples

`(EXAMPLE (match (circle 10 "solid" "green"))`
`  (rotate 37 (circle 10 "solid" "green")))`
	1	A	`; match :: Number, Image -> Image`

`(EXAMPLE (match (triangle 20 "solid" "blue") 3)`
`  (scale 3 (triangle 20 "solid" "blue")))`
	2		

`(EXAMPLE (match (circle 20 "outline" "gold"))`
`  (rotate 37 (circle 20 "outline" "gold")))`
	3	B	`; match :: Image, Number -> Image`

`(EXAMPLE (match 30 "red")`
`  (+ 30 (string-length "red")))`
	4		

`(EXAMPLE (match (circle 10 "solid" "orange")`
`22)`
`  (scale 22 (circle 10 "solid" "orange")))`
	5		

`(EXAMPLE (match 10 "blue")`
`  (+ 10 (string-length "blue")))`
	6	C	`; match :: Image -> Image`

`(EXAMPLE (match 5 (star 20 "solid" "red"))`
`  (rotate (- 90 5) (star 20 "solid" "red")))`
	7		

`(EXAMPLE (match (abs -4) "45") 4)`	8	D	`; match :: Number, String -> Number`

(optional)

Matching Examples and Contracts (3)

Match each Example on the left with its Contract on the right. NOTE: Multiple examples may match to the same Contract!

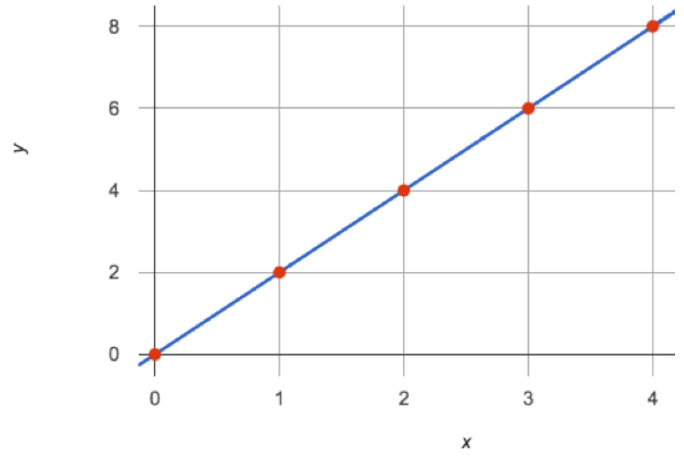
Contract			Examples
`(EXAMPLE (match 1.5) "greater than 1")`	1		
`(EXAMPLE (match 24) (star (* 24 2) "outline" "purple"))`	2		
`(EXAMPLE (match (string-length "tabletop")) "8")`	3	A	`; match :: Number -> String`
`(EXAMPLE (match (star 20 "outline" "red") 3) (* 3 (image-height (star 20 "outline" "red"))))`	4	B	`; match :: Number -> Image`
`(EXAMPLE (match (circle 10 "solid" "silver") 16) (* 16 (image-height (circle 10 "solid" "silver"))))`	5	C	`; match :: Number, Number -> Number`
`(EXAMPLE (match "triangle" "blue") (triangle 40 "outline" "blue"))`	6	D	`; match :: String, String -> Image`
`(EXAMPLE (match 30) (star (* 30 2) "outline" "purple"))`	7	E	`; match :: Image, Number -> Number`
`(EXAMPLE (match (string-length "coffee") (string-length "tea")) (+ 6 3))`	8		

(optional)

Notice and Wonder (Linearity)

Part 1

x	y
0	0
1	2
2	4
3	6
4	8



What do you Notice?	What do you Wonder?

Part 2

- What is the y-value for each table when x is 0?
- What is the next pair for each of these tables?

x	y
0	
1	2
2	3
3	4
4	5
5	6

independent	dependent
0	
1	20
2	17
3	14
4	11
5	8

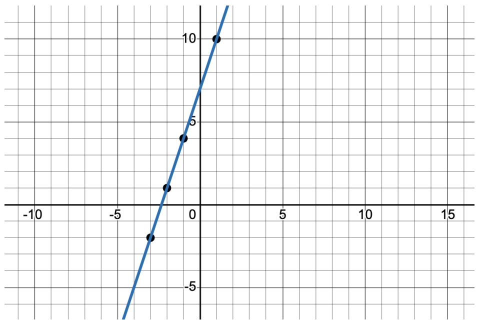
Matching Tables to Graphs

For each of the tables below, find the graph that matches.
Note: The scales on the graphs are not the same! Look at the axes to help you find the right match!

x	-1	0	1	2	2
y	4	7	10	13	16

1

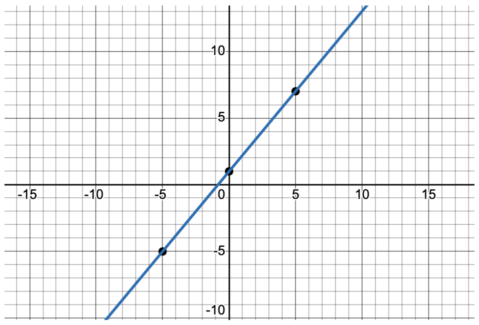
A



x	-5	-4	-3	-2	-1
y	9	8	7	5	5

2

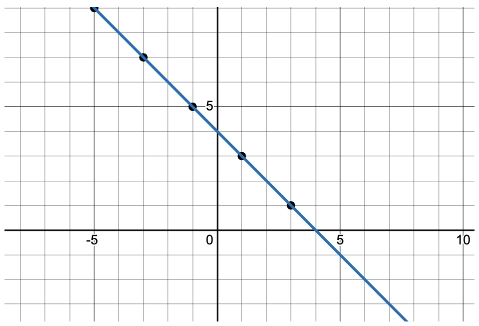
B



x	-2	-1	0	1	2
y	-10	-7	-4	-1	2

3

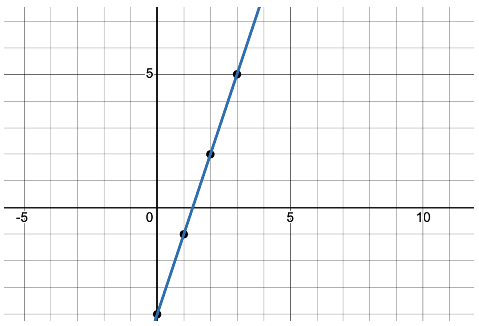
C



x	0	1	2	3	4
y	1	2.2	3.6	4.8	6

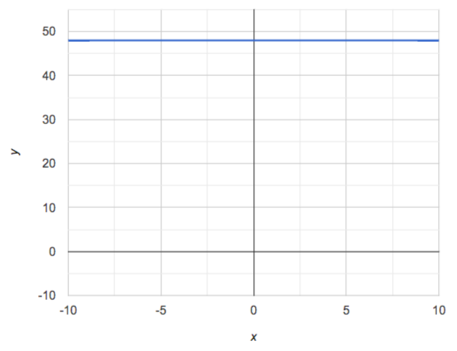
4

D

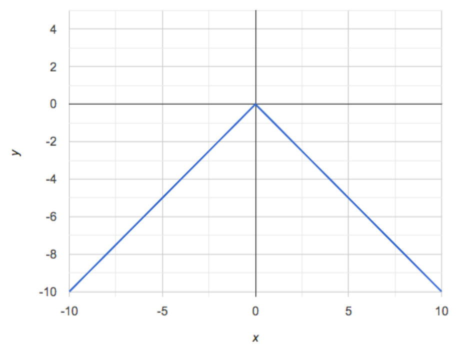


Are All Graphs Linear?

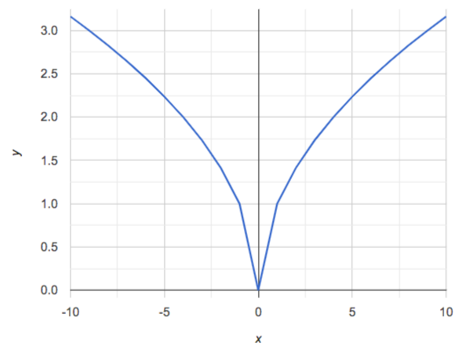
Beneath each graph circle **Linear** or **Not Linear**.



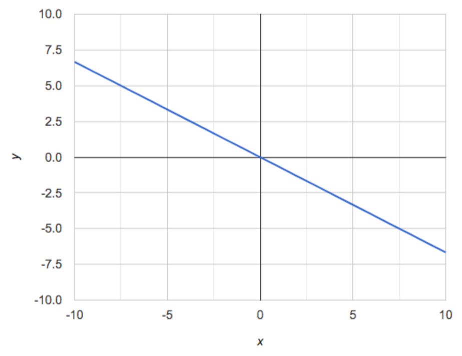
1) **Linear** or **Not Linear**?



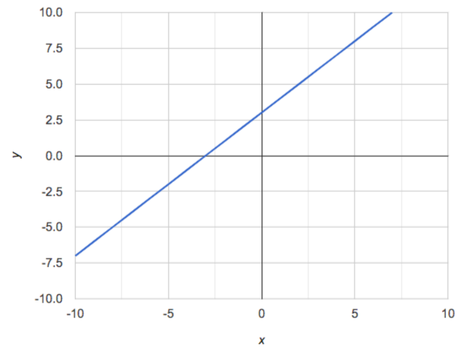
2) **Linear** or **Not Linear**?



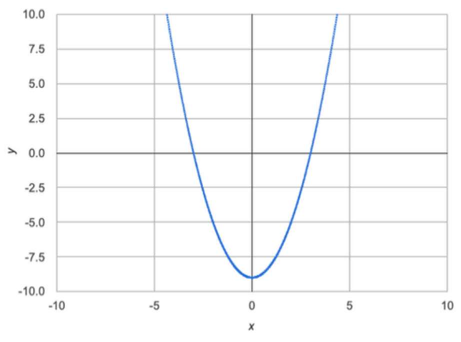
3) **Linear** or **Not Linear**?



4) **Linear** or **Not Linear**?



5) **Linear** or **Not Linear**?



6) **Linear** or **Not Linear**?

What do you Notice?	**What do you Wonder?**

Are All Tables Linear?

Look at the six tables shown below.

1) Extend as many of the tables as you can by adding the next (x,y) pair in the sequence.

2) If the table is linear, write down your prediction of what the y-value will be when x = 0.

3) If the table is not linear, write **not linear** instead of an answer for y.

A

x	-2	-1	0	1	2	
y	-2	-3	-4	-5	-6	

when x=0, y will equal ______________________

B

x	2	4	6	8	10	
y	-12	-16	-20	-24	-28	

when x=0, y will equal ______________________

C

x	1	2	3	4	5	
y	1	4	9	16	25	

when x=0, y will equal ______________________

D

x	5	6	7	8	9	
y	3	3	3	3	3	

when x=0, y will equal ______________________

E

x	1	2	3	4	5	
y	84	94	104	114	124	

when x=0, y will equal ______________________

F

x	-10	-9	-8	-7	-6	
y	$^{-1}/_{10}$	$^{-1}/_{9}$	$^{-1}/_{8}$	$^{-1}/_{7}$	$^{-1}/_{6}$	

when x=0, y will equal ______________________

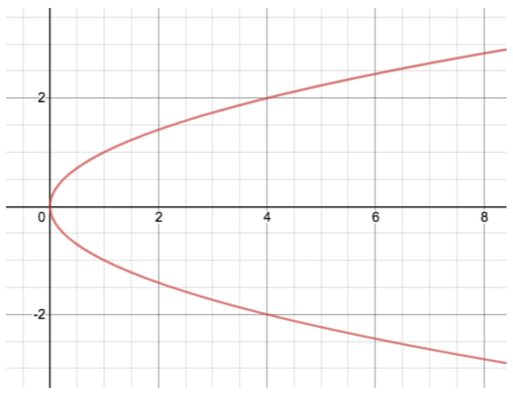
What do you Notice?	What do you Wonder?

Linear, Non-linear, or Bust?

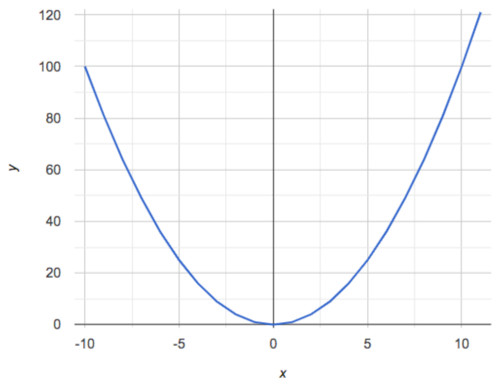
Circle whether each representation is of a linear function, a nonlinear function or is not a function at all!
Remember: Functions will pass the Vertical Line Test, meaning they'll have exactly one y-value for each x-value!

x	y
1	5
2	10
3	15
4	20
5	25
6	30
7	35

1) Linear Nonlinear Not a Function



2) Linear Nonlinear Not a Function



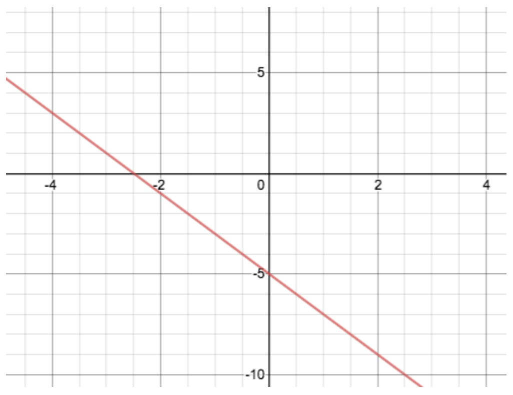
3) Linear Nonlinear Not a Function

x	y
1	1
2	4
3	9
4	16
5	25
6	36
7	49

4) Linear Nonlinear Not a Function

x	y
1	1
2	2
3	3
4	4
4	5
6	6
7	9

5) Linear Nonlinear Not a Function



6) Linear Nonlinear Not a Function

Slope & y-Intercept from Tables (Intro)

slope (rate): *how much y changes as x-increases by 1*
y-intercept: *the y-value when $x = 0$*

x	-1	0	1	2	3	4
y	-1	1	3	5	7	9

1) Compute the slope: ______________________

2) Compute the y-intercept: ______________________

3) What strategies did you use to compute the slope and y-intercept?

____

____

The slope and y-intercept in this table are harder to find, because the x-values don't go up by 1 and we can't see a value for $x = 0$.
Try filling in the points that have been skipped to compute the slope and y-intercept.

x	3	6	9	12
y	4	9	14	19

4) Compute the slope: ______________________

5) Compute the y-intercept: ______________________

The slope and y-intercept in this table are even harder to find, because the x-values are out of order!
Calculate the slope and y-intercept from *any* two points! Be sure to show your work.

x	3	20	5	9	1
y	5	56	11	23	-1

6) Compute the slope: ______________________

7) Compute the y-intercept: ______________________

Slope & y-Intercept from Tables (Practice)

x	-1	0	1	2	3	4
y	-1	2	5	8	11	14

1) slope: ______________________ y-intercept: ______________________

x	-2	-1	0	1	2	3
y	17	11	5	-1	-7	-13

2) slope: ______________________ y-intercept: ______________________

x	-3	-2	-1	0	1	2
y	0	$\frac{2}{3}$	$1\frac{1}{3}$	2	$2\frac{2}{3}$	$3\frac{1}{3}$

3) slope: ______________________ y-intercept: ______________________

x	-1	0	1	2	3	4
y	-7	-3	1	5	9	13

4) slope: ______________________ y-intercept: ______________________

x	-5	-4	-3	-2	-1	0
y	1	2.5	4	5.5	7	8.5

5) slope: ______________________ y-intercept: ______________________

x	-4	-3	- 2	-1	0	1
y	0	0.6	1.2	1.8	2.4	3

6) slope: ______________________ y-intercept: ______________________

x	1	2	3	4	5	6
y	5	3	1	-1	-3	-5

7) slope: ______________________ y-intercept: ______________________

x	-4	-2	0	2	4	6
y	0	4	8	12	16	20

★ slope: ______________________ y-intercept: ______________________

Identifying Slope in Tables

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

Can you identify the **slope** for the functions represented in each of these tables?
Note: Some tables may have their rows out of order!

1

x	y
-1	-3
4	12
8	21
9	24

slope/rate: ______________________

2

x	y
-5	35
-3	21
0	0
5	-35

slope/rate: ______________________

3

x	y
12	15
17	17
13	15.4
20	18.2

slope/rate: ______________________

4

x	y
1	39
4	31.5
3	34
7	24

slope/rate: ______________________

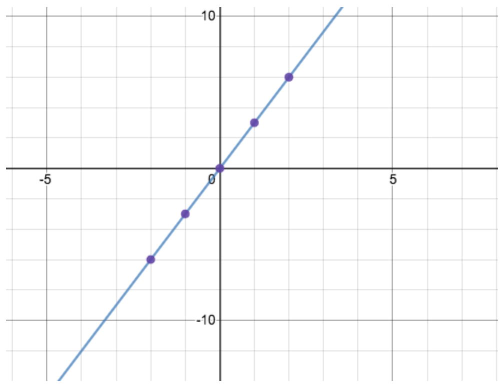
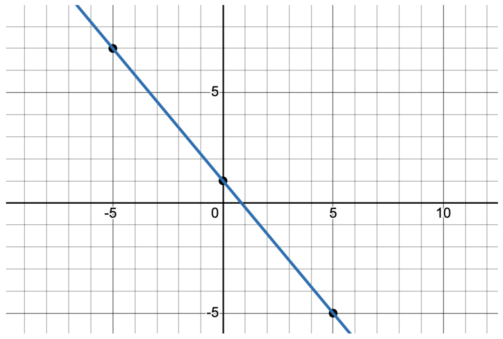
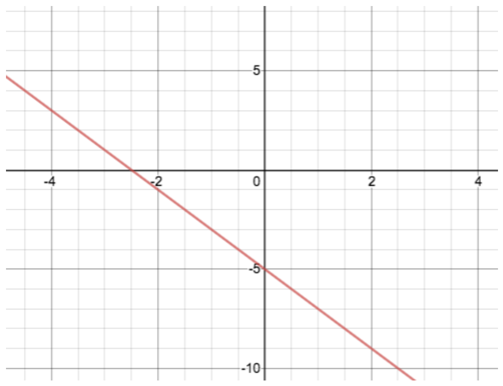
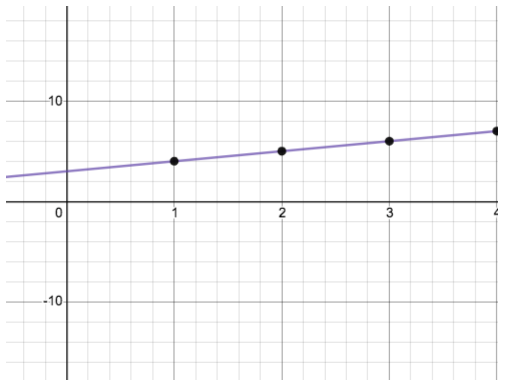
5

x	y
13	57
0	41.4
8	51
-2	39

slope/rate: ______________________

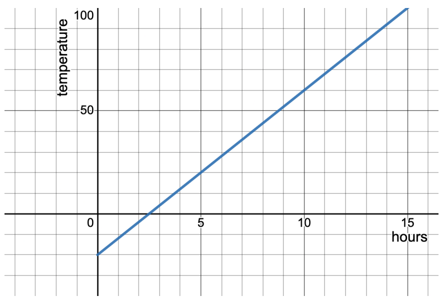
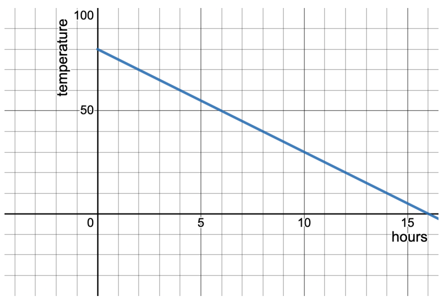
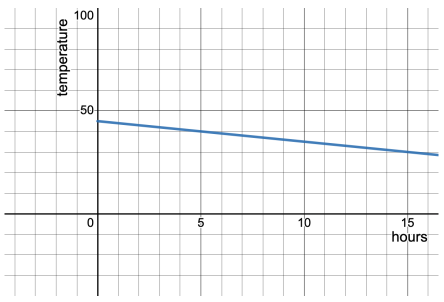
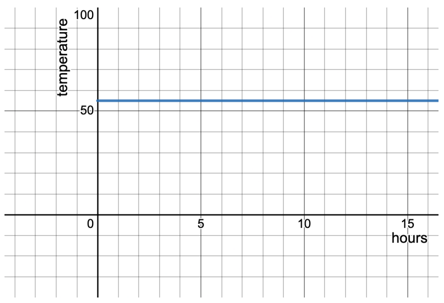
Identifying Slope and y-intercept in Graphs

Can you identify the **slope** and **y-intercept** for each of these graphs?

1		slope/rate: ________________ y-intercept: ________________
2		slope/rate: ________________ y-intercept: ________________
3		slope/rate: ________________ y-intercept: ________________
4		slope/rate: ________________ y-intercept: ________________

What Story does the Graph tell?

For each of the Graphs below, write the story that it tells. *(The first one has been done for you.)*

1		The temperature started at -20 degrees and increased by 8 degrees per hour.
2		
3		
4		

What Story does the Table tell?

For each of the Tables below, write the story that it tells.

1

maple syrup produced (gallons)	0	1	2	3	4
gallons of sap boiled	0	40	80	120	160

2

seconds on stove	0	10	20	30	40	50
water temp in deg F	50	59	68	77	86	95

3

tickets sold	0	10	20	30	40
profit in dollars	-560	-360	-160	40	240

4

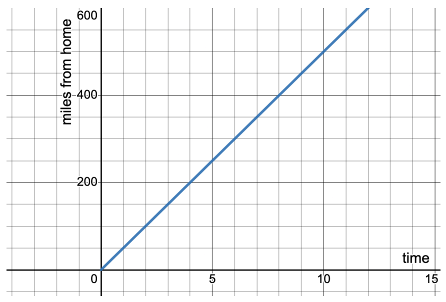
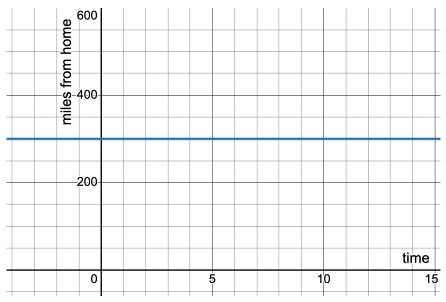
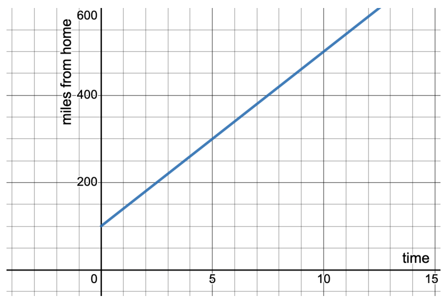
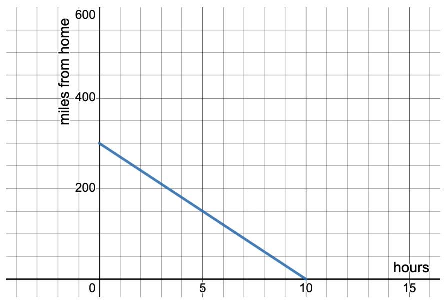
bowls served	0	10	20	30	40
gallons of gumbo in the pot	19	18	17	16	15

5

month	1	2	3	4	5	6	7	8	9	10	11	12
hours of daylight in Berlin	8.3	9.8	11.9	13.8	15.8	16.9	16.4	14.8	12.8	10.8	8.8	7.8

What Story does the Graph tell? (Miles from Home)

For each of the Graphs below, write the story that it tells.

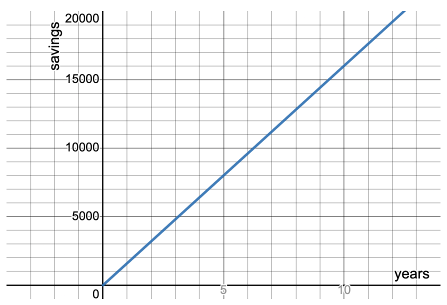
1		
2		
3		
4		

(optional)

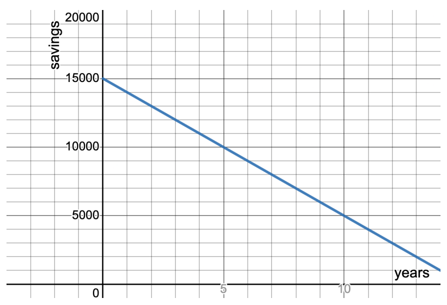
What Story does the Graph tell? (Savings)

For each of the Graphs below, write the story that it tells.

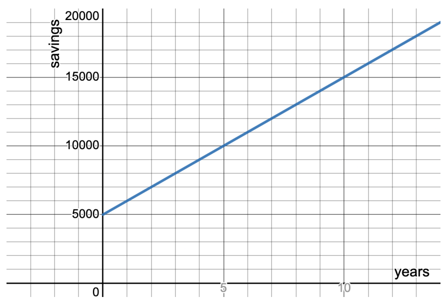
1



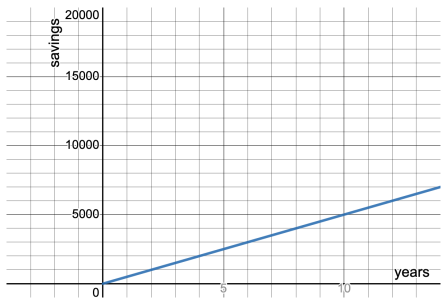
2



3



4

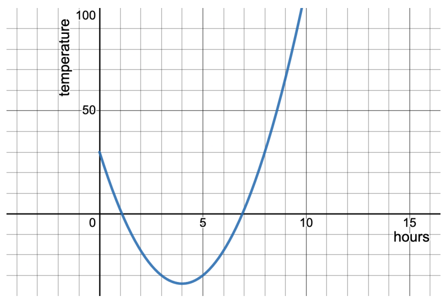


(optional)

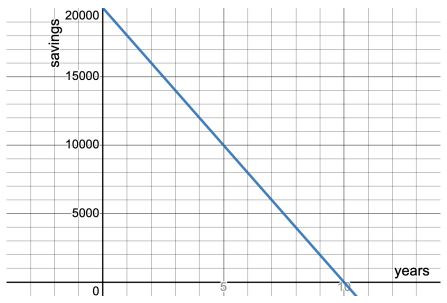
What Story does the Graph tell? (Challenge)

For each of the Graphs below, write the story that it tells.

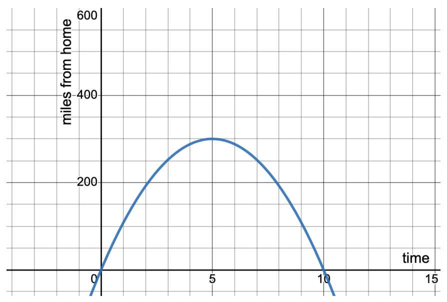
1



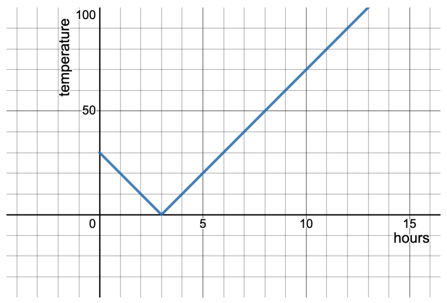
2



3



4



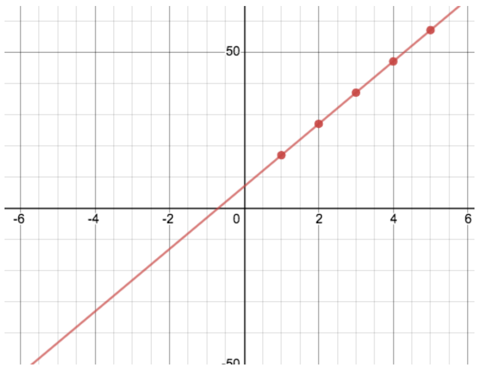
(optional)

Matching Tables to Graphs (Challenge)

For each of the tables below, find the graph that matches. **Note:** The tables are shown sideways to save space.
Note: The scales on the graphs are not the same! Look at the axes to help you find the right match!

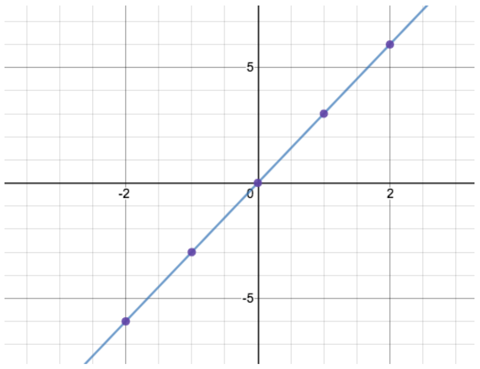
x	-3	-4	-1	-5	-2
y	3	4	1	5	2

1

A


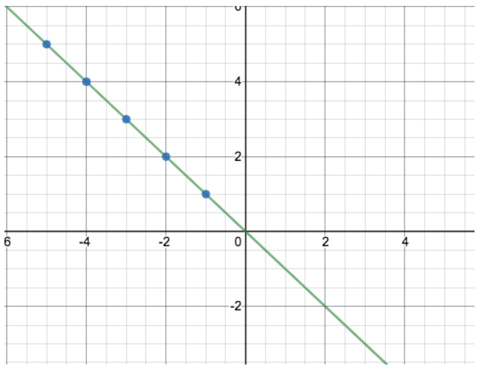
x	4	1	3	5	2
y	7	4	6	8	5

2

B


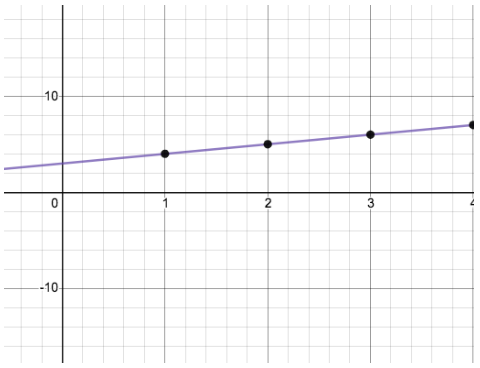
x	3	4	5	2	1
y	37	47	57	27	17

3

C


x	3	5	2	1	4
y	9	15	6	3	12

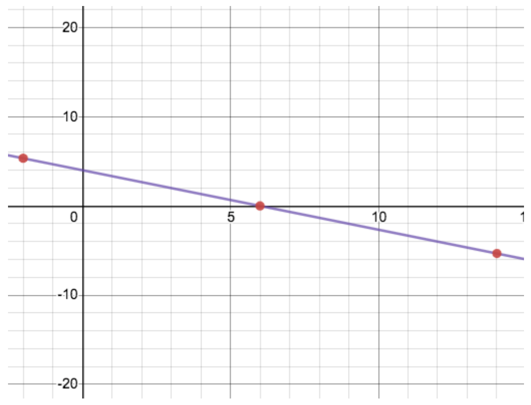
4

D


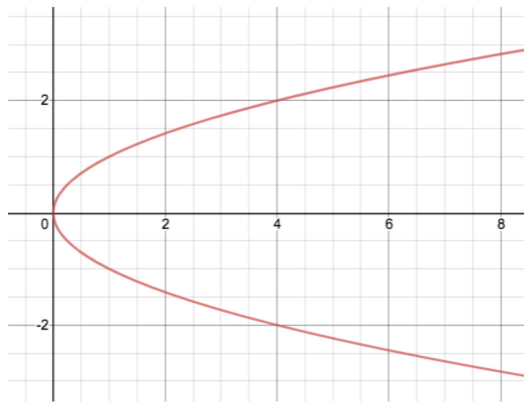
(optional)

Graphs: Linear, Non-linear, or Bust?

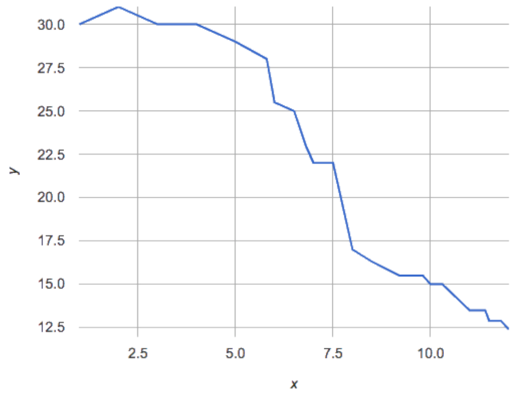
Decide whether each representation is of a linear function, a nonlinear function or is not a function at all!



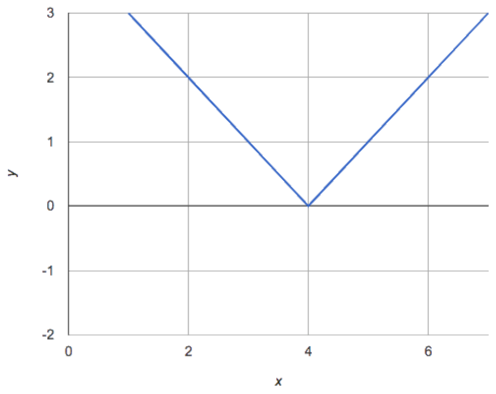
1) Linear Nonlinear Not a Function



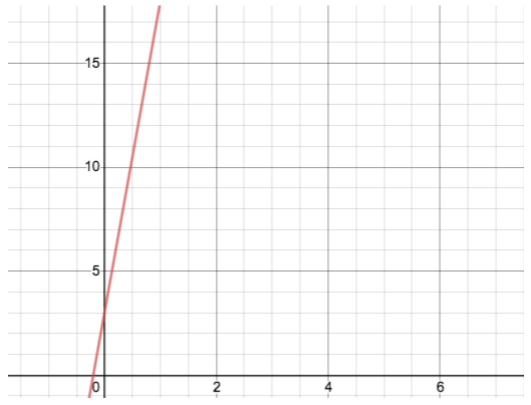
2) Linear Nonlinear Not a Function



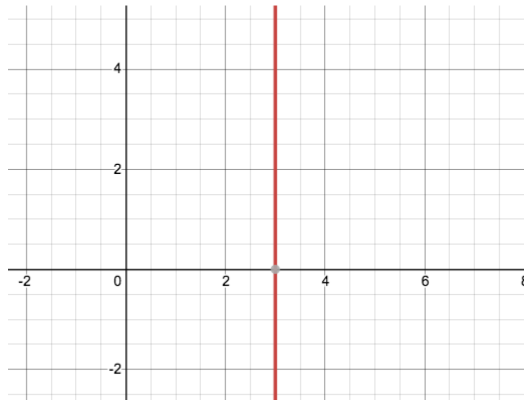
3) Linear Nonlinear Not a Function



4) Linear Function Nonlinear Not a Function



5) Linear Nonlinear Not a Function

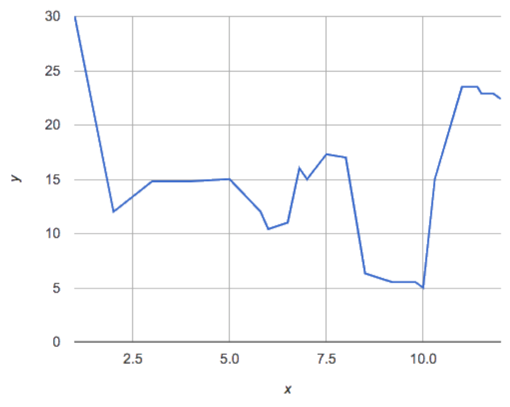


6) Linear Nonlinear Not a Function

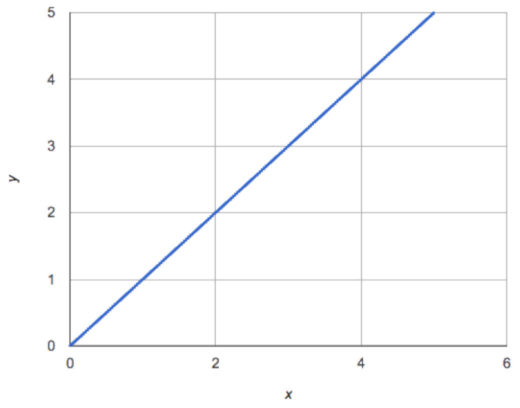
(optional)

Graphs: Linear, Non-linear, or Bust? (2)

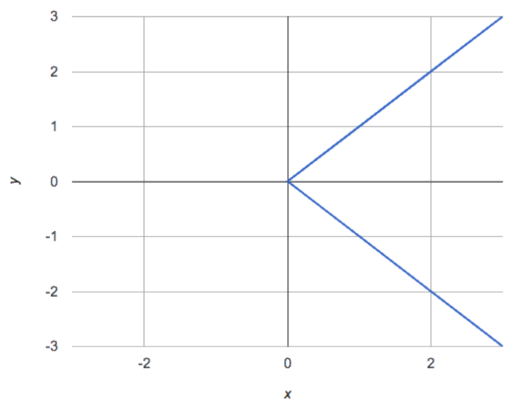
Decide whether each representation is of a linear function, a nonlinear function or is not a function at all!



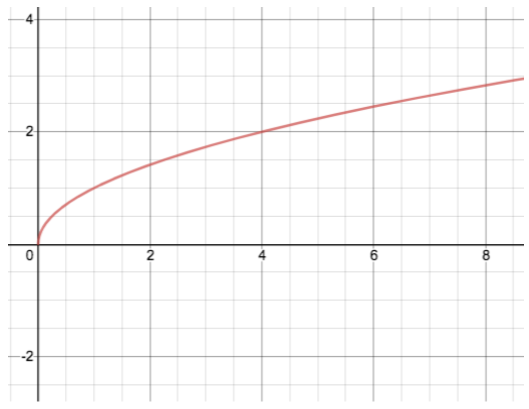
1) Linear Nonlinear Not a Function



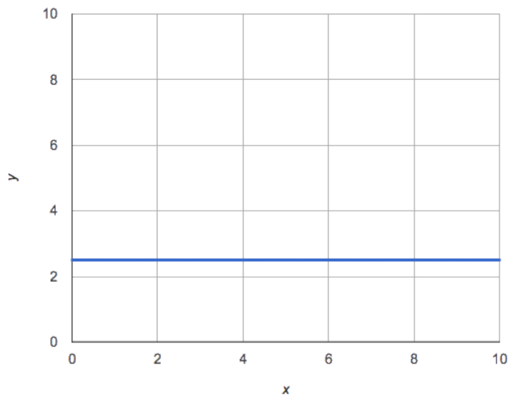
2) Linear Nonlinear Not a Function



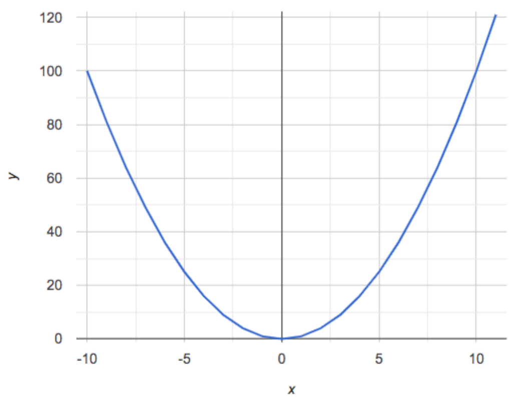
3) Linear Nonlinear Not a Function



4) Linear Nonlinear Not a Function



5) Linear Nonlinear Not a Function



6) Linear Nonlinear Not a Function

(optional)

Solving Word Problems in a Nutshell

Being able to see functions as Contracts, Examples or Definitions is like having three powerful tools. These representations can be used together to solve word problems! We call this **The Design Recipe**.

1) When reading a word problem, the first step is to figure out the **Contract** for the function you want to build. Remember, a Contract must include the Name, Domain and Range for the function!

2) Then we write a **Purpose Statement**, which is a short note that tells us what the function *should do*. Professional programmers work hard to write good purpose statements, so that other people can understand the code they wrote! Programmers work on teams; the programs they write must outlast the moment that they are written.

3) Next, we write at least two **Examples**. These are lines of code that show what the function should do for a *specific* input. Once we see examples of at least two inputs, we can *find a pattern* and see which parts are changing and which parts aren't.

4) To finish the Examples, we circle the parts that are changing, and label them with a short **variable name** that explains what they do.

5) Finally, we **define the function** itself! This is pretty easy after you have some examples to work from: we copy everything that didn't change, and replace the changeable stuff with the variable name!

Matching Word Problems and Purpose Statements

Match each word problem below to its corresponding purpose statement.

Annie got a new dog, Xavier, that eats about 5 times as much as her little dog, Rex, who is 10 years old. She hasn't gotten used to buying enough dogfood for the household yet. Write a function that generates an estimate for how many pounds of food Xavier will eat, given the amount of food that Rex usually consumes in the same amount of time.	**1**	**A**	Consume the pounds of food Rex eats and add 5.
Adrienne's raccoon, Rex, eats 5 more pounds of food each week than her pet squirrel, Lili, who is 7 years older. Write a function to determine how much Lili eats in a week, given how much Rex eats.	**2**	**B**	Consume the pounds of food Rex eats and subtract 5.
Alejandro's rabbit, Rex, poops about 1/5 of what it eats. His rabbit hutch is 10 cubic feet. Write a function to figure out how much rabbit poop Alejandro will have to clean up depending on how much Rex has eaten.	**3**	**C**	Consume the pounds of food Rex eats and multiply by 5.
Max's turtle, Rex, eats 5 pounds less per week than his turtle, Harry, who is 2 inches taller. Write a function to calculate how much food Harry eats, given the weight of Rex's food.	**4**	**D**	Consume the pounds of food Rex eats and divide by 5.

Writing Examples from Purpose Statements

We've provided contracts and purpose statements to describe two different functions. Write examples for each of those functions.

Contract and Purpose Statement

Every contract has three parts…

`; triple` : `Number` -> `Number`

function name — Domain — Range

`; Consumes a Number and triples it.`

what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*____________________*)* ____________________)

function name — input(s) — what the function produces

*(EXAMPLE (*____________________*)* ____________________)

function name — input(s) — what the function produces

Contract and Purpose Statement

Every contract has three parts…

`; upside-down` : `Image` -> `Image`

function name — Domain — Range

`; Consumes an image, and turns it upside down by rotating it 180 degrees.`

what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*____________________*)*

function name — input(s)

____________________)

what the function produces

*(EXAMPLE (*____________________*)* ____________________)

function name — input(s) — what the function produces

Fixing Purpose Statements

Beneath each of the word problems below is a purpose statement (generated by ChatGPT!) that is either missing information or includes unnecessary information.

- Write an improved version of each purpose statement beneath the original.
- Then, explain what was wrong with the ChatGPT-generated Purpose Statement.

1) **Word Problem:** *The New York City ferry costs $2.75 per ride. The Earth School requires two chaperones for any field trip. Write a function* `fare` *that takes in the number of students in the class and returns the total fare for the students and chaperones.*

ChatGPT's Purpose Statement: Take in the number of students and add 2.

Improved Purpose Statement: ______________________________

____

Problem with ChatGPT's Purpose Statement: ______________________________

____

2) **Word Problem:** *It is tradition for the Green Machines to go to Humpy Dumpty's for ice cream with their families after their soccer games. Write a function* `cones` *to take in the number of kids and calculate the total bill for the team, assuming that each kid brings two family members and cones cost $1.25.*

ChatGPT's Purpose Statement: Take in the number of kids on the team and multiply it by 1.25.

Improved Purpose Statement: ______________________________

____

Problem with ChatGPT's Purpose Statement: ______________________________

____

3) **Word Problem:** *The cost of renting an ebike is $3 plus an additional $0.12 per minute. Write a function* `ebike` *that will calculate the cost of a ride, given the number of minutes ridden.*

ChatGPT's Purpose Statement: Take in the number of minutes and multiply it by 3.12.

Improved Purpose Statement: ______________________________

____

Problem with ChatGPT's Purpose Statement: ______________________________

____

4) **Word Problem:** *Suleika is a skilled house painter at only age 21. She has painted hundreds of rooms and can paint about 175 square feet an hour. Write a function* `paint` *that takes in the number of square feet of the job and calculates how many hours it will take her.*

ChatGPT's Purpose Statement: Take in the number of square feet of walls in a house and divide them by 175 then add 21 years.

Improved Purpose Statement: ______________________________

____

Problem with ChatGPT's Purpose Statement: ______________________________

____

Word Problem: rocket-height

Directions: A rocket blasts off, and is now traveling at a constant velocity of 7 meters per second. Use the Design Recipe to write a function `rocket-height`, which takes in a number of seconds and calculates the height.

Contract and Purpose Statement

Every contract has three parts…

; __________ : ____________________ -> __________

function name | Domain | Range

; ______________________________

what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*____________________*)* ____________________)

function name | input(s) | what the function produces

*(EXAMPLE (*____________________*)* ____________________)

function name | input(s) | what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (______________________________)

function name | variable(s)

______________________________)

what the function does with those variable(s)

Rubric: Design Recipe

This rubric can be used for teachers to score students' Design Recipes or for peer review. If using this rubric for peer review, trade your Design Recipe with another student. Place this rubric and their Design Recipe side-by-side in front of you.

1) Go through the checklist in the left-hand column to assess their Contract. Check boxes or leave them blank depending on what you observe.

2) Once you have examined and analyzed the Contract, read the descriptive text (either "Wow!" or "Getting there") and check whichever one more accurately describes the work in front of you.

3) If the Design Recipe you're reviewing is "getting there," provide some descriptive feedback to help the student fix their work.

4) Repeat the process for the remaining sections of the Design Recipe.

The **CONTRACT**:	❏ Wow!	❏ Getting There…
❏ has correct function name ❏ has correct amount of Domain data types ❏ has correct data type(s) listed in the Domain ❏ has correct data type listed for the Range	The Contract you've written tells us a lot about how to use the function. In fact, we can figure out how to use your function just by looking at the Contract. You've included all essential information.	Something is missing from your Contract. It doesn't provide everything needed to understand the function. ***Here's what you need to do:***
The **PURPOSE STATEMENT**:	❏ Wow!	❏ Getting There…
❏ describes what the function consumes and produces ❏ describes how the result is computed, so that it can be combined to with the Contract to explain the Examples	The Purpose Statement is a concise and detailed restatement of the problem in your own words. It's a helpful explanation of what's happening in the problem.	Programmers and Mathematicians alike find it helpful to restate a problem in their own words. ***Your restatement is missing the following:***
The **EXAMPLES**:	❏ Wow!	❏ Getting There…
❏ have the correct function name ❏ have inputs that differ across Examples ❏ have the correct amount of Domain inputs ❏ have the correct expressions for what the function produces, using the given inputs ❏ have changeable parts circled and labeled	Your Examples not only help us to identify the pattern to define a function, they also let us double check that the functions we define do what we intend for them to do.	Your Examples do not help us to identify a pattern, or they don't allow us to double check our functions. ***Here's how you can improve that:***
The **DEFINITION**:	❏ Wow!	❏ Getting There…
❏ has the correct function name ❏ has the correct number, name, and order of variables (taken from the labels in the Examples section)	Your code correctly names the function, lists its variables, and states the expression to compute when the function is used!	Your Definition is missing something. ***Here's how to fix it:***

Writing Examples from Purpose Statements (2)

We've provided contracts and purpose statements to describe two different functions. Write examples for each of those functions.

Contract and Purpose Statement

Every contract has three parts…

`; half-image` : `Image` -> `Image`
function name / Domain / Range

`; Consumes an image, and produces that image scaled to half its size.`
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*____*)*
function name input(s)

____)
what the function produces

*(EXAMPLE (*____*)*
function name input(s)

____)
what the function produces

Contract and Purpose Statement

Every contract has three parts…

`; product-squared` : `Number Number` -> `Number`
function name / Domain / Range

`; Consumes two numbers and squares their product`
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*____________________________*)* ______________________________)
function name input(s) what the function produces

*(EXAMPLE (*____________________________*)* ______________________________)
function name input(s) what the function produces

Rocket Height Challenges

This page is designed to accompany work in the Rocket Height Starter File.

1) Can you make the rocket fly faster?

2) Can you make the rocket fly slower?

3) Can you make the rocket sink down instead of fly up?

4) Can you make the rocket accelerate over time, so that it moves faster the longer it flies?

5) Can you make the rocket blast off and then land again?

6) Can you make the rocket blast off, reach a maximum height of exactly 1000 meters, and then land?

7) Can you make the rocket blast off, reach a maximum height of exactly 1000 meters, and then land after exactly 100 seconds?

8) Can you make the rocket fly to the edge of the the universe?

Design Recipe Telephone

Most computer programs are written by huge teams! It is critical that each team member records their thinking with enough detail for other team members to be able to pick up where they left off. We're going to practice collaborative programming through an activity called Design Recipe Telephone.

1. Prepare the class and the materials

Choose which set of word problems you are going to start with and print enough copies so that each student will get one word problem.
Divide the class into groups of three.
Give each student within each group a different word problem from the set.

Word Problem Set 1:	Word Problem Set 2:	Option 3:
Design Recipe Telephone Set 1: g Design Recipe Telephone Set 1: h Design Recipe Telephone Set 1: r ★ *Once completed, the set of functions generated from these word problems can be used to fix the code in this Collaboration Starter File - For use with Design Recipe Telephone Set 1. If all the functions are defined correctly, the starter file will then generate a cool image!*	Design Recipe Telephone Set 2: symmetry Design Recipe Telephone Set 2: l-rect Design Recipe Telephone Set 2: right-trapezoid	Use any of the Design Recipe problems that students haven't solved before. ★ *There is a large collection of math problems that would work well with the Design Recipe in the Additional Exercises section of our Solving Word Problems with the Design Recipe lesson.*

2. Describe the rules for the activity

- In this activity, each person in your group will start with a different word problem. You will each be doing *one step of each Design Recipe problem*. After you complete your step, you will fold your paper to hide the part that you were looking at so that only *your work and the rest of the recipe* are visible. Then you will pass your work to the person to your right.
- The person who has received your paper will review your work and complete the next step based solely on what you wrote down for them. If they don't have the information they need, they will give the paper back to you for revision.
- Meanwhile, you will receive a different problem from the person to your left. If at any point your realize that the person before you didn't provide enough information, you may hand the paper back to them for revision.

Who's Doing What During Each Round of Design Recipe Telephone?		
Round 1 - Writing Contract and Purpose Statements from the Word Problem		
Student 1 - Problem A	Student 2 - Problem B	Student 3 - Problem C
everyone folds over the previous section, and passes their paper to the right		
Round 2 - Writing Examples *based solely on the Contract and Purpose Statement*		
Student 1 - Problem C	Student 2 - Problem A	Student 3 - Problem B
everyone folds over the previous section, and passes their paper to the right		
Round 3 - Writing Function Definitions *based solely on the Examples*		
Student 1 - Problem B	Student 2 - Problem C	Student 3 - Problem A

3. Practice makes perfect!

This activity can be repeated several times, or done as a timed competition between teams. The goal is to emphasize that each step - if done correctly - makes the following step incredibly simple.

4. Synthesize

The Design Recipe is a way of slowing down and thinking through each step of a problem.
If we already know how to get the answer, why would it ever be important to know how to do each step the slow way?

- *Sample Responses: Someday we won't be able to get the answer, and knowing the steps will help. We can help someone else who is stuck. We can work with someone else and share our thinking. We can check our work.*

(optional)

The Design Recipe (Restaurants)

Directions: Use the Design Recipe to write a function `split-tab` that takes in a cost and the number of people sharing the bill and splits the cost equally.

Contract and Purpose Statement

Every contract has three parts…

; ______________________ : ____ -> ____________
function name | Domain | Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (________________________________ *)* ________________________________)
function name | input(s) | what the function produces

(EXAMPLE (________________________________ *)* ________________________________)
function name | input(s) | what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name | variable(s)

____)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `tip-calculator` that takes in the cost of a meal and returns the 15% tip for that meal.

Contract and Purpose Statement

Every contract has three parts…

; ______________________ : ____ -> ____________
function name | Domain | Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (________________________________ *)* ________________________________)
function name | input(s) | what the function produces

(EXAMPLE (________________________________ *)* ________________________________)
function name | input(s) | what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name | variable(s)

____)
what the function does with those variable(s)

(optional)

The Design Recipe (Direct Variation)

Directions: Use the Design Recipe to write a function `wage`, that takes in a number of hours worked and returns the amount a worker will get paid if their rate is $10.25/hr.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Directions: On average, people burn about 11 calories/minute riding a bike. Use the Design Recipe to write a function `calories-burned` that takes in the number of minutes you bike and returns the number of calories burned. .

Contract and Purpose Statement

Every contract has three parts…

; __________ : *Number* -> *Number*
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

(optional)

The Design Recipe (Slope/Intercept)

Directions: For his birthday, James' family decided to open a savings account for him. He started with $50 and committed to adding $10 a week from his afterschool job teaching basketball to kindergartners. Use the Design Recipe to write a function `savings` that takes in the number of weeks since his birthday and calculates how much money he has saved.

Contract and Purpose Statement

Every contract has three parts…

;__________________:____->____________
function name Domain Range

;____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________________________*)* ________________________________)
function name input(s) what the function produces

*(EXAMPLE (*__________________________*)* ________________________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name variable(s)

____)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `moving` that takes in the days and number of miles driven and returns the cost of renting a truck. The truck is $45 per day and each driven mile is 15¢.

Contract and Purpose Statement

Every contract has three parts…

;__________________:____->____________
function name Domain Range

;____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________________________*)* ________________________________)
function name input(s) what the function produces

*(EXAMPLE (*__________________________*)* ________________________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name variable(s)

____)
what the function does with those variable(s)

The Design Recipe (Negative Slope/Intercept)

Directions: An Olympic pool holds 660,000 gallons of water. A fire hose can spray about 250 gallons per minute. Use the Design Recipe to write a function `pool` that takes in the number of minutes that have passed and calculates how much water is still needed to fill it.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Directions: The community arts fund awards a $1500 grant each month to support a new mural. They started with $50000 in their account. Use the Design Recipe to write a function `funds-available` that takes in the number of months and calculates how much money they have left.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

The Design Recipe (Geometry - Rectangles)

Directions: Use the Design Recipe to write a function `lawn-area` that takes in the length and width of a rectangular lawn and returns its area.

Contract and Purpose Statement

Every contract has three parts…

; ______________ : ______________________________ -> __________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________________ *)* ______________________)
function name input(s) what the function produces

(EXAMPLE (______________________ *)* ______________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (______________________________________)
function name variable(s)

____)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `rect-perimeter` that takes in the length and width of a rectangle and returns the perimeter of that rectangle.

Contract and Purpose Statement

Every contract has three parts…

; ______________ : ______________________________ -> __________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________________ *)* ______________________)
function name input(s) what the function produces

(EXAMPLE (______________________ *)* ______________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (______________________________________)
function name variable(s)

____)
what the function does with those variable(s)

(optional)

The Design Recipe (Geometry - Rectangular Prisms)

Directions: Use the Design Recipe to write a function `rectprism-vol` that takes in the length, width, and height of a rectangular prism and returns the Volume of a rectangular prism.

Contract and Purpose Statement

Every contract has three parts…

; ______________ : ______________________________ -> ______________
function name — Domain — Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*______________________________*)* ______________________________)
function name — input(s) — what the function produces

*(EXAMPLE (*______________________________*)* ______________________________)
function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name — variable(s)

____)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `rect-prism-sa` that takes in the width, length and height of a rectangular prism and calculates its surface area (the sum of the areas of each of its six faces)

Contract and Purpose Statement

Every contract has three parts…

; ______________ : ______________________________ -> ______________
function name — Domain — Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*______________________________*)*
function name — input(s)

____)
what the function produces

*(EXAMPLE (*______________________________*)*
function name — input(s)

____)
what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name — variable(s)

____)
what the function does with those variable(s)

(optional)

The Design Recipe (Geometry - Circles)

Directions: Use the Design Recipe to write a function `circle-area-dec` that takes in a radius and uses the decimal approximation of pi (3.14) to return the area of the circle.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name | Domain | Range

; ____________________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (__________ *)* __________)
function name | input(s) | what the function produces

(EXAMPLE (__________ *)* __________)
function name | input(s) | what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name | variable(s)

____________________)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `circumference` that takes in a radius and uses the decimal approximation of pi (3.14) to return the circumference of the circle.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name | Domain | Range

; ____________________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (__________ *)* __________)
function name | input(s) | what the function produces

(EXAMPLE (__________ *)* __________)
function name | input(s) | what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name | variable(s)

____________________)
what the function does with those variable(s)

The Design Recipe (Geometry - Cylinders)

Directions: Use the Design Recipe to write a function `circle-area` that takes in a radius and uses the fraction approximation of pi ($^{22}/_{7}$) to return the area of the circle.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `cylinder` that takes in a cylinder's radius and height and calculates its volume, making use of the function *circle-area* .

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

(optional)

The Design Recipe (Breaking Even)

Directions: The Swamp in the City Festival is ordering t-shirts. The production cost is $75 to set up the silk screen and $9 per shirt. Use the Design Recipe to write a function `min-shirt-price` that takes in the number of shirts to be ordered, *n*, and returns the minimum amount the festival should charge for the shirts in order to break even. (Assume that they will sell all of the shirts.)

Contract and Purpose Statement

Every contract has three parts…

; ______________________ : ____ -> ____________

function name   Domain   Range

; ____

what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*______________________________*)* ______________________________)

function name   input(s)   what the function produces

*(EXAMPLE (*______________________________*)* ______________________________)

function name   input(s)   what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)

function name   variable(s)

____)

what the function does with those variable(s)

The Design Recipe (Marquee & Cubing)

Directions: Use the Design Recipe to write a function `marquee` that takes in a message and returns that message in large gold letters.

Contract and Purpose Statement

Every contract has three parts…

;__________ : __________ -> __________
function name Domain Range

;__________
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `num-cube` that takes in a number and returns the cube of that number.

Contract and Purpose Statement

Every contract has three parts…

;__________ : __________ -> __________
function name Domain Range

;__________
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Design Recipe Telephone Set 1: g

Directions: Hali is decorating her tree house and is having a hard time fitting everything on the walls. She's figured out that if her artwork were 3/8 of the original size it would all fit. Help her by writing a function `g` to scale down any image to a size she can use!

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

★ NOTE ★ When writing examples, you can assume that we have predefined `image-a` and `image-b`.

(optional)

Design Recipe Telephone Set 1: h

Directions: Define a function h that will take an image and rotate it clockwise one-tenth of a turn. Hint: A full rotation is 360 degrees, which you may have heard people refer to in skateboarding or snowboarding tricks.

Contract and Purpose Statement

Every contract has three parts…

; __________ : `Image` -> `Image`
function name — Domain — Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (__________ *)* __________)
function name — input(s) — what the function produces

(EXAMPLE (__________ *)* __________)
function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name — variable(s)

__________)
what the function does with those variable(s)

★ NOTE ★ When writing examples, you can assume that we have predefined `image-a` and `image-b`.

(optional)

Design Recipe Telephone Set 1: r

A Contract worth remembering…

```
; regular-polygon :: Number, Number, String, String -> Image
; Takes in a size, the number of sides, a color, and a fill type and makes a shape with all equal sides
and all angles congruent.
```

Directions: Zora's favorite shape is a regular pentagon and they want to decorate a special box with pentagons of every color. Help them to realize their dream by writing a function `r` that takes in a color and returns a solid 5-sided regular polygon of size 300 in the given color.

Contract and Purpose Statement

Every contract has three parts…

; ____________ : *String* -> *Image*

function name | Domain | Range

; ____________________________

what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*____________*)* ____________)

function name | input(s) | what the function produces

*(EXAMPLE (*____________*)* ____________)

function name | input(s) | what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____________)

function name | variable(s)

____________)

what the function does with those variable(s)

(optional)

Design Recipe Telephone Set 2: symmetry

★NOTE★ When writing examples, you can assume that we have predefined `image-a` and `image-b`.

Directions: Nassim loves all things symmetrical. He figured out that if you flip an image horizontally and then place it beside the original image, you can turn any image into a symmetrical image. Help him to be more efficient by writing a new function `symmetry` that will take in any image and use it to make a new symmetrical image.

Contract and Purpose Statement

Every contract has three parts…

; ______________ : ______________________________ -> ______________
function name — Domain — Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*______________________________*)* ______________________________)
function name — input(s) — what the function produces

*(EXAMPLE (*______________________________*)* ______________________________)
function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name — variable(s)

____)
what the function does with those variable(s)

A Contract worth remembering:

```
; beside :: Image, Image -> Image
; places two images beside each other
```

(optional)

Design Recipe Telephone Set 2: l-rect

Directions: Ava loves purple rectangles that are 5 times as wide as they are tall. Help her out by writing a function `l-rect` that takes in a width and generates a solid rectangle that Ava would love.

Contract and Purpose Statement

Every contract has three parts…

; _____________ : __________________________ -> __________
function name · Domain · Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*________________________*)* ________________________)
function name · input(s) · what the function produces

*(EXAMPLE (*________________________*)* ________________________)
function name · input(s) · what the function produces

Definition

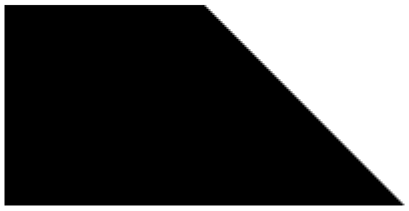
Write the definition, giving variable names to all your input values…

(define (____)
function name · variable(s)

____)
what the function does with those variable(s)

Design Recipe Telephone Set 2: right-trapezoid

★NOTE★An isosceles triangle has two sides that are the same length.



Directions: Zosia loves right-trapezoids composed of squares and isosceles-right-triangles. Write a function `right-trapezoid` that takes in the sidelength of the square and a color and returns a solid right-trapezoid.

Contract and Purpose Statement

Every contract has three parts…

; ________________ : ________________________________ -> ____________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________________________*)*
function name input(s)

____)
what the function produces

*(EXAMPLE (*__________________________*)*
function name input(s)

____)
what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name variable(s)

____)
what the function does with those variable(s)

A Contract worth remembering:

```
; right-triangle :: Number, Number, String, String -> Image
; Takes in 2 side lengths, a color, and a fill type and makes a right-triangle
```

Danger and Target Movement

Directions: Use the Design Recipe to write a function `update-danger`, which takes in the danger's x-coordinate and produces the next x-coordinate, which is 50 pixels to the left.

Contract and Purpose Statement

Every contract has three parts…

; ____________ : ______________________________ -> ____________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________________ *)* ______________________)
function name input(s) what the function produces

(EXAMPLE (______________________ *)* ______________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (______________________________________)
function name variable(s)

____)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `update-target`, which takes in the target's x-coordinate and produces the next x-coordinate, which is 50 pixels to the right.

Contract and Purpose Statement

Every contract has three parts…

; ____________ : ______________________________ -> ____________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________________ *)* ______________________)
function name input(s) what the function produces

(EXAMPLE (______________________ *)* ______________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (______________________________________)
function name variable(s)

____)
what the function does with those variable(s)

Problem Decomposition

Sometimes a problem is too complicated to solve all at once:

- Maybe there are too many variables.
- Maybe there is so much information that we can’t get a handle on it!
- Maybe we’ll be less likely to make mistakes if we think about the parts one at a time.

Problem Decomposition allows us to break complicated problems down into simpler pieces… and then solve by working with the pieces. There are two strategies:

- **Top-Down**:
  - Start with the "big picture", writing functions or equations that describe the connections between parts of the problem.
  - Then, work on defining those parts.
- **Bottom-Up**:
  - Start with the smaller parts, writing functions or equations that describe the parts we understand.
  - Then, connect those parts together to solve the whole problem.

You may find that one strategy works better for some types of problems than another, so make sure you’re comfortable using both of them!

Word Problems: revenue, cost

Directions: Use the Design Recipe to write a function `revenue`, which takes in the number of glasses sold at $1.75 apiece and calculates the total revenue.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `cost`, which takes in the number of glasses sold and calculates the total cost of materials if each glass costs $.30 to make.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Word Problem: profit

Directions: Use the Design Recipe to write a function `profit` that calculates total profit from glasses sold, which is computed by subtracting the total cost from the total revenue.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Profit - More than one Way!

Four students defined the same `revenue` and `cost` functions, shown below:

```
(define (revenue g) (* 1.75 g))
(define (cost g) (* 0.3 g))
```

They then came up with **four different definitions** for `profit`:

Khalil:	`(define (profit g) (- (* 1.75 g) (* 0.3 g)))`
Samaria:	`(define (profit g) (* (- 1.75 0.3) g))`
Alenka:	`(define (profit g) (* 1.45 g))`
Fauzi:	`(define (profit g) (- (revenue g) (cost g)))`

1) Which of these four definitions do you think is "best", and why?

2) If lemons get more expensive, which definitions of `profit` need to be changed?

3) If Sally raises her prices, which definitions of `profit` need to be changed?

4) Which definition of `profit` is the most flexible? Why?

Top Down or Bottom Up

Jamal's trip requires him to drive 20 mi to the airport, fly 2,300 mi, and then take a bus 6 mi to his hotel. His average speed driving to the airport is 40 mph, the average speed of an airplane is 575 mph, and the average speed of his bus is 15 mph.
Aside from time waiting for the plane or bus, how long is Jamal in transit?

Bear's Strategy:	**Lion's Strategy:**
$\text{Drive Time} = 20\text{ miles} \times \frac{1\text{ hour}}{40\text{ miles}} = 0.5\text{ hours}$ $\text{Fly Time} = 2300\text{ miles} \times \frac{1\text{ hour}}{575\text{ miles}} = 4\text{ hours}$ $\text{Bus Time} = 6\text{ miles} \times \frac{1\text{ hour}}{15\text{ miles}} = 0.4\text{ hours}$ $\text{In Transit Time} = \text{Drive Time} + \text{Fly Time} + \text{Bus Time}$ $0.5 + 4 + 0.4 = 4.9\text{ hours}$	$\text{In Transit Time} = \text{Drive Time} + \text{Fly Time} + \text{Bus Time}$ $\text{Drive Time} = 20\text{ miles} \times \frac{1\text{ hour}}{40\text{ miles}} = 0.5\text{ hours}$ $\text{Fly Time} = 2300\text{ miles} \times \frac{1\text{ hour}}{575\text{ miles}} = 4\text{ hours}$ $\text{Bus Time} = 6\text{ miles} \times \frac{1\text{ hour}}{15\text{ miles}} = 0.4\text{ hours}$ $0.5 + 4 + 0.4 = 4.9\text{ hours}$

1) Whose Strategy was Top Down? How do you know?

2) Whose Strategy was Bottom Up? How do you know?

3) Which way of thinking about the problem makes more sense to you?

What's happening with that Math?!

When calculating Jamal's drive time, we multiplied distance by speed. More specifically, we multiplied the starting value (20 miles) by $\frac{1\text{ hour}}{40\text{ miles}}$. Why? Why not reverse it, to use $\frac{40\text{ miles}}{1\text{ hour}}$, as stated in the problem?

Time is the desired outcome. Looking at the units, we can see that speed must have miles as its denominator to *cancel out* the miles in the starting value.

$$\frac{20\text{ miles}}{1} \times \frac{1\text{ hour}}{40\text{ miles}} = \frac{20\,\cancel{\text{miles}} \times 1\text{ hour}}{40\,\cancel{\text{miles}}} = \frac{20}{40}\text{hour} = \frac{1}{2}\text{hour}$$

Sally's Bike

We know that it costs Sally 30cents to make a cup of lemonade and she's selling each cup for $1.75. If the bike Sally wants costs $198 and sales tax in her town is 7 percent, how many cups of lemonade will Sally have to sell in order to buy the bike?
Use the open space below to find the answer, being sure to show your work!

(optional)

Inequalities

Sometimes we want to *ask questions* about data:

- Is `x` greater than `y`?
- Is one string equal to another?

These questions are answered with a new data type called a ***Boolean***.

Unlike Numbers, Strings, and Images, Booleans have only two possible values. A Boolean value is either `true` or `false`. You already know some functions that produce Booleans, such as `<` and `>`!

Our programming language has them, too. We can evaluate:

`(< 3 4)`	`(> 2 10)`	`(= -10 19)`
"3 is less than 4" is `true`	"2 is greater than 10" is `false`	"-10 is equal to 19" is `false`

We can also ask more complicated questions:

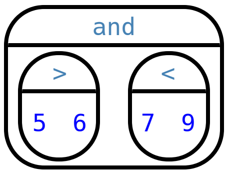
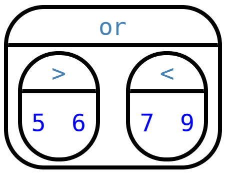
- Is the elephant small enough and light enough to ride in the boat?
- Do we have enough rice and enough time to make it for dinner?

Our programming language uses the `and` and `or` functions to combine to ***Simple Inequalities*** to make a ***Compound Inequality***.

- The `and` function will return true if **both** sub-expressions are `true`.
- The `or` function will return true if **at least one** sub-expression is `true`.

`(and (> 5 6) (< 7 9))`	`(or (> 5 6) (< 7 9))`
"5 is greater than 6 *and* 7 is less than 9"	"5 is greater than 6 *or* 7 is less than 9"
This will evaluate to `false`, because the expressions aren't both `true`.	This will evaluate to `true`, because at least one of the expressions is `true`.

The Circles of Evaluation work the same way with Booleans that they do with Numbers, Strings and Images.

Video games use Booleans for many things including:

- asking when a player's health is equal to zero
- determining whether two characters are close enough to bump into one another
- figuring out if a character's coordinates put it off the edge of the screen

Boolean Functions

Make a prediction about what each function in the Boolean Starter File does.

____

____

____

____

____

Now, experiment with the functions. Fill in the blanks below so that each of the five functions returns `true`.

1) `(odd? ____________)`

2) `(even? ____________)`

3) `(less-than-one? ____________)`

4) `(continent? ____________)`

5) `(primary-color? ____________)`

Fill in the blanks below so that each of the five functions returns `false`.

6) `(odd? ____________)`

7) `(even? ____________)`

8) `(less-than-one? ____________)`

9) `(continent? ____________)`

10) `(primary-color? ____________)`

All 5 of these functions produce Booleans. How would you describe what a Boolean is?

____

Simple Inequalities

Each inequality expression in the first column contains a number.
Decide whether or not that number is a solution to the expression and place it in the appropriate column.
Then identify 4 *solution* values and 4 *non-solution* values for `x`.

- **Solutions** will make the expression `true`.
- **Non-Solutions** will make the expression `false`.

You can see graphs of the solution sets of these inequalities and test out each of your lists in the Simple Inequalities Starter File.
The comments in the starter file will help you learn how it works!

★ Challenge yourself to use negatives, positives, fractions, decimals, etc. for your `x` values.

	Expression	4 solutions that evaluate to `true`	4 non-solutions that evaluate to `false`
a	`(> x 2)`		
b	`(<= x -2)`		
c	`(< x 3.5)`		
d	`(>= x -1)`		
e	`(> x -4)`		
f	`(<> x 2)`		

1) For which inequalities was the number from the expression part of the solution?

2) For which inequalities was the number from the expression not part of the solution?

3) For which inequalities were the solutions on the left end of the number line?

4) For which inequalities were the solutions on the right end of the number line?

Word Problem: hot?

Directions: Use the Design Recipe to write a function `hot?`, which takes in a temperature in Fahrenheit and determines if it is above 80 degrees

Contract and Purpose Statement

Every contract has three parts…

; ______________ : ______________________________ -> ______________
function name — Domain — Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________ ______________ *)* ______________)
function name — input(s) — what the function produces

(EXAMPLE (______________ ______________ *)* ______________)
function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values…

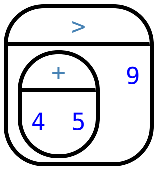
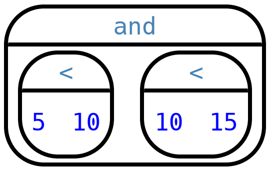
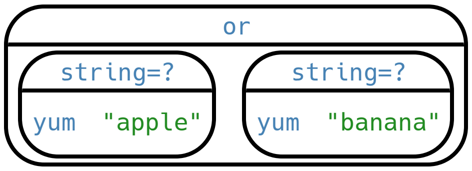
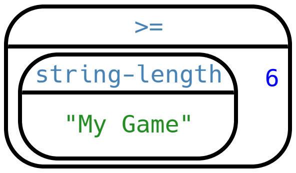
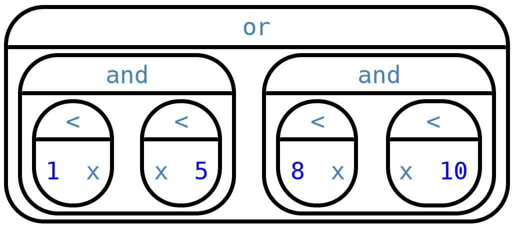
(define (______________ ______________)
function name — variable(s)

____)
what the function does with those variable(s)

(optional)

Converting Circles of Evaluation to Code

Convert each Circle of Evaluation on the left-hand side to Code.

	Circle of Evaluation	Code
1		
2		
3	 or string=? yum "apple" string=? yum "banana"	
4		
5	 or and < 1 x < x 5 and < 8 x < x 10	

Compound Inequalities - Practice

Create the Circles of Evaluation, then convert the expressions into Code in the space provided.

1) 2 is less than 5, and 0 is equal to 6

What will this evaluate to? Why? ______________________________

2) 6 is greater than 8, or -4 is less than 1

What will this evaluate to? Why? ______________________________

3) The String “purple” is the same as the String “blue”, and 3 plus 5 equals 8

What will this evaluate to? Why? ______________________________

4) Write the contracts for `and` & `or` in your Contracts page.

Compound Inequality Warmup

1) What are 4 solutions for $x > 5$?

2) What are 4 non-solutions for $x > 5$?

3) What are 4 solutions for $x \leq 15$?

4) What are 4 non-solutions for $x \leq 15$?

5) What 4 numbers are in the solution set of $x > 5$ **and** $x \leq 15$, making both of these inequalities true?

6) How would that be different from the solution set of $x > 5$ **or** $x \leq 15$, making at least one of these inequalities true?

Exploring Compound Inequalities

This page is designed to accompany the Compound Inequalities Starter File. When you click "Run" you will see 4 graphs. The first two are simple inequalities and the second two are compound inequalities.

1) What does `and-intersection` do?

2) Why is the dot on 5 red and the circle on 15 green?

3) Do you think every graph made with `and-intersection` will have a red dot at one end and a green dot at the other? Why or why not?

4) What does `or-union` do?

5) Why did the graph of this `or-union` result in the whole numberline being shaded blue?

6) Not all graphs of `or-union` will look like this. Can you think of a pair of inequalities whose union won't shade the whole graph?

Change the function definition on *line 8* to $x < 5$ **and the definition on *line 9* to** $x \geq 15$.
Before you click "Run", think about what the new graphs of `and-intersection` and `or-union` will look like. Then test them out.

7) What does the new `and-intersection` graph look like?

8) What does the new `or-union` graph look like?

9) Why is the dot for 5 still red and the dot for 15 still green?

10) Which of the 8 numbers from the list are part of the solution set? __________

How do you know? __________

11) Is 3 part of the solution set? ______ Explain. __________

12) Is 10 part of the solution set? ______ Explain. __________

Compound Inequalities: Solutions & Non-Solutions

For each Compound Inequality listed below, identify 4 *solutions* and 4 *non-solutions*, unless the solution set includes **all real numbers** or there are **no solutions**.

- Solutions for ***intersections*** (which use **and**) will make both of the expressions `true`.
- Solutions for ***unions*** (which use **or**) will make at least one of the expressions `true`.

Pay special attention to the numbers in the sample expression! Challenge yourself to use negatives, positives, fractions, decimals, etc.

The first two have been done for you - Answers will vary!

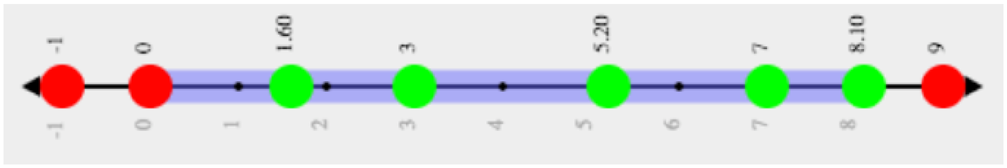
	Expression	4 solutions that evaluate to `true`	4 non-solutions that evaluate to `false`
a	`x > 5` and `x < 15`	6, 9.5, 12, 14.9	-2, 5, 15, 16.1
b	`x > 5` or `x < 15`	All real numbers	No non-solutions
c	`x <= -2` and `x > 7`		
d	`x <= -2` or `x > 7`		
e	`x < 3.5` and `x > -4`		
f	`x < 3.5` or `x > -4`		
g	`x >= -1` and `x > -5`		
h	`x >= -1` or `x > -5`		
i	`x < -4` and `x > 2`		

1) Could there ever be a union with *no solutions*? Explain your thinking.

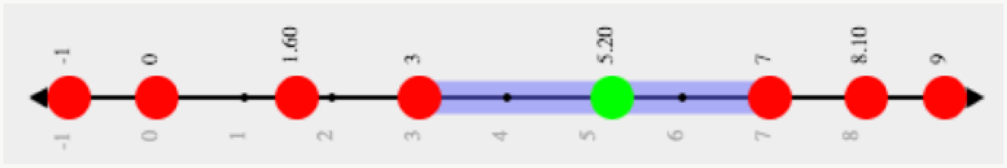
2) Could there ever be an intersection whose solution is *all real numbers*? Explain your thinking.

Compound Inequality Functions

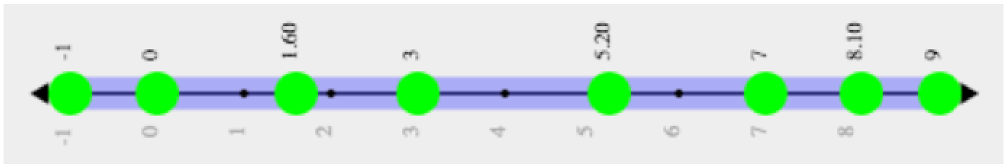
Each of the plots below was generated using the code `(inequality comp-ineq (list -1 0 1.6 3 5.2 7 8.1 9))`. Using the numbers 3 and 7, write the code to define `comp-ineq` for each plot.
Note: The example is defined using 0 and 8.1 rather than 3 and 7.



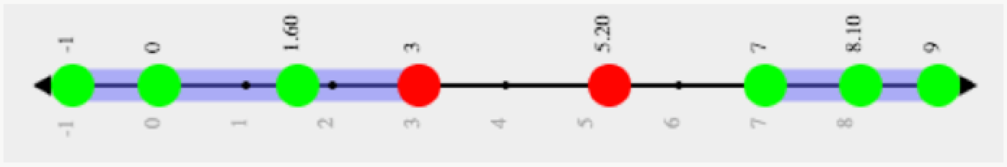
code: `(define (comp-ineq x) (and (> x 0) (<= x 8.1)))`



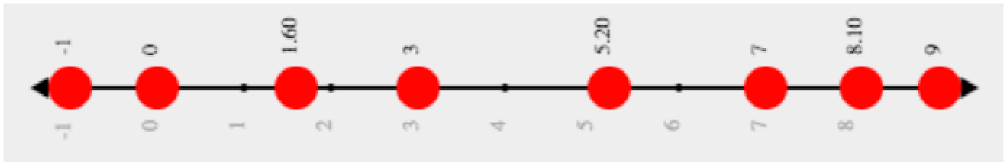
code: ______________________________



code: ______________________________



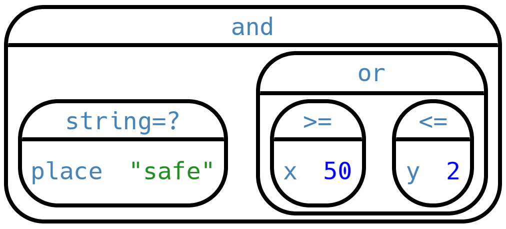
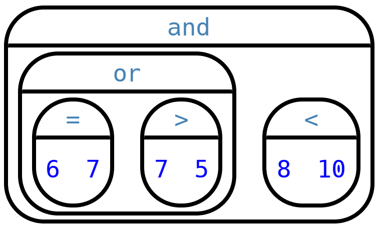
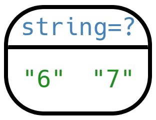
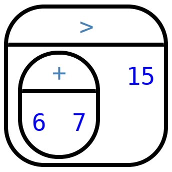
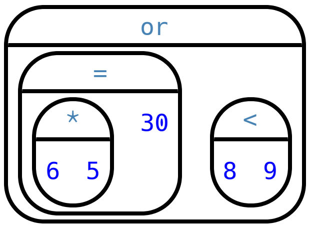
code: ______________________________



code: ______________________________

Converting Circles of Evaluation with Booleans to Code 2

For each Circle of Evaluation on the left-hand side, write the code for the Circle on the right-hand side.

	Circle of Evaluation	Code
1	 and string=? place "safe" or >= x 50 <= y 2	
2		
3		
4		
5		

(optional)

Sam the Butterfly

Open the [Sam the Butterfly Starter File](#) starter file and click "Run". (Hi, Sam!) Move Sam around the screen using the arrow keys.

1) What do you Notice about the program?

____

____

2) What do you Wonder?

____

____

3) What do you see when Sam is at (0,0)? Why is that?

____

4) What changes as the butterfly moves left and right?

____

5) Sam is in a 640 × 480 yard. Sam's mom wants Sam to stay in sight... *How far to the left and right can Sam go and still remain visible?*

____

____

6) Write an inequality to complete each of the following statements:

Sam hasn't gone off the left edge of the screen as long as...

____

Sam hasn't gone off the right edge of the screen as long as...

____

7) Draw the Circle of Evaluation for each inequality you wrote above.

8) Translate each of the Circles of Evaluation into code.

code: ______________________________

code: ______________________________

Left and Right

Directions: Use the Design Recipe to write a function `safe-left?`, which takes in an x-coordinate and checks to see if it's greater than -50.

Contract and Purpose Statement

Every contract has three parts…

; ______________________ : ____ -> ______________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*______________________________________*)* ______________________________________)
function name input(s) what the function produces

*(EXAMPLE (*______________________________________*)* ______________________________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name variable(s)

____)
what the function does with those variable(s)

Directions: Use the Design Recipe to write a function `safe-right?`, which takes in an x-coordinate and checks to see if it is less than 690.

Contract and Purpose Statement

Every contract has three parts…

; ______________________ : ____ -> ______________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*______________________________________*)* ______________________________________)
function name input(s) what the function produces

*(EXAMPLE (*______________________________________*)* ______________________________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name variable(s)

____)
what the function does with those variable(s)

Word Problem: onscreen?

Directions: Use the Design Recipe to write a function `onscreen?`, which takes in an x-coordinate and checks to see if Sam is safe on the left while also being safe on the right.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

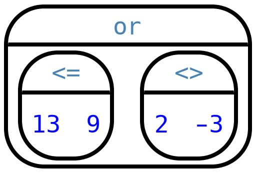
(define (__________)
function name variable(s)

__________)
what the function does with those variable(s)

Warmup: Coding Compound Inequalities

Remember:

- some useful code for writing inequalities: `>=` `<=` `<>`
- `and` expressions return true when both sub-expressions return true
- `or` expressions return true when at least one sub-expression returns true `==`

Expression	Circles of Evaluation	Code
13 is less than or equal to 9, or 2 is not equal to -3		`(or (<= 13 9) (<> 2 -3))`
1) What will this evaluate to? Why? True. The 2nd expression is true; `or` expressions return true if at least one subexpression is true.		
3 is greater than or equal to 5, and 5 is less than 8		
2) What will this evaluate to? Why? ______		
6 is less than or equal to 6, or 12 is greater than -7		
3) What will this evaluate to? Why? ______		
3 is not equal to 2, and 3 + 5 is equal to 8		
4) What will this evaluate to? ______		

(optional)

Onscreen - More than One Way

Nokosee's Thinking

```
(define (safe-bottom? y) (>= y -30))
(define (safe-top? y) (<= y 510))
(define (onscreen? y) (and (safe-bottom? y) (safe-top? y)))
```

Sabra's Thinking

```
(define (safe-bottom? y) (> y -40))
(define (safe-top? y) (< y 520))
(define (onscreen? y) (and (> y -40) (< y 520)))
```

1) Nokosee and Sabra have different strategies for keeping Sam on the screen. How does Nokosee's strategy work?

2) How does Sabra's strategy work?

3) What's an advantage of Nokosee's strategy?

4) What's an advantage of Sabra's strategy?

5) Which strategy do you prefer? Why?

Piecewise Functions in a Nutshell

- Sometimes we want to build functions that act differently for different inputs. For example, suppose a business charges $10/pizza, but only $5/pizza for orders of six or more. How could we write a function that computes the total price based on the number of pizzas?
- In math, **Piecewise Functions** are functions that can behave one way for part of their Domain, and another way for a different part. In our pizza example, our function would act like $cost(pizzas) = 10 * pizzas$ for anywhere from 1-5 pizzas. But after 5, it acts like $cost(pizzas) = 5 * pizzas$.
- Piecewise functions are divided into "pieces". Each piece is divided into two parts:
  1. How the function should behave
  2. The domain where it behaves that way
- Our programming language can be used to write piecewise functions, too! Just as in math, each piece has two parts:

```
(define (cost pizzas)
  (cond
    [(>= pizzas 6) (* 5 pizzas)])
```

Piecewise functions are powerful, and let us solve more complex problems. We can use piecewise functions in a video game to add or subtract from a character’s x-coordinate, moving it left or right depending on which key was pressed.

Red Shape - Explore

1) Open the Red Shape Starter File, and read through the code you find there. This code contains new programming that you haven't seen yet! Take a moment to list everything you Notice, and then everything you Wonder…

What do you Notice?	What do you Wonder?

2) What happens if you click "Run" and type `(red-shape "ellipse")`?

3) Add **another example** for "triangle".

4) Add another line of code to the definition, to define what the function should do with the input "triangle".

5) Come up with some new shapes, and add them to the code. Make sure you include examples or you will get an error message!

6) In your own words, describe how *piecewise functions* work in this programming environment.

Word Problem: red-shape

Directions: A friend loves red shapes so we've decided to write a program that makes it easy to generate them. Write a function called `red-shape` which takes in the name of a shape and makes a 20-pixel, solid, red image of the shape.

Contract and Purpose Statement

Every contract has three parts…

; red-shape : *String* -> *Image*
function name — Domain — Range

; Given a shape name, produce a solid, red, 20-pixel image of the shape.
what does the function do?

Examples

Write some examples, then circle and label what changes…

	function name	input(s)	what the function produces
(EXAMPLE (	*red-shape*	*"circle"*)	*(circle 20 "solid" "red")*)
(EXAMPLE (	*red-shape*	*"triangle"*)	*(triangle 20 "solid" "red")*)
(EXAMPLE (	*red-shape*	*"rectangle"*)	*(rectangle 20 20 "solid" "red")*)
(EXAMPLE (	*red-shape*	*"star"*)	*(star 20 "solid" "red")*)

Definition

Write the definition, giving variable names to all your input values…

(define (______)
function name — variable(s)

(*cond*

[______ ______]

[______ ______]

[______ ______]

[______ ______]

[______ ______]))

Decide & Defend - Piecewise Onto Functions

Joy and Marianna have written two different sets of code to accomplish the same goal of helping a caterer direct people with dietary restrictions to a menu item that works for them. Look at the code below.

Joy's Code:

```
(define (entree diet)
  (cond
    [(string=? diet "gluten-free") salmon]
    [(string=? diet "kosher") salmon]
    [(string=? diet "lactose-int") salmon]
    [(string=? diet "nut allergy") lasagna]
    [(string=? diet "vegan") stir-fry]
    [(string=? diet "vegetarian") stir-fry]
    [else (text "unknown diet" 20 "red")]))
```

Marianna's Code:

```
(define (entree diet)
  (cond
    [(or (or (string=? diet "gluten-free") (string=? diet "kosher")) (string=? diet "lactose-int"))
salmon]
    [(or (string=? diet "vegan") (string=? diet "vegetarian")) stir-fry]
    [else (text "unknown diet" 20 "red")]))
```

Whose method do you like better? Why?

Word Problem: Mood Generator

NOTE: This file uses emojis. Even though emojis look like images, they are actually characters in a string! They can be accessed from your keyboard, just like any other character.

Directions: They say a picture is worth a thousand words. Write a function `mood` that translates moods into emojis so that we can "see" what someone is feeling.

Contract and Purpose Statement

Every contract has three parts…

`; mood`	:	`String`	->	`String`
function name		Domain		Range

`; Consumes a mood and produces the emoji for that mood.`

what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (	`mood`	`"happy"`	)	`"😀 "`	)
	function name	input(s)		what the function produces
(EXAMPLE (	`mood`	`"sad"`	)	`"😥 "`
	function name	input(s)		what the function produces
(EXAMPLE (	`mood`	`"angry"`	)	`"😡 "`
	function name	input(s)		what the function produces
(EXAMPLE (	`mood`	`"sick"`	)	`"🤮 "`
	function name	input(s)		what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)

function name variable(s)

(`cond`

[______________________________ ______________________________]

[______________________________ ______________________________]

[______________________________ ______________________________]

[______________________________ ______________________________]

[______________________________ ______________________________]))

Alice's Restaurant - Explore

Alice's code has some new elements we haven't seen before, so let's experiment a bit to figure out how it works! **Open the Alice's Restaurant Starter File, click "Run", and try using the `cost` function in the Interactions window.**

1) What does `(cost "hamburger")` evaluate to? ______________________________

2) What does `(cost "pie")` evaluate to? ______________________________

3) What if you ask for `(cost "fries")`? ______________________________

4) Explain what the function is doing in your own words. ______________________________

____

____

5) What is the function's name? ____________ Domain? ____________ Range? ____________

6) What is the name of its variable? ______________________________

7) Alice says onion rings have gone up to $3.75. Change the `cost` function to reflect this.

8) Try adding menu items of your own. What's your favorite? ______________________________

9) For an unknown food item, the function produces the String `"That's not on the menu!"` Is this a problem? Why or why not?

____

____

10) Suppose Alice wants to calculate the price of a hamburger, *including a 5% sales tax*. Draw a Circle of Evaluation for the expression below.

(optional)

Word Problem: alices-restaurant

Directions: Alice's Restaurant has hired you as a programmer. They offer the following menu items: hamburger ($6.00), onion rings ($3.50), fried tofu ($5.25) and pie ($2.25). Write a function called `alices-restaurant` which takes in the name of a menu item and outputs the price of that item.

Contract and Purpose Statement

Every contract has three parts…

; _______________ : ______________________________ -> __________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (____________________ *)* ____________________)
function name input(s) what the function produces

(EXAMPLE (____________________ *)* ____________________)
function name input(s) what the function produces

(EXAMPLE (____________________ *)* ____________________)
function name input(s) what the function produces

(EXAMPLE (____________________ *)* ____________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name variable(s)

(*cond*

[____________________ ____________________]

[____________________ ____________________]

[____________________ ____________________]

[____________________ ____________________]

[____________________ ____________________]))

(optional)

Word Problem: update-player

Directions: The player moves by 20 pixels each time the up or down key is pressed. Write a function called `update-player`, which takes in the player's y-coordinate and the name of the key pressed ("up" or "down"), and returns the new y-coordinate.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (`update-player 300 "up"`) __________)
function name input(s) what the function produces

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

(EXAMPLE (__________ *)* __________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

(*`cond`*

[__________ __________]

[__________ __________]

[__________ __________]))

Challenges for update-player

For each of the challenges below, see if you can come up with two EXAMPLEs of how it should work!

1) **Warping** - Program one key to "warp" the player to a set location, such as the center of the screen.

```
(EXAMPLE (update-player ______________ ______________)
  ______________)
(EXAMPLE (update-player ______________ ______________)
  ______________)
```

2) **Boundaries** - Change `update-player` such that `PLAYER` cannot move off the top or bottom of the screen.

```
(EXAMPLE (update-player ______________ ______________)
  ______________)
(EXAMPLE (update-player ______________ ______________)
  ______________)
```

3) **Wrapping** - Add code to `update-player` such that when `PLAYER` moves to the top of the screen, it reappears at the bottom, and vice versa.

```
(EXAMPLE (update-player ______________ ______________)
  ______________)
(EXAMPLE (update-player ______________ ______________)
  ______________)
```

4) **Hiding** - Add a key that will make `PLAYER` seem to disappear, and reappear when the same key is pressed again.

```
(EXAMPLE (update-player ______________ ______________)
  ______________)
(EXAMPLE (update-player ______________ ______________)
  ______________)
```

Challenge: Character Movement in Two Dimensions

You have all the tools you need to begin this challenge if:

- your game is working
- both the `Danger` and `Target` return to the screen
- your `Player` moves up and down with the arrow keys

```
; update-danger :: Number -> Number
; consumes danger’s x-coordinate and produces the next x-coordinate
```

The `update-danger` function only moves our `DANGER` left or right.

Suppose we wanted to write a new function, `update-danger-2` that moves the `DANGER` *diagonally* ...

1) What, if anything will have to change about the Domain? ______________________

2) What, if anything, will have to change about the Range? ______________________

Since an (x, y) coordinate has two Numbers, one idea might be to write the Contract this way:

```
; update-danger-2 :: Number Number -> Number Number
; consumes danger’s x- and y-coordinate, and produces the next x- and next y-coordinate
```

...But that Contract breaks an important rule about functions: Given an input, all functions must produce one output!

We need some way to package two Numbers together into a single value...

Fortunately, WeScheme has another ***data type***, called a ***Posn*** .
The Posn data type utilizes two Numbers to describe a *single* "position"!

We can make a Posn to represent the position (100, 200) with the following code:

```
(make-posn 100 200)
```

3) What expression will make a Posn representing the origin? ______________________

4) Write the Contract for the `make-posn` function on the line below.

____

Challenge: Character Movement in Two Dimensions (2)

Directions: On the lines below, write the new Contract and Purpose for `update-danger-2`, so that it produces a Posn instead of a Number. Then complete the Design Recipe.

Contract and Purpose Statement

Every contract has three parts...

; ________________ : ________________________________ -> ____________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes...

(EXAMPLE (________________________ *)* ________________________)
function name input(s) what the function produces

(EXAMPLE (________________________ *)* ________________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (____)
function name variable(s)

____)
what the function does with those variable(s)

Adding Your New Function to Your Game File

1) Find `update-danger` in your game file.
Directly beneath it, add `update-danger-2` (including Contract, Purpose, Examples, and Definition) to your game file.

2) Scroll down to the very end of your game file and find the following **PROVIDED CODE**.

```
(define g (make-game TITLE TITLE-COLOR
                     BACKGROUND
                     DANGER update-danger
                     TARGET update-target
                     PLAYER update-player
                     mystery update-mystery
                     distances-color line-length distance
                     collide? onscreen?))
(play g)
```

Change `update-danger` to `update-danger-2` in the list and click "Run".

- This change will tell your program to use your new function with 2D movement, instead of the original function.
- *Note: If, at any point, you would like to go back to using the original function, all you have to do is change this list so that it says* `update-danger` *instead of* `update-danger-2` *and click "Run" again!*

Challenge: update-player-2

Double-check:

- Is your game working?
- Do both the `Danger` and `Target` return to the screen?
- Does your `Player` move up and down with the arrow keys?
- Have you completed Challenge: Character Movement in Two Dimensions?
- Does your `Danger` move diagonally?

...then you have all the tools you need to work through this Design Recipe and get your player moving in all four directions!

Directions: Write a new function `update-player-2` that takes in the player's x-coordinate, y-coordinate, and an arrow key (described by a String) and moves the player to a new Posn. Your goal is to get all 4 arrow keys working as you would expect them to by moving the player 50 pixels in the corresponding direction!

Contract and Purpose Statement

Every contract has three parts...

; ______________________ : ______________________________________ -> ______________
function name — Domain — Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes...

*(EXAMPLE (*______________________________*)* ______________________________)
function name — input(s) — what the function produces

*(EXAMPLE (*______________________________*)* ______________________________)
function name — input(s) — what the function produces

*(EXAMPLE (*______________________________*)* ______________________________)
function name — input(s) — what the function produces

*(EXAMPLE (*______________________________*)* ______________________________)
function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values...

(define (____)
function name — variable(s)

(*cond*

[______________________ ______________________]

[______________________ ______________________]

[______________________ ______________________]

[______________________ ______________________]

[______________________ ______________________]

Once you complete this Design Recipe:
Follow the directions on Challenge: Character Movement in Two Dimensions (2) for adding your new function to your game file, this time changing `update-player` to `update-player-2`.

★ *Once you've mastered 2-dimensional movement, you might want to add secret functionality for some of your favorite letters on the keyboard...*

Line Length Explore

Sign in to WeScheme and open your Game File.

Defining `line-length`

Find the definition for the `line-length` function and consider the code you see.

1) What do you Notice?

2) What do you Wonder?

Using `line-length`

Click Run, and practice using `line-length` in the ***Interactions Area*** *with different values for* `a` *and* `b`.

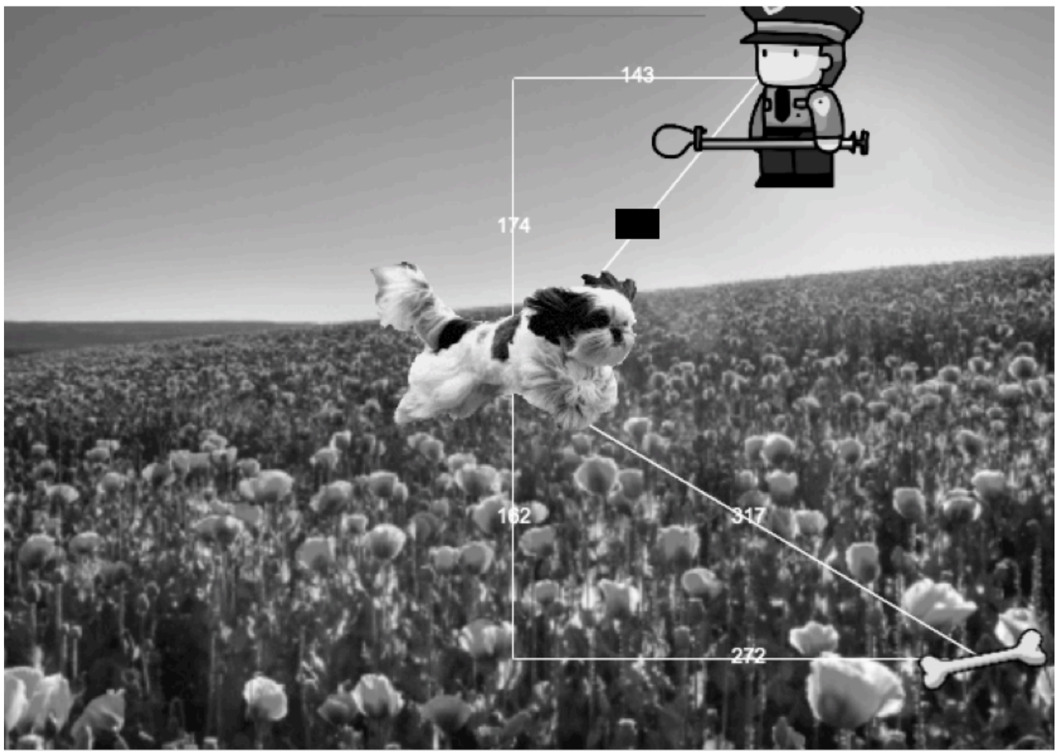
3) What does the `line-length` function *do?*

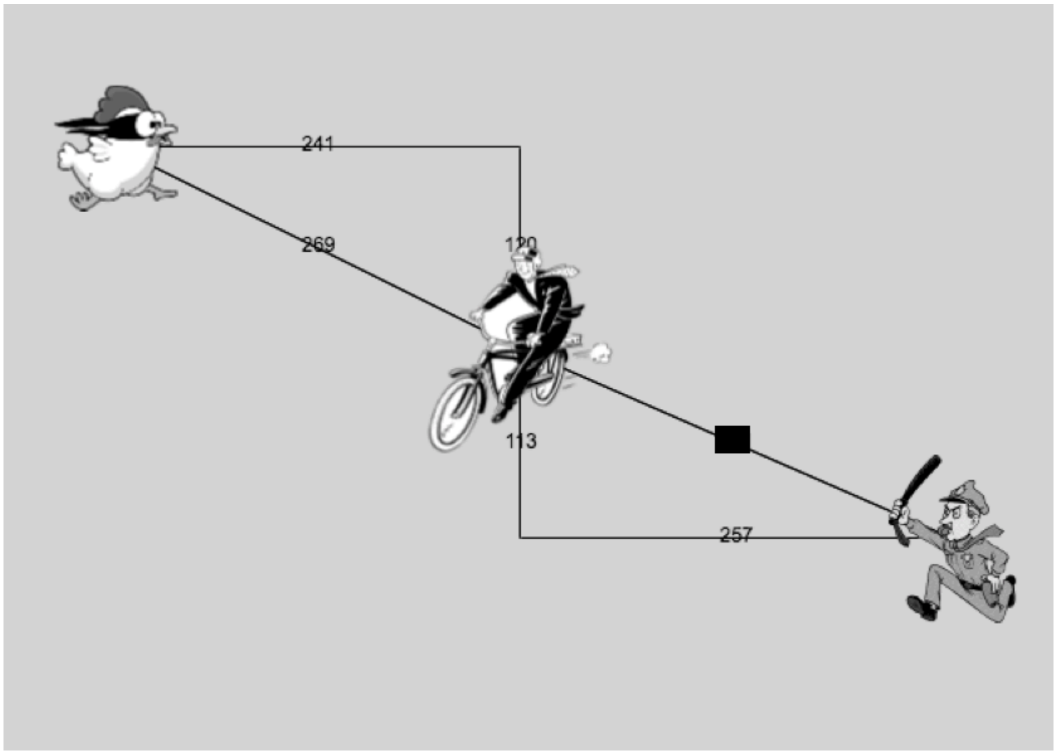
4) Why does it use conditionals?

5) Why is the distance between two points always positive?

Writing Code to Calculate Missing Lengths

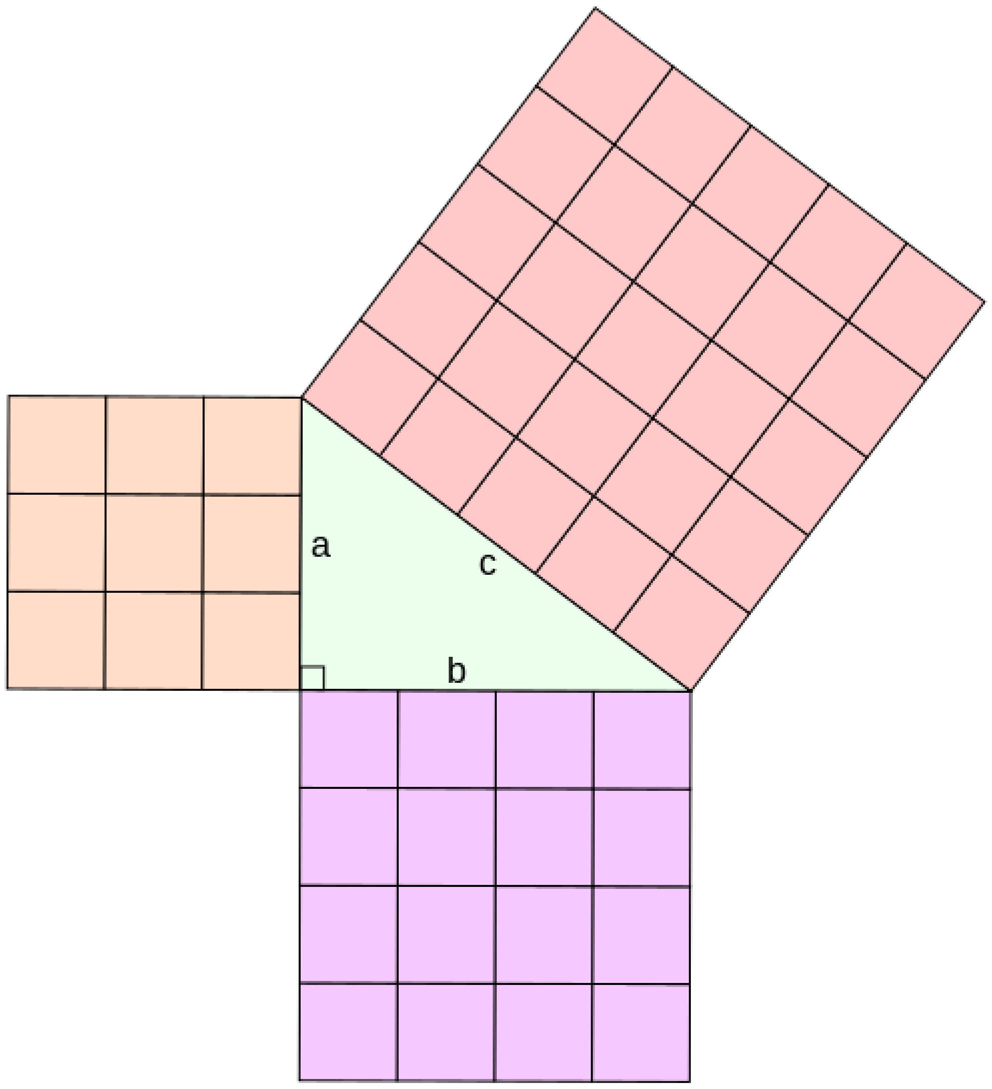
In each of the game screenshots below, one of the distance labels has been hidden. Write the code to generate the missing distance on the line below each image. *Hint: Remember the Pythagorean Theorem!*





Proof Without Words

Long ago, mathematicians realized that there is a special relationship between the three squares that can be formed using the sides of a right triangle.



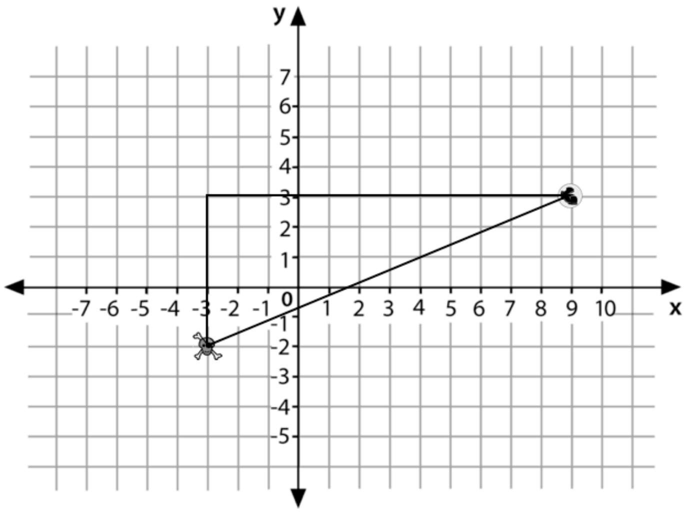
How would you describe the relationship you've observed between the three squares whose side-lengths are determined by the lengths of the sides of a right triangle?

Distance on the Coordinate Plane

Reading Code:

Distance between the Pyret and the boot:

```
(sqrt (+ (sqr (line-length 9 -3)) (sqr (line-length 3 -2))))
```



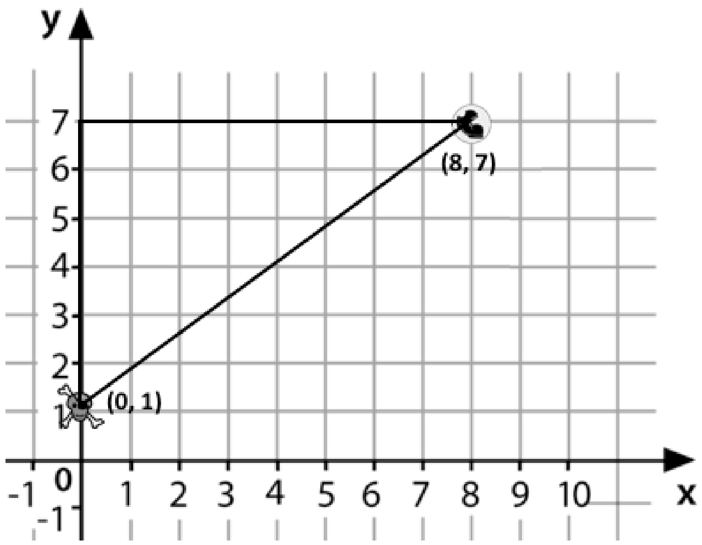
1) Where do the 9 and -3 come from? ______________________

2) Where to the 3 and -2 come from? ______________________

3) Explain how the code works. ______________________

____

Writing Code



Now write the code to find the distance between this boot and pyret.

____

____

Circles of Evaluation: Distance between (0, 2) and (4, 5)

Suppose your player is at (0, 2) and a character is at (4, 5)…

1) Identify the values of x_1 , y_1 , x_2 , and y_2

x_1	y_1	x_2	y_2
(x-value of 1st point)	*(y-value of 1st point)*	*(x-value of 2nd point)*	*(y-value of 2nd point)*

What is the distance between your player and the character?

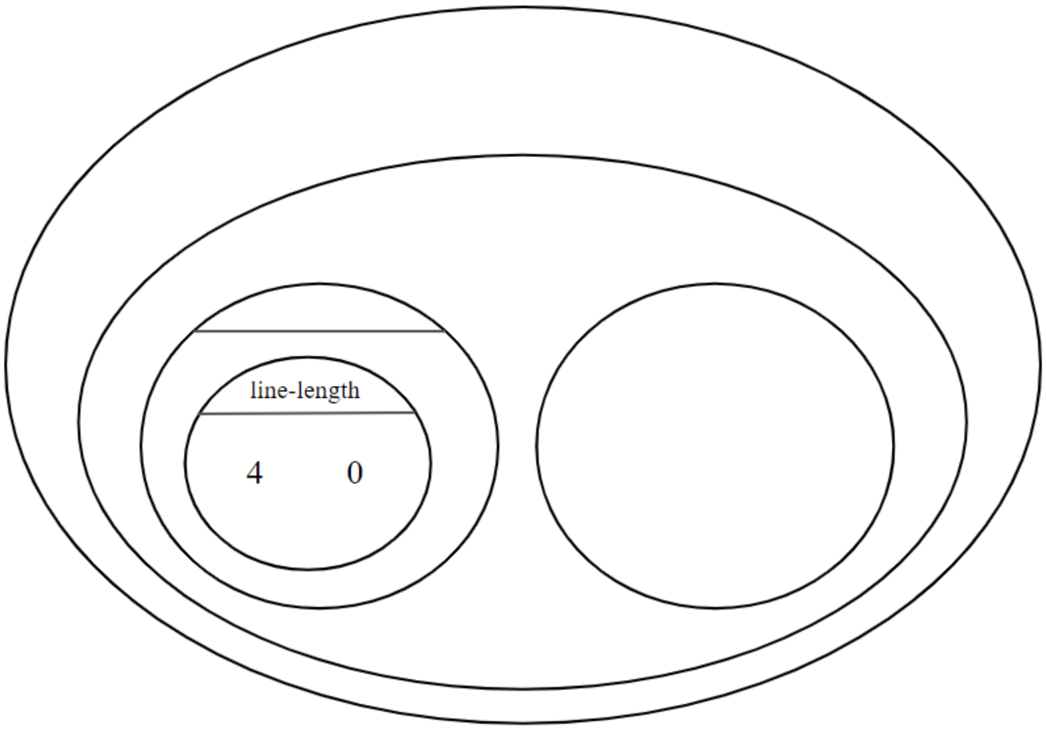
- We can use `line-length` to computer the horizontal and vertical distances and then use those to find the diagonal distance.
  - The horizontal distance between x_1 and x_2 is computed by `(line-length x2 x1)`.
  - The vertical distance between y_2 and y_1 is computed by `(line-length y2 y1)`.
- The hypotenuse of a right triangle with legs the lengths of those distances is computed by: $\sqrt{\text{line-length}(x_2, x_1)^2 + \text{line-length}(y_2, y_1)^2}$
- So, when we substitute these points in, the distance between them will be computed by:

$$\sqrt{\text{line-length}(4, 0)^2 + \text{line-length}(5, 2)^2}$$

2) The points are (0,2) and (4,5). Why aren't we using `(line-length 0 2)` and `(line-length 4 5)`?

---

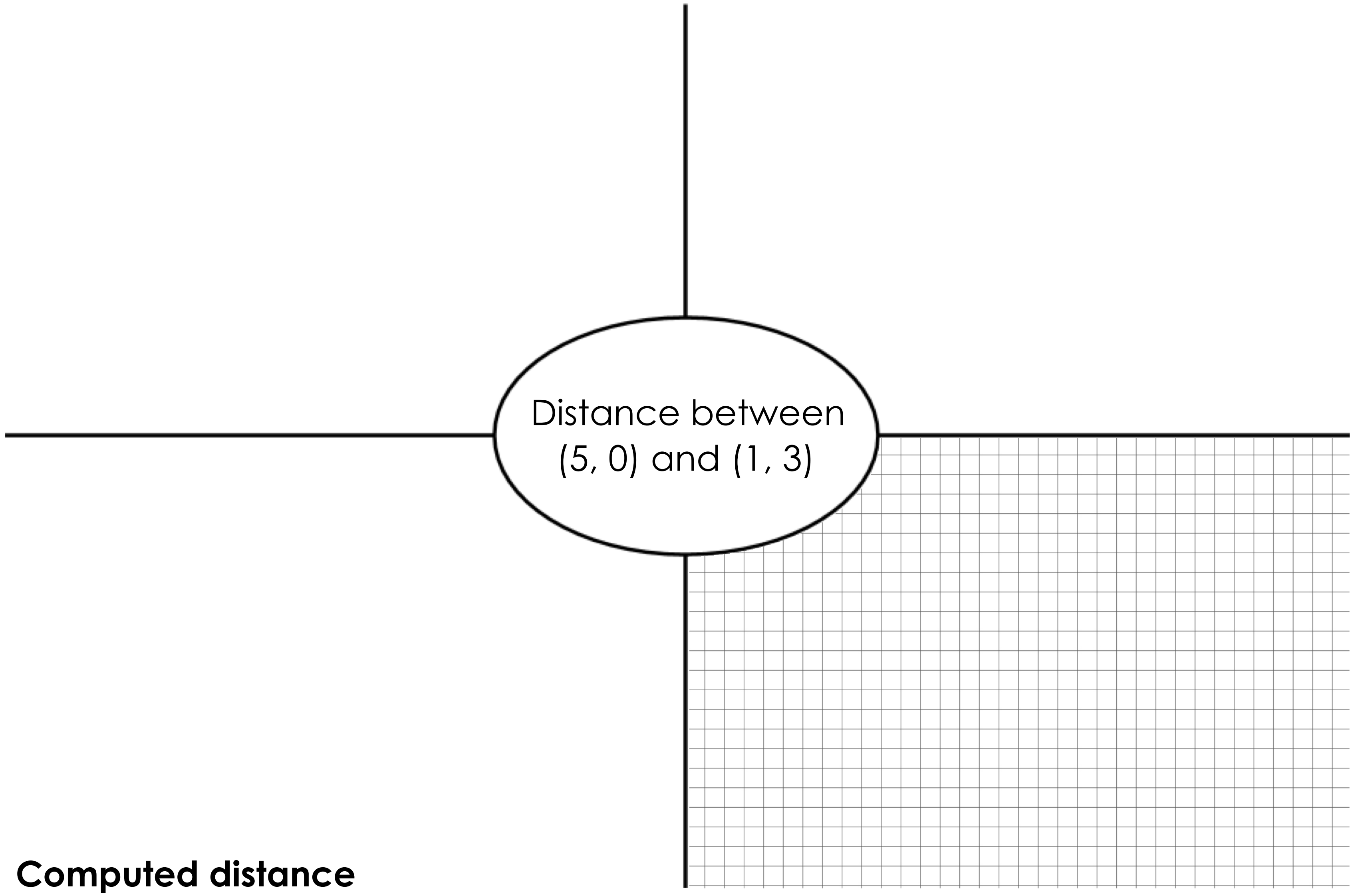
3) Translate the expression above, for (0,2) and (4,5) into a Circle of Evaluation below.
Hint: In our programming language `sqr` *is used for* x^2 *and* `sqrt` *is used for* $\sqrt{x}$



4) Convert the Circle of Evaluation to Code below.

Circle of Evaluation

Code

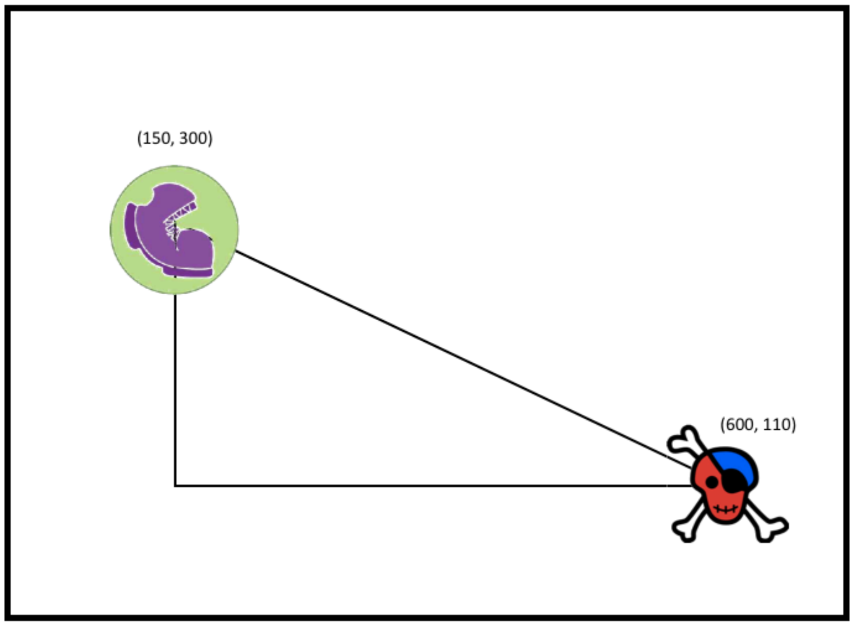


Computed distance between (5, 0) and (1, 3)

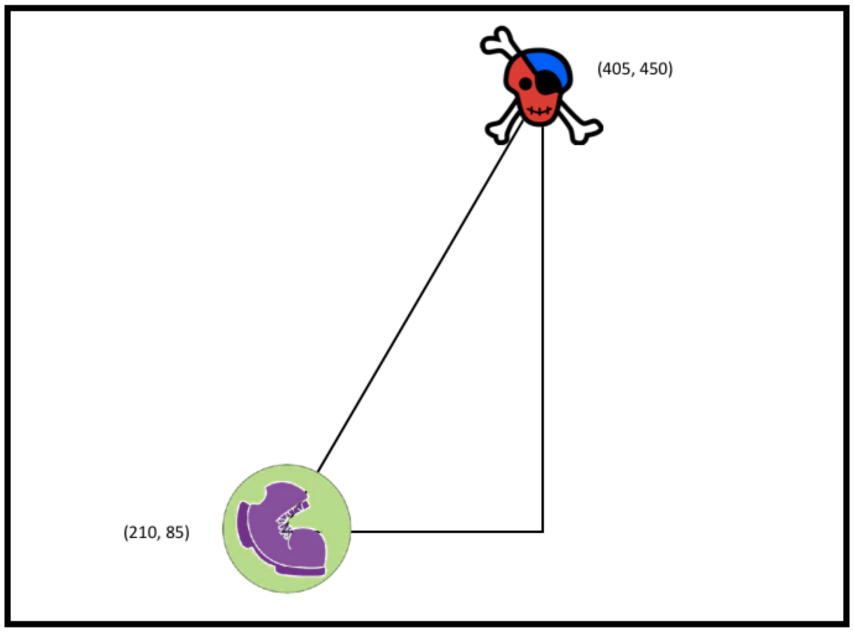
Graph

Distance From Game Coordinates

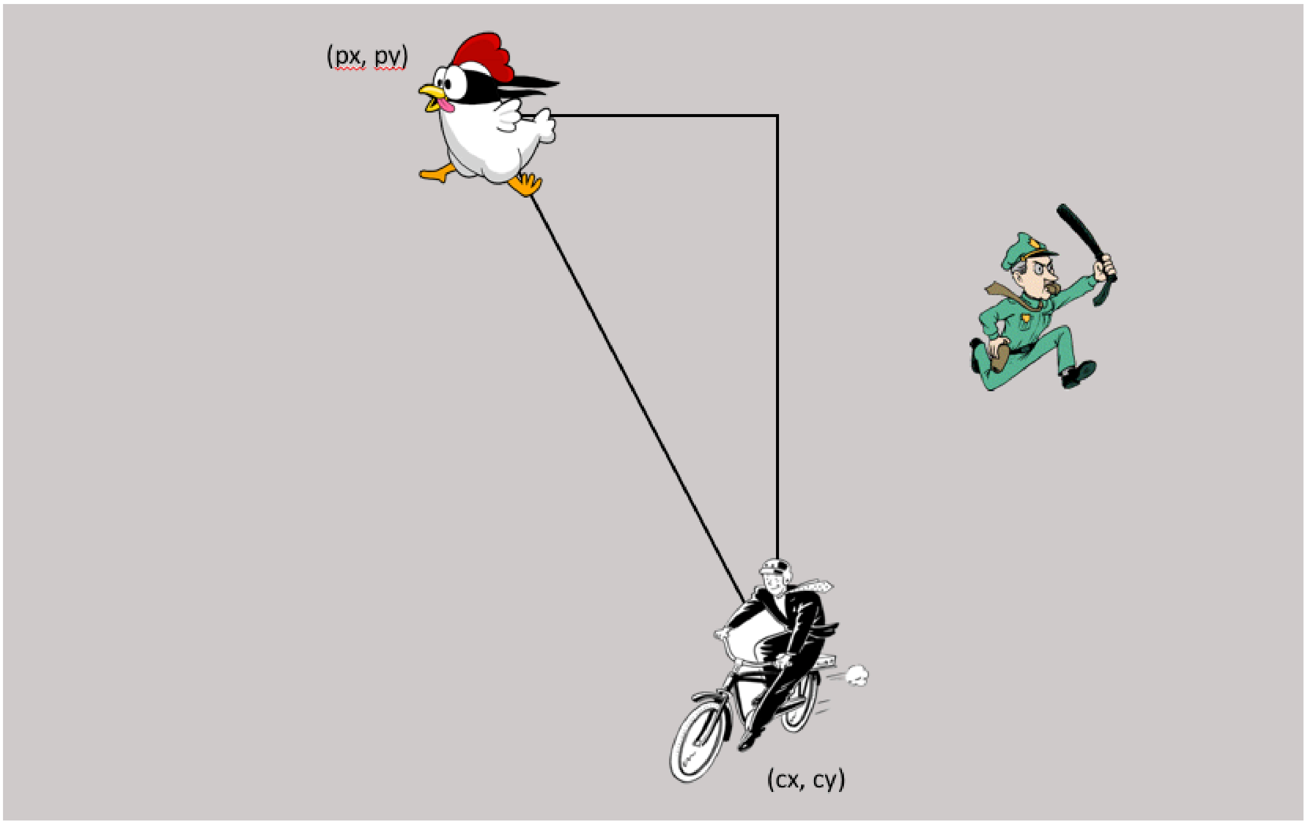
For each of the game screenshots, write the code to calculate the distance between the indicated characters. *The first one has been done for you.*



```
(sqrt (+ (sqr (line-length 600 150)) (sqr (line-length 110 300))))
```



Distance (px, py) to (cx, cy)



Directions: Use the Design Recipe to write a function `distance`, which takes in FOUR inputs: `px` and `py` (the x- and y-coordinate of the Player) and `cx` and `cy` (the x- and y-coordinates of another character), and produces the distance between them in pixels.

Contract and Purpose Statement

Every contract has three parts…

; ______________________ : ____ -> ______________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________________________________ *)* ______________________________________)
function name input(s) what the function produces

(EXAMPLE (______________________________________ *)* ______________________________________)
function name input(s) what the function produces

Definition

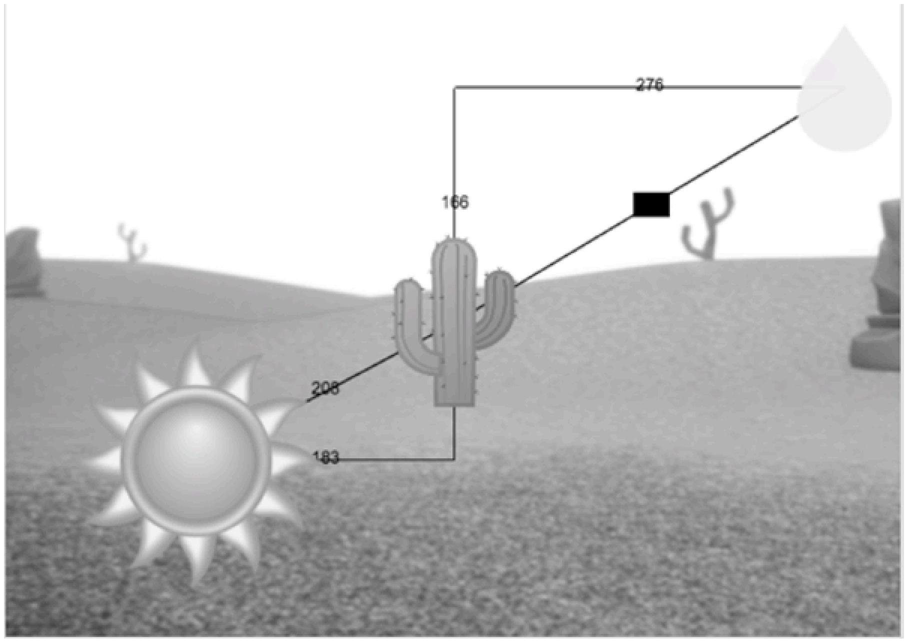
Write the definition, giving variable names to all your input values…

(define (____)
function name variable(s)

____)
what the function does with those variable(s)

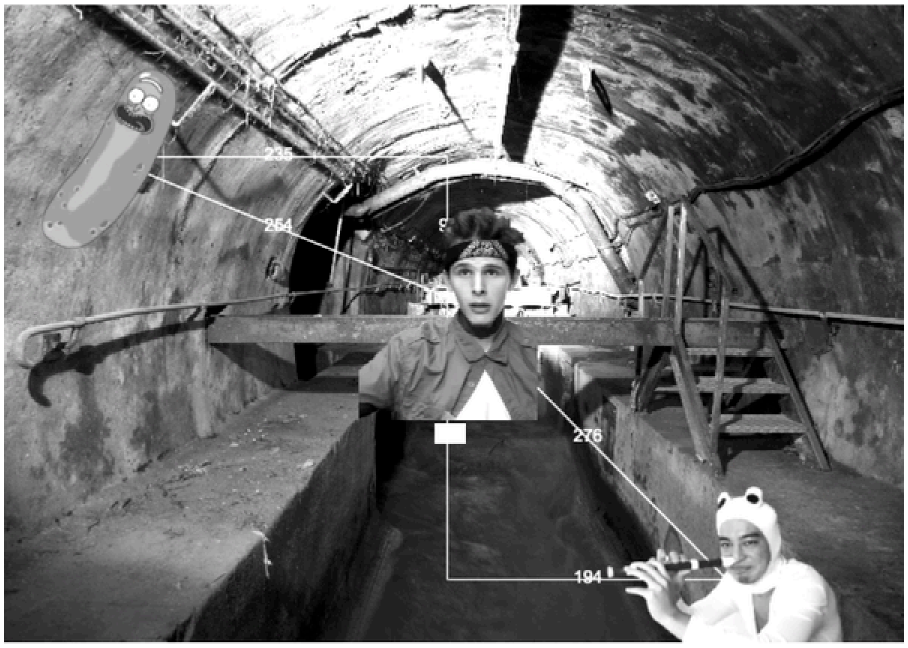
Comparing Code: Finding Missing Distances

For each of the game screenshots below, the math and the code for computing the covered distance is shown. Notice what is similar and what is different about how the top and bottom distances are calculated. Think about why those similarities and differences exist and record your thinking.



$\sqrt{166^2 + 276^2}$

```
(sqrt (+ (sqr 166) (sqr 276)))
```



$\sqrt{276^2 - 194^2}$

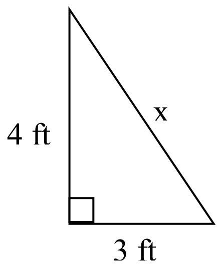
```
(sqrt (- (sqr 276) (sqr 194)))
```

Name:___________________________________ Date: _________ Pythagorean Theorem Practice

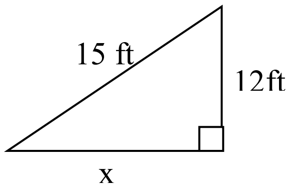
Label the hypotenuse of the triangle c. In each triangle find the length of the side marked *x* to the nearest unit (foot, cm, etc.). Show your work.

$$a^2 + b^2 = c^2$$

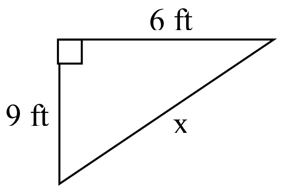
1.



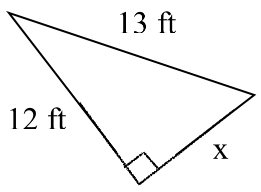
2.



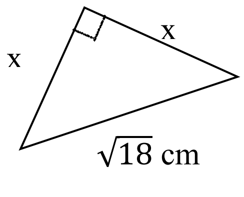
3.



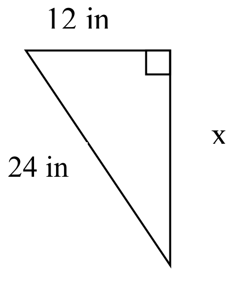
4.



5.

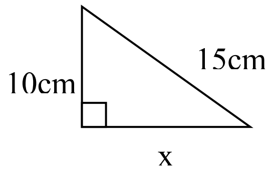


6.

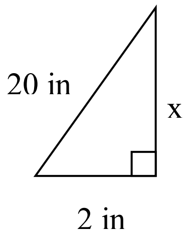


(optional)

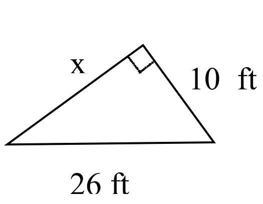
7.



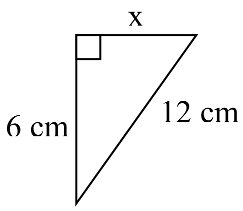
8.



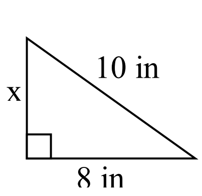
9.



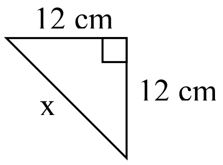
10.



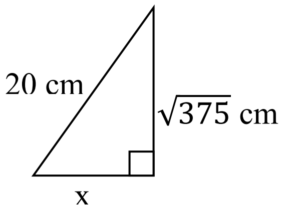
11.



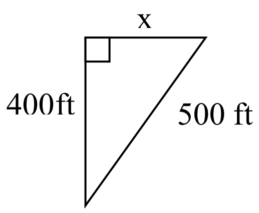
12.



13.



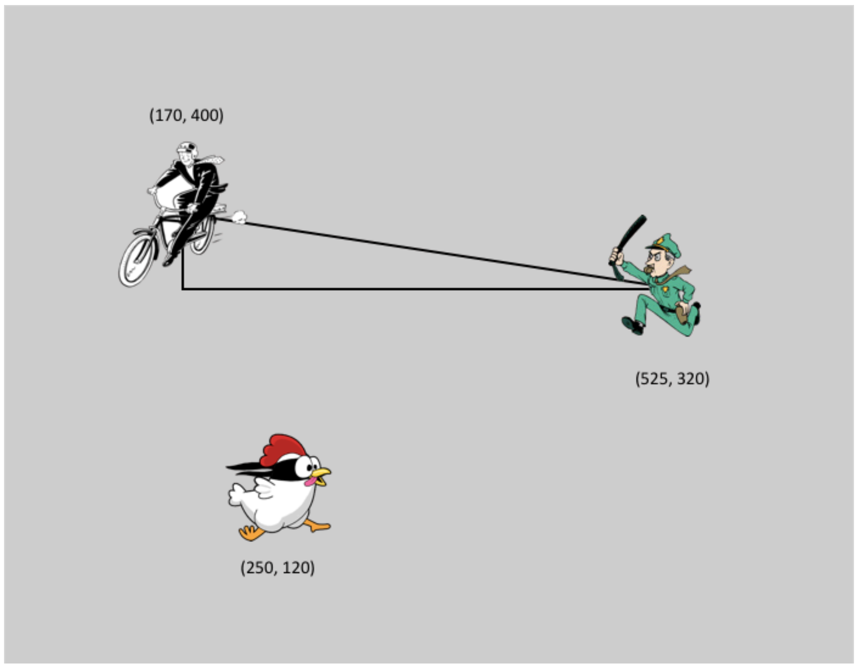
14.

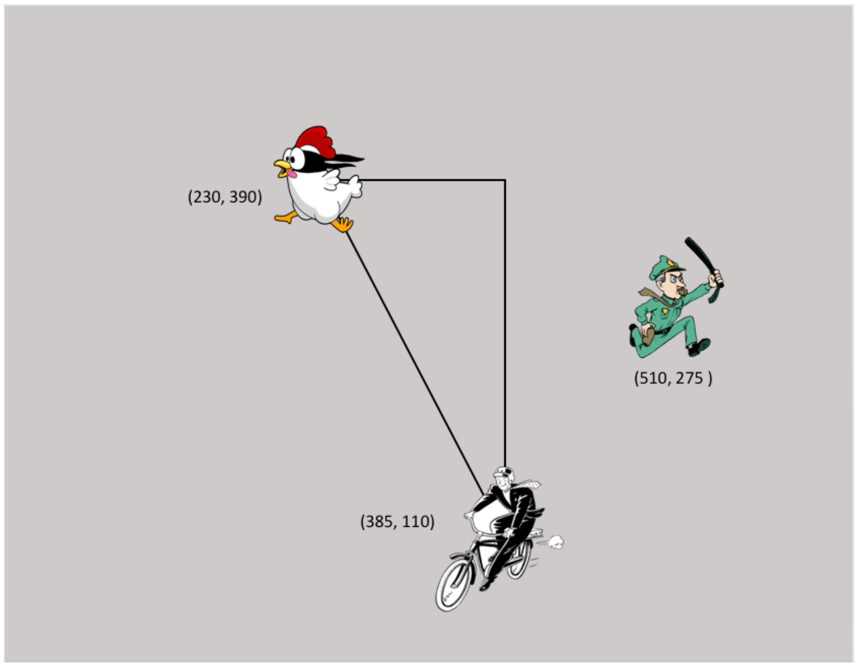


(optional)

Distance From Game Coordinates 2

For each of the game screenshots below, write the code to calculate the distance between the indicated characters. *Refer to Distance from Game Coordinates for an Example.*





(optional)

Word Problem: line-length

Directions: Write a function called `line-length`, which takes in two numbers and returns the **positive difference** between them. It should always subtract the smaller number from the bigger one. If they are equal, it should return zero.

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________
function name Domain Range

; __________
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (`line-length` `10 5`*)* `(- 10 5)`*)*
function name input(s) what the function produces

(EXAMPLE (`line-length` `2 8`*)* `(- 8 2)`*)*
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)
function name variable(s)

(`cond`

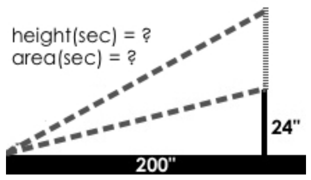
[__________ __________]

[__________ __________]))

(optional)

Top Down / Bottom Up

A retractable flag pole starts out 24 inches tall, and grows taller at a rate of 0.6in/sec. An elastic is anchored 200 inches from the base and attached to the top of the pole, forming a right triangle. Using a top-down or bottom-up strategy, define functions that compute the *height* of the pole and the *area* of the triangle after a given number of seconds.



Directions: Define your first function (`height` or `area`) here.

Contract and Purpose Statement

Every contract has three parts…

; ______________ : ______________________________ -> ______________
function name — Domain — Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________ *)* ______________)
function name — input(s) — what the function produces

(EXAMPLE (______________ *)* ______________)
function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name — variable(s)

____)
what the function does with those variable(s)

Directions: Define your second function (`height` or `area`) here.

Contract and Purpose Statement

Every contract has three parts…

; ______________ : ______________________________ -> ______________
function name — Domain — Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (______________ *)* ______________)
function name — input(s) — what the function produces

(EXAMPLE (______________ *)* ______________)
function name — input(s) — what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name — variable(s)

____)
what the function does with those variable(s)

Word Problem: collision?

Directions: Use the Design Recipe to write a function `collision?`, which takes in FOUR inputs: `px` and `py` (the x- and y-coordinate of the Player) and `cx` and `cy` (the x- and y-coordinates of another character), and makes use of the `distance` function to check if they are close enough to collide.

Contract and Purpose Statement

Every contract has three parts…

; ________________ : ________________________________ -> __________
function name Domain Range

; ____
what does the function do?

Examples

Write some examples, then circle and label what changes…

(EXAMPLE (____________________ *)* ____________________)
function name input(s) what the function produces

(EXAMPLE (____________________ *)* ____________________)
function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (____)
function name variable(s)

____)
what the function does with those variable(s)

Design Recipe

Directions:

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________

function name Domain Range

; __________

what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)

function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)

function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)

function name variable(s)

__________)

what the function does with those variable(s)

Directions:

Contract and Purpose Statement

Every contract has three parts…

; __________ : __________ -> __________

function name Domain Range

; __________

what does the function do?

Examples

Write some examples, then circle and label what changes…

*(EXAMPLE (*__________*)* __________)

function name input(s) what the function produces

*(EXAMPLE (*__________*)* __________)

function name input(s) what the function produces

Definition

Write the definition, giving variable names to all your input values…

(define (__________)

function name variable(s)

__________)

what the function does with those variable(s)

Contracts for Algebra (Wescheme)

Contracts tell us how to use a function, by telling us three important things:

1. The **Name**
2. The **Domain** of the function - what kinds of inputs do we need to give the function, and how many?
3. The **Range** of the function - what kind of output will the function give us back?

For example: The contract `triangle :: (Number, String, String) -> Image` tells us that the name of the function is `triangle`, it needs three inputs (a Number and two Strings), and it produces an Image.
With these three pieces of information, we know that typing `(triangle 20 "solid" "green")` will evaluate to an Image.

Name		Domain		Range
`; *`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Number`
`(* 1 2)`				
`; +`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Number`
`(+ 1 2)`				
`; -`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Number`
`(- 1 2)`				
`; /`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Number`
`(/ 1 2)`				
`; <`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Boolean`
`(< 3 4) ; produces true`				
`; <=`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Boolean`
`(<= 3 3) ; produces true, because 3 is equal to 3`				
`; =`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Boolean`
`(= 3 4) ; produces false`				
`; >`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Boolean`
`(> "a" "b") ; produces false`				
`; >=`	::	($\underset{a}{\underline{\text{Number}}}$, $\underset{b}{\underline{\text{Number}}}$)	->	`Boolean`
`(>= 3 4) ; produces false, because 3 is neither greater-than nor equal-to 4`				
`; above`	::	($\underset{above}{\underline{\text{Image}}}$, $\underset{below}{\underline{\text{Image}}}$)	->	`Image`
`(above (circle 10 "solid" "black") (square 50 "solid" "red"))`				
`; and`	::	($\underset{condition1}{\underline{\text{Boolean}}}$, $\underset{condition2}{\underline{\text{Boolean}}}$)	->	`Boolean`
`(and (> 0 1)  (= 4 4)) ; produces false because both conditions must be true`				
`; beside`	::	($\underset{left}{\underline{\text{Image}}}$, $\underset{right}{\underline{\text{Image}}}$)	->	`Image`
`(beside (circle 10 "solid" "black") (square 50 "solid" "red"))`				

Name		Domain		Range
; circle	::	$(\underset{\text{radius}}{\underline{\text{Number}}}, \underset{\text{fill-style}}{\underline{\text{String,}}}, \underset{\text{color}}{\underline{\text{String}}})$	->	Image
(circle 50 "solid" "purple")				
; ellipse	::	$(\underset{\text{width}}{\underline{\text{Number}}}, \underset{\text{height}}{\underline{\text{Number}}}, \underset{\text{fill-style}}{\underline{\text{String}}}, \underset{\text{color}}{\underline{\text{String}}})$	->	Image
(ellipse 100 50 "outline" "orange")				
; expt	::	$(\underset{\text{base}}{\underline{\text{Number}}}, \underset{\text{power}}{\underline{\text{Number}}})$	->	Number
(expt 3 4) ; three to the fourth power				
; flip-horizontal	::	$(\underline{\text{Image}})$	->	Image
(flip-horizontal (text "Lion" 50 "maroon"))				
; flip-vertical	::	$(\underline{\text{Image}})$	->	Image
(flip-vertical (text "Orion" 65 "teal"))				
; image-url	::	$(\underset{\text{url}}{\underline{\text{String}}})$	->	Image
(image-url "https://bootstrapworld.org/images/icon.png")				
; isosceles-triangle	::	$(\underset{\text{size}}{\underline{\text{Number}}}, \underset{\text{vertex-angle}}{\underline{\text{Number}}}, \underset{\text{fill-style}}{\underline{\text{String}}}, \underset{\text{color}}{\underline{\text{String}}})$	->	Image
(isosceles-triangle 50 20 "solid" "grey")				
; or	::	$(\underset{\text{condition1}}{\underline{\text{Boolean}}}, \underset{\text{condition2}}{\underline{\text{Boolean}}})$	->	Boolean
(or (> 1 0) (= 4 4)) ; produces true if one or more conditions are be true				
; overlay	::	$(\underset{\text{top}}{\underline{\text{Image}}}, \underset{\text{bottom}}{\underline{\text{Image}}})$	->	Image
(overlay (circle 10 "solid" "black") (square 50 "solid" "red"))				
; radial-star	::	$(\underset{\text{points}}{\underline{\text{Num}}}, \underset{\text{outer}}{\underline{\text{Num}}}, \underset{\text{inner}}{\underline{\text{Num}}}, \underset{\text{fill-style}}{\underline{\text{Str}}}, \underset{\text{color}}{\underline{\text{Str}}})$	->	Image
(radial-star 6 20 50 "solid" "red")				
; rectangle	::	$(\underset{\text{width}}{\underline{\text{Number}}}, \underset{\text{height}}{\underline{\text{Number}}}, \underset{\text{fill-style}}{\underline{\text{String}}}, \underset{\text{color}}{\underline{\text{String}}})$	->	Image
(rectangle 100 50 "outline" "green")				
; regular-polygon	::	$(\underset{\text{size}}{\underline{\text{Number}}}, \underset{\text{vertices}}{\underline{\text{Number}}}, \underset{\text{fill-style}}{\underline{\text{String}}}, \underset{\text{color}}{\underline{\text{String}}})$	->	Image
(regular-polygon 25 5 "solid" "purple")				
; rhombus	::	$(\underset{\text{size}}{\underline{\text{Number}}}, \underset{\text{top-angle}}{\underline{\text{Number}}}, \underset{\text{fill-style}}{\underline{\text{String}}}, \underset{\text{color}}{\underline{\text{String}}})$	->	Image
(rhombus 100 45 "outline" "pink")				
; right-triangle	::	$(\underset{\text{leg1}}{\underline{\text{Number}}}, \underset{\text{leg2}}{\underline{\text{Number}}}, \underset{\text{fill-style}}{\underline{\text{String}}}, \underset{\text{color}}{\underline{\text{String}}})$	->	Image
(right-triangle 50 60 "outline" "blue")				
; rotate	::	$(\underset{\text{degrees}}{\underline{\text{Number}}}, \underset{\text{img}}{\underline{\text{Image}}})$	->	Image
(rotate 45 (star 50 "solid" "darkblue"))				
; scale	::	$(\underset{\text{factor}}{\underline{\text{Number}}}, \underset{\text{img}}{\underline{\text{Image}}})$	->	Image
(scale 1/2 (star 50 "solid" "lightblue"))				

Name		Domain		Range
; sqr	::	(Number)	->	Number
(sqr 4)				
; sqrt	::	(Number)	->	Number
(sqrt 4)				
; square	::	($\underset{\text{size}}{\text{Number}}$, $\underset{\text{fill-style}}{\text{String}}$, $\underset{\text{color}}{\text{String}}$)	->	Image
(square 50 "solid" "red")				
; star	::	($\underset{\text{radius}}{\text{Number}}$, $\underset{\text{fill-style}}{\text{String}}$, $\underset{\text{color}}{\text{String}}$)	->	Image
(star 50 "solid" "red")				
; star-polygon	::	($\underset{\text{size}}{\text{Number}}$, $\underset{\text{point-count}}{\text{Number}}$, $\underset{\text{step-count}}{\text{Number}}$, $\underset{\text{fill-style}}{\text{String}}$, $\underset{\text{color}}{\text{String}}$)	->	Image
(star-polygon 100 10 3 "outline" "red")				
; string-contains?	::	($\underset{\text{haystack}}{\text{String}}$, $\underset{\text{needle}}{\text{String}}$)	->	Boolean
(string-contains? "hotdog" "dog")				
; string-length	::	(String)	->	Number
(string-length "rainbow")				
; sum	::	($\underset{\text{table-name}}{\text{Table}}$, $\underset{\text{column}}{\text{String}}$)	->	Number
undefined				
; text	::	($\underset{\text{message}}{\text{String}}$, $\underset{\text{size}}{\text{Number}}$, $\underset{\text{color}}{\text{String}}$)	->	Image
(text "Zari" 85 "orange")				
; translate	::	($\underset{\text{front}}{\text{Image}}$, $\underset{\text{x-coordinate}}{\text{Number}}$, $\underset{\text{y-coordinate}}{\text{Number}}$, $\underset{\text{behind}}{\text{Image}}$)	->	Image
(translate (circle 10 "solid" "black") 10 10 (square 50 "solid" "red"))				
; triangle	::	($\underset{\text{size}}{\text{Number}}$, $\underset{\text{fill-style}}{\text{String}}$, $\underset{\text{color}}{\text{String}}$)	->	Image
(triangle 50 "solid" "fuchsia")				
; triangle/asa	::	($\underset{\text{top-left-angle}}{\text{Number}}$, $\underset{\text{left-side}}{\text{Number}}$, $\underset{\text{bottom-angle}}{\text{Number}}$, $\underset{\text{fill-style}}{\text{String}}$, $\underset{\text{color}}{\text{String}}$)	->	Image
(triangle/asa 90 200 10 "solid" "purple")				
; triangle/sas	::	($\underset{\text{bottom-R-side}}{\text{Number}}$, $\underset{\text{top-R-angle}}{\text{Number}}$, $\underset{\text{top-side}}{\text{Number}}$, $\underset{\text{fill-style}}{\text{String}}$, $\underset{\text{color}}{\text{String}}$)	->	Image
(triangle/sas 50 20 70 "outline" "darkgreen")				

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