

Algebra 2 Fall 2025 Student Workbook - Pyret Edition



Workbook v1.0

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Introduction to Programming in a Nutshell

The **Editor** is a software program we use to write Code. Our Editor allows us to experiment with Code on the right-hand side, in the **Interactions Area**. For Code that we want to *keep*, we can put it on the left-hand side in the **Definitions Area**. Clicking the "Run" button causes the computer to re-read everything in the Definitions Area and erase anything that was typed into the Interactions Area.

Data Types

Programming languages involve different data types, such as Numbers, Strings, Booleans, and even Images.

- Numbers are values like 1, 0.4, 1/3, and -8261.003.
 - Numbers are usually used for quantitative data and other values are usually used as categorical data.
 - In Pyret, decimals *must* start with a zero. For example, 0.22 is valid, but .22 is not.
- Strings are values like "Emma", "Rosanna", "Jen and Ed", or even "08/28/1980".
 - All strings *must* be surrounded by quotation marks.
- Booleans are either true or false.

All values evaluate to themselves. The program 42 will evaluate to 42, the String "Hello" will evaluate to "Hello", and the Boolean false will evaluate to false.

Operators

Operators (like +, -, *, <, etc.) work the same way in Pyret that they do in math.

- Operators are written between values, for example: 4 + 2.
- In Pyret, operators must always have spaces around them. 4 + 2 is valid, but 4+2 is not.
- If an expression has different operators, parentheses must be used to show order of operations. 4 + 2 + 6 and 4 + (2 * 6) are valid, but 4 + 2 * 6 is not.

Applying Functions

Functions work much the way they do in math. Every function has a name, takes some inputs, and produces some output. The function name is written first, followed by a list of *arguments* in parentheses.

- In math this could look like f(5) or g(10, 4).
- In Pyret, these examples would be written as f(5) and g(10, 4).
- Applying a function to make images would look like star(50, "solid", "red").
- There are many other functions in Pyret, for example sqr, sqrt, triangle, square, string-repeat, etc.

Functions have *contracts*, which help explain how a function should be used. Every Contract has three parts:

- The Name of the function literally, what it's called.
- The Domain of the function what type(s) of value(s) the function consumes, and in what order.
- The Range of the function what type of value the function produces.

Strings and Numbers

Make sure you've loaded <u>code.pyret.org (CPO)</u>, clicked "Run", and are working in the **Interactions Area** on the right. Hit Enter/return to evaluate expressions you test out.

Strings

String values are always in quotes.

- Try typing your name (in quotes!).
- Try typing a sentence like "I'm excited to learn to code!" (in quotes!).
- Try typing your name with the opening quote, but without the closing quote. Read the error message!
- Now try typing your name without any quotes. Read the error message!

1) Explain what you understand about how strings work in this programming language.

Numbers

2) Try typing 42 into the Interactions Area and hitting "Enter". Is 42 the same as "42"? Why or why not?

3) What is the largest number the editor can handle?

4) Try typing 0.5. Then try typing .5. Then try clicking on the answer. Experiment with other decimals.

Explain what you understand about how decimals work in this programming language.

5) What happens if you try a fraction like 1/3?

6) Try writing **negative** integers, fractions and decimals. What do you learn?

Operators

7) Just like math, Pyret has <i>operators</i> like $+, -, \star$ and /.
Try typing in $4 + 2$ and then $4+2$ (without the spaces). What can you conclude from this

3) Type in the following expressions, one at a time : 4 + 2 * 6	$(4 + 2) \times 6 4 + (2 \times 6)$	What do you notice?
--	--------------------------------------	---------------------

9) Try typing in 4 + "cat", and then "dog" + "cat". What can you conclude from this?

Booleans

Boolean-producing expressions are yes-or-no questions, and will always evaluate to either true ("yes") or false ("no"). What will the expressions below evaluate to? Write down your prediction, then type the code into the Interactions Area to see what it returns.

	Prediction	Result			Prediction	Result	
1) 3 <= 4			2) "a" > "b"				
3) 3 == 2			4) "a" < "b"				
5) 2 < 4			6) "a" == "b"				
7) 5 >= 5			8) "a" <> "a"				
9) 4 >= 6			10) "a" >= "a"				
11) 3 <> 3			12) "a" <> "b"				
13) 4 <> 3			14) "a" >= "b"				
15) In your own words	15) In your own words, describe what < does.						
16) In your own words	s, describe what $\geq d$	Des					
17) In your own words, describe what <> does							
				Prediction	1:	Result:	
18) string-contai	.ns("catnap", "c	at")					
19) string-contai	.ns("cat", "catn	ap")					
20) In your own words returns true?	s, describe what stri	ng-contains does	s. Can you generate a	nother expres	sion using string-o	contains that	
★ There are infinite st	tring values ("a", "aa", "	aaa") and infinite nu	Imber values out the	re (2,-1,0,-1,	2). But how many d	ifferent Boolean	

values are there?

Applying Functions

Open <u>code.pyret.org (CPO)</u> and click "Run". We will be working in the Interactions Area on the right.

Test out these two expressions and record what you learn below:

- regular-polygon(40, 6, "solid", "green")
- regular-polygon(80, 5, "outline", "dark-green")

1) You've seen data types like Numbers, Strings, and Booleans. What data type did the regular-polygon function produce?

2) How would you describe what a regular polygon is?

3) The regular-polygon function takes in four pieces of information (called arguments). Record what you know about them below.

	Data Type	Information it Contains
Argument 1		
Argument 2		
Argument 3		
Argument 4		

There are many other functions available to us in Pyret. We can describe them using *contracts*. The Contract for regular-polygon is: # regular-polygon :: Number, Number, String, String -> Image

- Each Contract begins with the function name: in this case regular-polygon
- Lists the data types required to satisfy its Domain: *in this case* Number, Number, String, String
- And then declares the data type of the Range it will return: in this case Image

Contracts can also be written with more detail, by annotating the Domain with variable names :

regular-polygon ::	(Number	, Number ,	String ,	String) -> Image
	size	number-of-sides	fill-style	color	-

4) We know that a square is a regular polygon because

#

★ Where else have you heard the word *contract* used before?

Practicing Contracts: Domain & Range

Note: The contracts on this page are not defined in Pyret and cannot be tested in the editor.

is-beach-weather
Consider the following Contract: # is-beach-weather :: Number, String -> Boolean
1) What is the Name of this function?
2) How many arguments are in this function's Domain ?
3) What is the Type of this function's first argument ?
4) What is the Type of this function's second argument ?
5) What is the Range of this function?
3) What is the Type of this function's first argument ?

6) Circle the expression below that shows the correct application of this function, based on its Contract.

A. is-beach-weather(70,	90)	
<pre>B.is-beach-weather(80,</pre>	100,	"cloudy")
C. is-beach-weather("sur	nny",	90)

D. is-beach-weather(90, "stormy weather")

cylinder

Consider the following Contract:
cylinder :: Number, Number, String -> Image
7) What is the Name of this function?
0) U su
8) How many arguments are in this function's Domain ?
9) What is the Type of this function's first argument ?
10) What is the Type of this function's second argument ?
11) What is the Type of this function's third argument ?
12) What is the Range of this function?

13) Circle the expression below that shows the correct application of this function, based on its Contract.

A. cylinder("red", 10, 60)
B. cylinder(30, "green")
C. cylinder(10, 25, "blue")
D. cylinder(14, "orange", 25)

Matching Expressions and Contracts

Match the Contract (left) with the expression that uses it correctly (right). Note: The contracts on this page are not defined in Pyret and cannot be tested in the editor.

Contract		Expression
<pre># make-id :: String, Number -> Image</pre>	1 A	<pre>make-id("Savannah", "Lopez", 32)</pre>
<pre># make-id :: String, Number, String -> Image</pre>	2 В	make-id("Pilar", 17)
<pre># make-id :: String -> Image</pre>	з с	: make-id("Akemi", 39, "red")
<pre># make-id :: String, String -> Image</pre>	4 D) make-id("Raïssa", "McCracken")
<pre># make-id :: String, String, Number -> Image</pre>	5 E	: make-id("von Einsiedel")

Contract		Expression
<pre># is-capital :: String, String -> Boolean</pre>	6 A	<pre>show-pop("Juneau", "AK", 31848)</pre>
<pre># is-capital :: String, String, String -> Boolean</pre>	7 В	show-pop("San Juan", 395426)
<pre># show-pop :: String, Number -> Image</pre>	8 C	<pre>is-capital("Accra", "Ghana")</pre>
<pre># show-pop :: String, String, Number -> Image</pre>	9 D	show-pop(3751351 , "Oklahoma")
<pre># show-pop :: Number, String -> Number</pre>	10 E	is-capital("Albany", "NY", "USA")

Contracts for Image-Producing Functions

Log into <u>code.pyret.org (CPO)</u> and click "Run". Experiment with each of the functions listed below in the interactions area. Try to find an expression that produces an image. Record the contract and example code for each function you are able to use!

Name	Domain		Range
<pre># triangle</pre>	:: Number, String, String	->	Image
<pre>triangle(80, "solid",</pre>	"darkgreen")		
# star	::	->	
# circle	::	->	
<pre># rectangle</pre>	::	->	
# text	::	->	
# square	::	->	
# rhombus	::	->	
# ellipse	::	->	
<pre># regular-polygon</pre>	::	->	
<pre># right-triangle</pre>	::	->	
<pre># isosceles-triangle</pre>	::	->	
<pre># radial-star</pre>	::	->	
<pre># star-polygon</pre>	::	->	
<pre># triangle-sas</pre>	::	->	
<pre># triangle-asa</pre>	::	->	

Catching Bugs when Making Triangles

Learning about a Function through Error Messages

1) Type triangle into the Interactions Area of <u>code.pyret.org (CPO)</u> and hit "Enter". What do you learn?

2) We know that all functions will need an open parenthesis and at least one input! Type triangle(80) in the Interactions Area and hit Enter/return. Read the error message. What hint does it give us about how to use this function?

3) Using the hint from the error message, experiment until you can make a triangle. What is the contract for triangle?

4) Read the explanation below. Then explain the difference in your own words.

syntax errors - when the computer cannot make sense of the code because of unclosed strings, missing commas or parentheses, etc. contract errors - when the function isn't given what it needs (the wrong type or number of arguments are used)

The difference between syntax errors and contract errors is:

Finding Mistakes with Error Messages

The following lines of code are all BUGGY! Read the code and the error messages below. See if you can find the mistake WITHOUT typing it into Pyret.

5) triangle(20, "solid" "red") Pyret didn't understand your program around triangle(20, "solid" **"red"**)

This is a ______ error. The problem is that ______

6) triangle(20, "solid")

This application expression errored:

triangle(20 , "solid")

<u>2 arguments</u> were passed to the <u>operator</u>. The <u>operator</u> evaluated to a function accepting 3 parameters. An <u>application expression</u> expects the number of parameters and <u>arguments</u> to be the same.

This is a ______ error. The problem is that ______

7) triangle(20, 10, "solid", "red")

This <u>application expression</u> errored: **triangle**(20, 10, "solid", "red") <u>4 arguments</u> were passed to the <u>operator</u>. The <u>operator</u> evaluated to a function accepting 3 parameters. An <u>application expression</u> expects the number of parameters and <u>arguments</u> to be the same.

This is a ______ error. The problem is that ______

8) triangle (20, "solid", "red")

Pyret thinks this code is probably a function call: triangle (20, "solid", "red")

Function calls must not have space between the *function expression* and the *arguments*.

This is a ______ error. The problem is that ____

Using Contracts

For questions 1,2,4,5,8 & 9, use the contracts provided to find expressions that will generate images similar to the ones pictured. Test your code in <u>code.pyret.org (CPO)</u> before recording it.

	<pre># ellipse :: (Number ,</pre>	_, <u>Number</u> , <u>String</u> , <u>String</u>) -> Image height fill-style	
1)			
2)			
3)	Write an expression using ellipse to produce a circle.		

	<pre># regular-polygon :: (Numb side-le</pre>	ber , <u>Number</u> , ength , <u>number-of-sides</u> ,	<u>String</u> , <u></u>	String) -> Image ^{color}
4)				
5)				
6)	Use regular-polygon to write an expression for a square!			
7)	How would you describe a regular polygon to a friend?			

	<pre># rhombus :: (Number</pre>	_, <u>Number</u> ,,	String , St fill-style c	ring) -> Image ^{olor}
8)				
9)				
10)	Write an expression to generate a rhombus that is a square!			

Triangle Contracts

Respond to the questions. Go to <u>code.pyret.org (CPO)</u> to test your code.

triangle :: (<u>Number</u>, <u>String</u>, <u>String</u>) -> Image # isosceles-triangle :: (<u>Number</u>, <u>Number</u>, <u>String</u>, <u>String</u>) -> Image

2) Why do you think triangle only needs one number, while right-triangle and isosceles-triangle need two numbers?

3) Write right-triangle expressions for the images below using 100 as one argument for each.



4) Write isosceles-triangle expressions for the images below using 100 as one argument for each.



5) Write 2 expressions that would build **right-isosceles** triangles. Use **right-triangle** for one expression and **isosceles-triangle** for the other expression.



6) Which do you like better? Why?

Composing with Circles of Evaluation

Notice and Wonder Suppose we want to see the text "Diego" written vertically in yellow letters of size 150. Let's use Circles of Evaluation to look at the structure:			
We can start by generating the Diego image.	And then use the rotate function to rotate it 90 degrees.		
text "Diego" 150 "yellow" →	90 text "Diego" 150 "yellow"		
<pre>text("Diego", 150, "yellow")</pre>	<pre>rotate(90, text("Diego", 150, "yellow"))</pre>		
1) What do you Notice?			
2) What do you Wonder?			
Let's Rotate an Image of Your Name! Suppose you wanted the computer to show your name in your favorite color	and rotate it so that it's diagonal		
Write your name (any size), in your favorite color	rotate the image so that it's diagonal		
3) Draw the circle of evaluation:	4) Draw the circle of evaluation:		
5) Convert the Circle of Evaluation to code:	6) Convert the Circle of Evaluation to code:		

Circle of Evaluation to Code (Scaffolded)

Complete the Code by Filling in the Blanks!

Finish the Code by filling in the blanks.

1) Circle 5 "solid" "tan" 9 "solid" "red"			
overlay(circle(, "solid",	_),	_(9,	_, "red"))

Complete the Code by adding Parentheses

For each Circle of Evaluation, finish the Code by adding parentheses and commas.





rotate 8 above star 5 "solid" "gold" triangle 3 "solid" "green"



beside rotate 9 triangle 5 "solid" "blue" circle 8 "outline" "red"

Modeling Data

A Quick Review...

When viewing a cloud of points on a scatter plot, sometimes we can see a pattern in the data.

- If the points cluster around a straight line, it might mean there's a *linear relationship* between the *explanatory variable* (x) and *response variable* (y).
- The line can slope up or down, indicating a *positive* relationship (where the two variables increase together) or *negative* relationship (where the response variable decreases as the explanatory variable increases).

These lines are known as models for the data. The straight-line function describing a linear relationship is called a linear model.

- With a good model, the point cloud will hug the line tightly. A poor model will have lots of points that stray far from the line.
- Models *summarize* the data. For most datasets that means there will be data points that are not *exactly* on the line! And sometimes the line of best fit won't even pass through a single point in the dataset.
- We can use linear regression (lr-plot in Pyret) to compute the best possible linear model for a dataset, known as the line of best fit.

S: Measuring Error in a Model

Differences between the predicted y-value and actual y-value for each x-value are called residuals .	Predicted datapoint
A residual tells us "how wrong" the model was at that particular point.	Residual
Data = Model + Error	Actual datapoint

We can summarize the error for *all* the points in a dataset using the **Standard Deviation of the Residuals** - known as **S** - to get a sense of how much to trust the predictions made by a model.

- **S** is expressed in terms of *units of the response variable* (y) and tells us how much error we expect in predictions made from the model. (*e.g. up to* \$8000, 5 years, 11 inches, etc.)
- The closer the data points are to the model, the smaller the residuals are, and the smaller **S** will be.
- If the S-value for a model is zero, it fits the data perfectly!
- When we compare two models for the same dataset, the one with the lower *S-value* fits better.
- We have no way of knowing whether or not *S-values* represent a small or large amount of error until we consider them in relation to the range of the dataset! (*e.g. errors of \$20,000 are huge in the context of median salary, but small in the context of national budgets.*)

How could we Measure Whether a Model is a Good Fit? (Lizards)

Summarize the Relationship You See

Below is a sample of liz	zards from the Animals	Dataset.						
name	pounds	weeks		7		•(
Amy	5	7						
Aries	4	7		6 🔵 —			•	
Boss	6	5	eeks					
Brittany	3	8	ž	5				
Buck	7	4					T	
Butterscotch	1	6		4				
Chico	8	4		4				•
Coconut	6	6						
Dodger	5	8		3	2	4	6	
Dylan	7	3			2	4	0	c

1) Use a straightedge to draw the line of best fit that best summarizes the relationship you see in the data on the scatter plot.

2) Describe how you decided where to draw the line.

Comparing Models

3) Cy and Jo drew the two lines below. Do you think cy(x) or jo(x) is a better model for this data? Why?





4) What could we measure, to calculate how much better of a model it is?

5) Neither of these models is the best possible model! What would have to be true of a third model, for us to know that it was a better fit than

these two?

Introducing fit-model

These data visualizations were generated using the <u>Lizard Sample Starter File</u>. They can be viewed as interactive charts by uncommenting the final lines in the Definitions Area and clicking "Run".



What do you Notice?	What do you Wonder?

1) How is the fit-model plot for cy's model similar to the fit-model plot for jo's model?

2) How is the fit-model plot for cy's model different from the fit-model plot for jo's model? _____

3) What do you think the three terms in the legend refer to?

- Model:
- Data: ____
- Residuals:

Considering **S** in Context

For each model below, decide whether you agree that the model is a good fit. Then rank the models from 1 (best fit) to 8 (worst fit).

How good is the model?	Ranking
 A data scientist is working with data from animals at a shelter. The range of days to adoption in this dataset are from 0 to 400. An S value of 300 means predicted adoption times could be off by 300 days. 	
Ithat this model is a good fit.	
 2) A student is exploring a dataset on climate change. The range of Arctic Sea Ice is from 3,920,000 to 7,670,000 square kilometers An S value of 300 means predicted Arctic Sea Ice coverage could be off by 300 square kilometers. I that this model is a good fit. 	
 3) A data scientist is working with data from US public schools. The range of graduates per school per year is 2 to 2003. An S value of 300 means predicted graduate values could be off by 300 students. I that this model is a good fit. 	
 4) A student is exploring a dataset on earthquakes. The range of earthquake depths in this dataset are from 4200m to 664000m. An S value of 300 means predicted earthquake depths could be off by 300 meters. 	
 5) A student is exploring a dataset on arrests in Los Angeles. The age range in this dataset is from 0 to 92. An S value of 1 means predicted ages could be off by 1 year. Ithat this model is a good fit. 	
 6) A data scientist is working with data about snowflakes. The range of snowflake weights is from 0.001 grams to 0.02 grams. An S value of 1 means predicted values could be off by 1 gram. Ithat this model is a good fit. 	
 7) A data scientist is working with data from animals at a shelter. The range of ages is from 0.5 years to 16 years. An S value of 1 means predicted ages could be off by 1 year. I that this model is a good fit. 	
 8) A student is working with a dataset of adult blue whales. The range of weights is 200,000 to 330,000 pounds. An S value of 1 means predicted weights could be off by 1 pound. Ithat this model is a good fit strongly agree, agree, disagree, strongly disagree 	

Interpreting our Models

Cy's Model: $cy(x) = -3x + 23$
This model predicts that lizards weighing 0 pounds will be adopted in and that,
for every additional pound a lizard weighs, will by by x-variable units y-variable increase/decrease rate of change
The error in the model is described by an S-value of about I that
this model is a good fit considering that in this dataset range fromtototo
Jo's Model: $jo(x) = -0.8x + 10.4$
This model predicts that lizards weighing 0 pounds will be adopted in and that,
for every additional,,
The error in the model is described by an <i>S-value</i> of about I that
this model is a good fit considering that in this dataset range from to
Comparing Models
1) Is Jo's model better or worse than Cy's model?
2) How much error do we expect in predictions made with Jo's model than predictions made with Cy's model?
Percent Change = $\frac{\text{Difference between the S-values}}{\text{S-value for Cy's model}} \times 100 =$
We expect predictions made with Jo's model to have percent error than predictions made with Cy's model!
My Model
If your teacher had you complete <u>From Lines to Functions</u> , write the function you defined for your model on the line below and then complete the interpretation. If your class did not define models for the lines you drew, you can skip this section.
my-model $(x) =$
This model predicts that lizards weighing 0 pounds will be adopted in, and that,
for every additional,,
The error in the model is described by an S-value of about I
this model is a good fit considering that in this dataset range fromtototo

Exploring the States Dataset

Open the State Demographics Starter File.

Then, click "Run" and type states-table into the Interactions Area on the right to see the dataset.

What do you Notice about this dataset?	What do you Wonder about this dataset?

Describing Income

1) Find the two states with the lowest **median** income and complete the table below with their information.

State	Median Household Income	Per-capita Income

2) Would a model built from two states with low median-income be likely to fit the rest of the data well? Why or Why not?

3) Why do you think states typically have higher median-income than per-capita income?

Definitions in the Code

The two lines of code under # Define some rows extract rows 0 and 1 from the table, and define them as alabama and alaska.

4) Type alabama into the Interactions Area. What do you get back?

5) Underneath the definition of those rows, **add new definitions** for california, michigan, and one other state. Click "Run", so that Pyret reads your new definitions and test them out in the Interactions Area to make sure you defined them correctly!

 \star Add any additional Notices or Wonderings you have about this dataset to the table at the top.

Looking for Patterns

Open the State Demographics Starter File.

What columns do you think might have a relationship?

(e.g. is the number of veterans related to the land-area? Is the population in 2010 related to the population in 2020?)

1) I think that	2) I think that
might be related to	might be related to
because	because

What do you predict the relationships will look like? (e.g. Negative? Positive? Curves?)

Label the axes on the graphs below with your variables from 1 and 2. Then sketch how you imagine they might be related.



If there's a pattern, describe it so that someone else could sketch it. What does it mean? If there isn't a pattern, what does that mean?

5)	6)

More Relationships

Pyret allows us to generate many scatter plots, quite quickly! Take advantage of that to identify two relationships that appear to be strong. Hint: If you're working in the Interactions Area of Pyret, you can hit the up arrow to bring back previously used lines of code.

7) There appears to be a _	negative / positive	_relationship between _	explanatory variable (x-axis)	and	response variable (y-axis)
8) There appears to be a _	negative / positive	relationship between	explanatory variable (x-axis)	and	response variable (y-axis)

★ What have you learned about our state and the others we decided to focus on?

Build a Model from Samples: College Degrees v. Income

The Alabama - Alaska Model

Open the <u>State Demographics Starter File</u>.

1) Record the	epct-college	-or-higher and med	lian-income value	es for the alabama and ala	ska rows, as ((x,y) pairs below:	:
()	()
AL p	ct-college-or-higher	AL median-income	/	AK pct-college-or-hig	her Ak	(median-income	/
2) Compute t as the "predic	he equation of th ctor function", or	e line passing between "line of best fit"), which	thesetwopointsus predictsmedian-	ing the space below. This lin income as a function of pct	e will be your :-college-o	linear model (als r-higher.	so known
3) Write the o	complete model k	pelow (in both Function	and Pyret notation):			
al-ak(x) =	slope(m)	x +y-intercept / verti	ical shift	fun al-ak(x): (* x)	+	end
4) What med	ian income does t	this model expect a stat	te without ANY colle	ege graduates (0%) to earn?			
5) What does A moo	this model predi lel built from Alas	ct? ska and Alabama expec _ and predicts that a 1 p	ts that median incon	ne for a state without ANY o college degrees x-variable units	college graduat is associated w	tes (0%) will be vith a	
	y-units	increase / decrease	in	riable units			
Return to you (If there are al	ur copy of the stan ny errors or warnir	rter file and define al- ngs, fix them and click "Ri	ak on line 44 of the un" again.)	Definitions Area. Then Click	۲ "Run".		
6) In the Inter	ractions Area, try	plugging in the pct-c	ollege-or-high	er value for Alabama by typ	oingal-ak(22	2.6).	
• Ho	ow well does it pr	edict the correct media	an income for Alabai	ma?			
• W	hat would you ty	pe to predict median in	come for Alaska?				
• Ho Co	ow well does it pronsider: If it doesn'	edict the correct media t predict it perfectly, why	an income for Alaska might that be?	?			
Try different	pct-college-	or-higher values fro	om <i>othe</i> r states, to s	ee how well our Alabama-A	aska model fits	s the rest of the c	country.
7) Identify a s	state for which th	is model works well: _					
8) Identify a s	state for which th	is model works poorly:					
Another	Model						
9) Look at the	e scatter plot. Ima	agine you were going to	define another mo	del. Identify two other states	that would lik	ely generate a be	etter fit.

and

Fitting Models: College Degrees v. Income

Open your copy of the <u>State Demographics Starter File</u>.

The al-ak Model			
Type fit-model(states-tabl Area, then find the points represent <i>look</i> !	e, "state", "pct-college-or- ting AL and AK along the predictor line.	higher", "median-income", Hint: You know their coordinates and t	al-ak) in the Interactions hey will help you know where to
1) What do you Notice?			
2) What do you Wonder?			
3) Find S in the upper left corner. V	Vhat is the S value (the number after S	S)?	
4) With median income ranging from	m to lowest median income highest median	, what does the S -value of t nincome	he al-ak model tell us?
Comparing Models			
5) Use fit-model to find the $S-v_{s}$	alue for the mi-ca model.		
6) Is the mi-ca model better or wo	orse than the al-ak model?		
7) How much error do v	we expect in predictions made with the	mi-ca model than predictions mad	e with the al-ak model?
Percent Change = $\frac{\text{Diffe}}{3}$	$\frac{100}{\text{S-value for al-ak model}} \times 100$	= mi-ca model predictions	are expected to have
percent	error than al-ak model predic	tions!	
A Model of Your Own			
8) Identify two other states that you	u think would make a better model:	and	
Add two new definitions for these	states to your <u>State Demographics Sta</u>	arter File.	
9) Record the college-or-high	er and median-income valuesfort	nese states, as (x,y) pairs below:	
(,,,) median-income	(,,,,,,,) median-income
10) Derive your model and write it l	below (in both Function and Pyret notat	tion), then fit the model and record th	ne <i>S</i> -value:
other(x) = slope(m)	X +		
<pre>fun other(x): (</pre>	* x) +	end S:	_
11) Adjust the slope and y-intercep below:	t of your model to get the smallest $ S$ po	ossible. Write the best model you fin	d (and corresponding S)
<pre>fun best(x): (</pre>	* x) +	end	-
12) How much error do w	e expect in predictions made with your	model than predictions made with t	ne mi-ca model?%

Optimizing and Interpreting Linear Models

Open your copy of the <u>State Demographics Starter File</u> and click "Run".						
Build a Model Computationally						
1) Evaluate lr-plot(states-table, "state", "pct-college-or-higher", "median-income").Whatis S?						
2) On the line below, write the optimal linear model that was computed through linear regression:						
$optimal(x) = \underbrace{x + \underbrace{y-intercept/vertical shift}}_{y-intercept/vertical shift} \qquad fun optimal(x): (\underbrace{x + \underbrace{y-intercept/vertical shift}}_{y-intercept/vertical shift}$						
Interpret the al-ak Model						
We started with a model based on Alabama and Alaska (fun $al-ak(x)$: (5613.67 * x) + -83616.02 end), and got an S-value of ~36164.68. We can interpret this as follows:						
Alabama-Alaska model predicts that a 1 percent increase in percent college degrees is sensible name x-axis units explanatory variable (x)						
associated with a <u>~5614 dollar</u> increase in rate of change in y-units increase / decrease in response variable (y)						
Withmedian household incomeranging from\$39,031to\$73,538and an S -value ofresponse variable (y)lowest y-valuehighest y-value						
~36,164.68 dollars , I strongly disagree that this model is a good fit.						
Interpreting the Optimal Model The increase in increase in is						
associated with a in						
rate of change in y-units increase / decrease response variable (y)						
With ranging from to and an S -value of response variable (y) lowest y-value						
<i>S</i> -value <i>y</i> -axis units <i>y</i> -axis u						
5) What median income does this model predict when 30% of a state's population has attended college?						
6) Why doesn't it make sense to use this model to predict median income for a state with no college attendees (0%)?						
• What if it goes from 50% to 51%?						
 What if it goes from 90% to 91%?						
 Does this model predict the same increase in income for every additional 1% college-or-higher? Why or why not? 						

Which Form is Best?

For each set of data provided below,

- Decide which form of the line would be the easiest to build from the available information.
- Write a definition of the linear model in that form.
- Translate the definition into Pyret notation.



Exploring the Aaron Judge Dataset

For this page, you'll need to open the <u>Aaron Judge Starter File</u> on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. **Read the comments at the top of the file**, which describe what each column in the dataset means.

1) What is the name of the table being loaded in this file?

Lookingfo	or Possible Relatio	nships								
We can look for	r potential relationships	between quantitat	ive columns b	by build	ing scatte	r plots. Le	et's see w	hat the	y reveal.	
2) Make a scatt	er plot comparing	pitch speed explanatory variab	ar	nd	hi respo	t speed	e	<u> .</u>		
• Doe:	s there appear to be a rel	ationship betweer	npitch-spe	ed and	lhit-spe	eed? If so	o, describe	e it		
• Base • V	ed on the scatter plot, wh Why?	at hit speed would	l you predict f	or a ba	ll pitched a	it 95mph	?		_	
• How	/ confident are you in you	r prediction?	not at all	1	2	3	4	5	very	
3) Make a scatt	er plot comparing	pitch speed explanatory variab	ar	nd	hit respo	distance	le			
• Does	s there appear to be a rel	ationship betweer	npitch-spe	ed and	lhit-dis	stance?	lf so, des	cribe it.		
• Base • V	ed on the scatter plot, wh	at hit distance wou	uld you predic	t for a	ball pitche	d at 95m	ph?			
• How	r confident are you in you	r prediction?	not at all	1	2	3	4	5	very	
4) Make a scatt • Doe:	er plot comparing s there appear to be a rel	bat angle explanatory variab ationship betweer	ar le bat-angle	nd e and h	hit respo	distance onse variabl	^{le} so, descril	 be it		
• Base	ed on the scatter plot, wh	at distance would	you predict fo	or a bal	hit at an a	ngle of 6	0 degrees	5?		
• V • How	Why? v confident are you in you	r prediction?	not at all	1	2	3	4	5	very	
5) Make a scatt	er plot comparing	bat angle explanatory variab	ar le	nd	hi respo	t speed	le escribe it			
					cc-speed	1. 11 30, 0				
• Base • V	ed on the scatter plot, wh Why?	at speed would yo	u predict for a	a ball hi	t at an ang	le of 20 c	legrees?_			
How How O What is happed	v confident are you in you pening on lines 21 - 36 of	r prediction? the starter file?	not at all	1	2	3	4	5	very	
		_								

What Shape of model would Fit the Data?

For this page, you'll need to open the <u>Aaron Judge Starter File</u> on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.

Note: For this page we will be focusing on just the curve balls, so make sure you are writing code using curve-table

Fitting Linear Models

1) Use lr-plot(curve-table, "id", "bat-angle", "hit-distance") to find the optimal linear model.

What is S for this model?

Write the model below, in both math and Pyret notation.

	y = X +y-intercept/vertical shift	fun y(x): (* x) + end
2) Sketc	h the linear model from your lr-plot below!	3) What do you Notice?
hit-distance		4) What do you Wonder?
		5) Is the best-possible linear model a good fit? Why or why not?
	bat-angle	

Fitting Curves



') How would you describe the pattern?									
8) At what bat-angle is the curve's "peak"?									
9) Based on your best-guess curve, predict the hit-distance for new hits at each bat-angle below.									
• 15 degrees:									
• 35 degrees:									
• 50 degrees:									

What Kind of Model? (Descriptions)

Decide whether each situation sounds like it would be better modeled by a linear or quadratic function, and circle your answer.							
1) A car is 50 miles away, traveling at 65mph. How far away is the car after each hour?							
Linear	Quadratic						
2) The data plan for a cell phone bill costs $5/gb$, plus $15/mo$. How much is the bill for a given month, after x number of gigabytes?							
Linear	Quadratic						
3) A cannonball is fired from the deck of the S.S. Parabola, and arcs throug	gh the sky before hitting its target, 17 miles away.						
Linear	Quadratic						
4) The area of a circle, as its radius increases.							
Linear	Quadratic						
5) The circumference of a circle, as its radius increases.							
Linear	Quadratic						
6) A ball is dropped from the top of the Empire State Building, and starts falling faster and faster. How far has it dropped after x seconds?							
Linear	Quadratic						
\star A ball is dropped from the top of the Empire State Building, and starts falling faster and faster. How fast is it moving after x seconds?							

Linear

Quadratic

What Kind of Model? (Tables)

Decide whether each representation is best described by a linear model, a quadratic model, some *other* model, or no model at all! Record how you decided by showing any work that you feel is useful or writing an explanation.

For Class Discussion:

4	х	0	1	2	3	4	5	6	Linear Quadratic
1	у	5	6	9	14	21	30	41	Some other model/pattern No model
	х	0	1	2	3	4	5	6	Linear
2	у	0	3	6	9	12	15	18	Some other model/pattern
									Nomodel
For Ir	depen	dent Pr	actice:						
	x	1	2	3	4	5	6	7	Linear
3	у	1	3	5	7	9	11	13	Some other model/pattern
	x	-3	-2	-1	0	1	2	3	Linear
4	у	-23	-38	-47	-50	-47	-38	3 -23	Some other model/pattern
									No model
	×	2	2	1	0	1	2	2	Linear
5	X	-3	2	1	2	1	1	1	Quadratic Some other model/pattern
	y	1	2	-	2	1	1	1	No model
									Linear
6	X	1	2	3	4	5	6	7	Quadratic Some other model/pattern
	У	2	5	10	17	26	37	50	No model
7	х	1	2	3	4	5		6	Linear Quadratic
	У	10	100	1000	10000	1000	000	1000000	Some other model/pattern No model
0	х	1	2	3	4	5	6	7	Linear Quadratic
0	у	100	102	105	109	114	12	0 127	Some other model/pattern No model

Parabolas

1) Sketch a *parabola* on each of the grids below that matches the description.

one x-intercept, opens down

two x-intercepts, opens up

no x-intercepts







2) Label the vertex, root(s), and y-intercept of the parabola below with:

A) the coordinates

B) the vocabulary word that describes it



3) Draw a dotted line representing the *axis of symmetry* and label it with the equation that defines it.

Graphing Quadratic Models

For this page, you'll need to have **Exploring Quadratic Functions (Desmos)** open to **Slide 1: Transforming Parabolas**. The parabola you'll see is the graph of $g(x) = x^2$. Another, **identical** parabola is hiding behind it. This second parabola is written in Vertex Form: $f(x) = a(x - h)^2 + k$. Each model setting starts at the value that makes f(x) equivalent to g(x).

1) Use the starting values of a, h, and k you see in Desmos, to complete this equation: $g(x) = x^2 = f(x) =$ _____ (x -____)² + ____

Magnitude *a*

2) In the first square below, make a sketch of the original graph you see (a = 1, h = 0, k = 0).

Then try changing the value of *a* to 10, 0.1, 0, -10 and -2, graphing each parabola in the squares below. Label the vertex "V" and any roots with "R"!



3) What does a tell us about a parabola?

Translation

Horizontal Translation h

Set a back to 1. Change the value of h to -5 and 5, graphing each parabola in the squares below.





Label the vertex "V" and any roots "R"!

4 What does *h* tell us about a parabola?

Vertical Translation k

Set h back to 0. Change the value of k to -5 and 5, graphing each parabola in the squares below.





k = 5

Label the vertex "V" and any roots "R"!

5 What does *k* tell us about a parabola?

Modeling Bat Angle v. Hit Distance (Curveballs)

Vertex form of	a Quadratic Model							
Vertex Form:	$f(x) = a(x-h)^2 + k$	 <i>a</i> : determines whether the parabola opens up or down and how steep the curve is <i>h</i> : <i>horizontal shift</i> (also the x-coordinate of the vertex! <i>h</i> is often 0) <i>k</i> : <i>vertical shift</i> (also the y-coordinate of the vertex!) 						
1) We've determined	that the optimum bat angl	e is around 30 degrees. What variable in the equation should we replace with 30?						
2) What y-coordinate	?) What y-coordinate of the vertex (<i>vertical shift</i>) would best match the shape of the curve?							
3) Does it make sense for <i>a</i> to be negative or positive for this curve?								
Fitting the Mo For this section, you'll r	del Visually in Desm need to have Exploring Quad	າດຣ dratic Functions(Desmos) open to Slide 3: Fitting a Model: Bat Angle v Hit Distance (Curveballs).						
4) Using your thinking the sliders until you'v	g about the values of a , h , e landed on the best mode	, and k from above, adjust the sliders to fit a quadratic model to the data. Continue adjusting I you can. Record your values for a , h , and k below.						
a:	<i>h</i> :	k:						
5) Using the values of fun curve(x)	<i>a</i> , <i>h</i> , and <i>k</i> you decided : (*	on in the Desmos file, define your quadratic function below in Pyret notation.						
Testing how G Return to your copy of	ood the Model is in I the <u>Aaron Judge Starter File</u> ,	Pyret , adjust the definition for curve(x) on line 42, and click "Run".						
6) Use fit-model to Hint: If you for	ofit curve(x) to the cur got the contract for fit-mc	rve-table data. What <i>S</i> -value did you get? odel, <i>look it up in the <u>Contracts Pages</u>!</i>						
7) The S -value for th	e optimal linear model was	s about 104 feet.						
My quadratic	model should do a little b	at predicting hit distances from bat angles.						
What does this	s model actually me	an?						
After experimenting,	I came up with a quadratic	model for this dataset showing that is correlated to						
The vertex of the para	abola drawn by this model	is a at about, which means that, which means that						
Before this point, as b	pat angle increases, hit dist	ance After this point, as the angle increases hit distance						
The error in the mode	el is described by an S-valu	e of about Ithatthat						
this model is a good fi	t considering that	in this dataset range fromtoto y-variablehighest y-valuehighest y-value						

What can we learn from each form of a Quadratic Function?

Draw arrows from each form to the information which you can easily learn from it.

roots (x-intercepts)

Factored Form:

y = -2(x-5)(x+6)

coordinates of vertex

Standard Form:

 $y = 3x^2 + 5x + 6$

the parabola opens up / down

Vertex Form:

 $y = 2(x - 5)^2 + 17$

y-intercept

axis of symmetry

What Kind of Model? (Definitions)

Circle whether each representation below describes a **linear** or **quadratic** function, or neither. If the function is quadratic, fill in all the information that you can *easily read from the form*, without any calculation!

1)	$f(x) = 3x^2 + 22$	2) $g(x) = 2(x-11)(x-243)$
	Linear Quadratic Neither	Linear Quadratic Neither
If it's Quadratic		If it's Quadratic
Ţ	Form: Factored, Vertex, Standard Parabola opens: up/down	Form: Factored, Vertex, Standard Parabola opens: up/down
-	axis of symmetry (x=) Vertex (x, y)]	axis of symmetry (x=) Vertex (x, y)]
	Roots (write both coords)	Roots (write both coords)
3)	h(y) = 100 - $4y$	4) $z(x) = rac{3}{5} \; x + 7$
	Linear Quadratic Neither	Linear Quadratic Neither
lf it	's Quadratic	If it's Quadratic
٦	Form: Factored, Vertex, Standard Parabola opens: up/down	Form: Factored, Vertex, Standard Parabola opens: up/down
-	axis of symmetry (x=) Vertex (x, y)]	axis of symmetry (x=) Vertex (x, y)]
	Roots (write both coords)	Roots (write both coords)
5)	<pre>fun graph(x): 12 * x end</pre>	6) fun m(p): 2 * (p - 5) * (p - 16) end
	Linear Quadratic Neither	Linear Quadratic Neither
If it's Quadratic		If it's Quadratic
٦	Form: Factored, Vertex, Standard Parabola opens: up/down	Form: Factored, Vertex, Standard Parabola opens: up/down
-	axis of symmetry (x=) Vertex (x, y)]	axis of symmetry (x=) Vertex (x, y)]
	Roots (write both coords)	Roots (write both coords)
7)	$r(s) = 42(s - 10)^2 - 3$	8) fun f(x): (2 * sqr(x - 1)) + 15 end
	Linear Quadratic Neither	Linear Quadratic Neither
lf it	's Quadratic	If it's Quadratic
٦	orm: Factored, Vertex, Standard Parabola opens: up/down	Form: Factored, Vertex, Standard Parabola opens: up/down
-	axis of symmetry (x=) Vertex (x, y)]	axis of symmetry (x=) Vertex (x, y)]
	Roots (write both coords)	Roots (write both coords)

Build a Model from Samples - Batting Data

We're going to compute a quadratic function from 3 representative points in the curve-table we've been working with. While our model would be most accurate if we kept track of all of the decimal values, for today we are going to simplify our calculations by doing some rounding.



3) In the space below - or on another sheet of paper - solve this series of equations for *a*, *b*, and *c*. As we solve the series, we'll likely get some very long decimal values. You can round these values to the thousandths place as we work.

4) Write your finished model in function and Pyret notation below.



5) Update the function definition for f on line 57 of the <u>Aaron Judge Starter File</u> and test it out using fit-model!
Exploring the Covid Dataset

For this page, you'll need to h make a copy of the file that's	ave the <u>Covid Sprea</u> just for you.	<u>d Starter File</u> ope	n on your computer.	r. If you haven't already, select Save a Copy from the "File" menu to
1) Take a look at the Defini	tions Area and find	the "Notes on C	olumns". What is th	the start date for the data in this table?
2) Click "Run", and evaluate	ecovid-tablein	the Interactions	s Area. What do yo	ou notice?
3) Evaluate MA1 in the Inte	ractions Area. Wha	nt does it return?		
4) Evaluate CT1. What info	rmation do you lea	rn?		
5) Evaluate NH1. Why is it '	unbound" and how	could we make	it work?	
6) Define three new Rows	called NH1, RI1 and	d VT1, for New H	Hampshire, Rhode I	e Island and Vermont. Click "Run" and test them out.
a. How many people	e in Vermont had te	ested positive by	June 10th, 2020?	?
b. How many people	e in New Hampshir	e tested positive	e by June 10th, 202)20?
c. How many people	e in Rhode Island te	ested positive by	June 10th, 2020?	?
7) In Pyret, make a scatter Sketch the resulting scatte	plot from covid-1 r plot below.	able showing	the relationship be	etween day and positive, and using state as your labels.
000000 e				
				8) In which state did the number of cases grow fastest?
200000				
150000				9) In which state did the number of cases grow <i>slowest</i> ?
				10) Are these strong or weak relationship(s)?
-50	50	100	da 150	day
-00	90	190	190	

11) What do you Notice? _____

12) What do you Wonder? _____

Linear Models for Covid in Massachusetts

For this page, you'll need to have the <u>Covid Spread Starter File</u> open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.

1) Evaluate covid-table and MA-table in the Interactions Area. How are these tables different?

2) Discuss lines 27-34 with your partner. What do you think is going on here?

3) Make a scatter plot from the MA-table, using state as the labels, and day and positive as the x- and y-axis. Sketch the plot below.



day

4) Use lr-plot to obtain the *best-possible linear model* for the MA Covid data, graph it on the plot above, and define it below:

y =		<i>S</i> =			
The optimized linear model for this dataset predicts	an increase / decrease	of about	slope	y-variable	per
The error in the model is described by an S-valu	e of abouts	units	, whicl	is a poor, ok, goo	fit considering that d
in this dataset range from	to lowest y-value	highest y-va	alue		
Change the definition of the linear function in the	Covid Spread Starte	<u>er File</u> to mat	ch the mode	l produced by l r	-plot and "Save".
Do you think a linear function is a good fit for this da	taset? Why or why n	ot?			
	is a starally doine?				
what do you think the code that defines MA- Lab Le	is actually doing? _				

Quadratic Models for Covid in Massachusetts

Fitting the Model Visually $f(x) = a(x - h)^2 + k$ For this section, you'll need to have **Slide 1: Quadratic Model for MA** of **Modeling Covid Spread (Desmos)** open on your computer.

1) Try changing the values of a, h and k to find three promising quadratic models, graphing each one and labeling your values on the grids below.

$a = \underbrace{\begin{array}{c} 40000 \\ h = \\ k = \underbrace{\\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	$a = \underbrace{\frac{00000}{10000}}_{00000}$	50 1 <u>1</u> 0 1 <u>2</u> 6	a = h = k =	-100000	10 126 20
2) Do your quadratic models open up or dowr	n?What	does that tell us abou	t a?		
3) Describe one of your models: Where is the	vertex?(,,) What is the h	norizontal shift?	The vertica	al shift?
4) Which quadratic form would be the easiest	to fit to this data?	standard \Box	factored \Box	vertex□	
Fitting the Model Programmatic For this section, open your copy of the <u>Covid Spre</u>	cally $f(x) = a(x - h)$ ead Starter File.	$(k)^{2} + k$			
5) In the space below, define quadratic1 to	o be the first model ye	ou fit in Desmos.			
<pre>fun quadratic1(x): (a</pre>	* (sqr(x	(–h))) +	k	end
 6) Return to <u>Covid Spread Starter File</u> and uport Then click "Run" to load your updated definiti 7) Use fit-model to determine the <i>S</i>-value 	date the definitions fo on. e of each model using	or quadraticl,qu gthe MA-table.	adratic2,and q	uadratic3.	
Hint: If you forgot the contract for fit- S for quadratic1:	model,lookitupintl S for quadra	he <u>Contracts Pages</u> ! tic2:	<i>S</i> for	quadratic3: _	
What does this model actually m	nean?				
After experimenting, the best quadratic mode	el I came up with for t	his dataset shows tha	tare c are c	orrelated to	iable
The vertex of the parabola drawn by this mod	el is a minima or ma	at about xima?	, W	hich predicts that	
The error in the model is described by an S-va	lue of abouts	units	Istrongly	ragree, agree, disagree,	strongly disagree
that this model is a good fit considering that _	y-variable	_ in this dataset rang	e from lowest y	-value toh	ighest y-value
Are Quadratic Models a Good Fi	t for This Data?				
8) Would you feel good about making predicti	ions based on these n	nodels? Why or why n	ot?		

What Kind of Model? (Tables)

Decide whether each table is best described by a linear, quadratic, or exponential model.

If you chose exponential: What is the growth factor? Doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?

HINT: Can you draw the arrows to calculate the first difference? The second? What does it mean if neither one is constant?

		x	У				x	У	
		1	5				0	10	
		2	10				1	100	
		3	15				2	1000	
		4	20				3	10000	
		5	25				4	100000	
		6	30				5	1000000	
		7	35				6	10000000	
1)	Linear	Quadratic	Expone	ential factor	2)	Linear	Quadratic	Expone	ntial factor
		x	у				x	у	
		70	-169				-3	36	
		71	-126				-2	16	
		72	-81				-1	4	
		73	-34				0	0	
		74	15				1	4	
		75	66				2	16	
		76	119				3	36	
3)	Linear	Quadratic	Expone	ential factor	4)	Linear	Quadratic	Expone	ntial factor
		x	у				x	у	
		0	3				-5	466656	
		1	6				-4	7776	
		2	12				-3	1296	
		3	24				-2	216	
		4	48				-1	36	
		5	96				0	6	
		6	192				1	1	
5)	Linear	Quadratic	Expone	ential	*	Linear	Quadratic	Expone	ential

Graphing Exponential Models: $f(x) = a(b)^{x} + k$

For this page, you'll need to have **Modeling Covid Spread (Desmos)** open to **Slide 3: Exploring Exponential Models**. The curve you'll see is the graph of g(x), an exponential function. Another, **identical** curve f(x) is hiding behind it.

1) Use the starting values of a, b, and k in Desmos to complete the following equation: $g(x) = f(x) = (-b)^{x} + (-b)$

Base *b*

2) Make sure k = 0 and a = 1. Experiment with b. For what values of b is the function *undefined*, with the line disappearing? ______ **3)** Keeping a = 1 and k = 0, change b to 0.5, 1, and 2, graphing each curve below. For each curve, label the coordinates at x=1, 2, and 3.



4) What does b tell us about an exponential function, when b > 1?

5) What does b tell us about an exponential function, when 0 < b < 1?

Vertical Shift...and Horizontal Asymptote k

6) Keeping a = 1 and b = 2, try changing the value of k to -10, 0, and 10, graphing each curve in the squares below. For each curve, find and label the y-value where the curve is "most horizontal", then **draw a horizontal line at that y-value**.



7) What does k tell us about an exponential function?

Initial Value *a*

8) Set k = 0 and b = 2. Change the value of a to 10, 2, and -5, graphing each curve in the squares below. For each curve, label the y-intercept (x=0).



9) What does *a* tell us about an exponential function?

What Kind of Model? (Graphs & Scatter Plots)

Are these scatter plots best be described by linear, quadratic, or exponential models? If it's exponential, draw the asymptote!



What Kind of Model? (Descriptions)

Decide whether each situation is best described by a linear, quadratic, or exponential function. If the function is exponential: Identify the growth factor and the initial value.

Car Values

A particular kind of car sells for \$32,000, and its resale value drops by 12.5% each year.

1) Is the function increasing or decreasing?

2) When the car is brand-new (x=0), how much is it worth?

3) How much is it worth after...

(1 year) x=1	(2 years) x=2	x=3	x =4

4) What is the **form** of this function? linear \Box quadratic \Box exponential \Box

5) If it's exponential,

Fill in the blanks to write a function that shows the value of the car after a given number of years:

f(x) = initial value	a ×grow	/th factor	<u>ь</u> х +	horizontal asymptote k
ls it exponential	growth ? \Box	or	decay ? 🗆	

Lemonade Stand

Sally is selling lemonade, for \$1.25 a glass in hopes of finally be able to get the power drill she's been wanting. She starts with \$20 cash.

6) Is the function increasing or decreasing?

7) When Sally starts the day (x=0), how many dollars does she have?

8) How many dollars will she have after...

(first sale) x=1	(second sale) x=2	x =3	x = 4

9) What is the form of this function?	🗆 linear	□quadratic	exponential

10) If it's exponential,

Fill in the blanks to write a function that shows how much Sally has saved after a given number of sales:

 $f(x) = \underbrace{\qquad}_{\text{initial value } a} \times \underbrace{\qquad}_{\text{growth factor } b} \times + \underbrace{\qquad}_{\text{horizontal asymptote } k}$

Is it exponential growth? \Box or decay? \Box

What Kind of Model? (Descriptions 2)

Decide whether each situation is best described by a linear, quadratic, or exponential function. If the function is exponential: Identify the growth factor and the initial value.

High Fives

Two students started a club. At every meeting, students in attendance must high-five each of the other students. Club membership has been growing steadily by one student each meeting.

1) Is the function increasing or decreasing?

2) When the 2 students started the club (x=0), how many high-fives happened?

3) How many high-fives happen at the subsequent meetings...

(3 students) x=1	(4 students) x=2	x=3	x=4

4) What is the **form** of this function? linear
quadratic
exponential

5) If it's exponential,

Fill in the blanks to write a function that shows the how many high-fives happen for a given number of students:

f(x) =		×		х	+	
0.1.1	initial value <i>a</i>		growth factor b			horizontal asymptote k

Is it exponential growth ? \Box or decay ? \Box

Going Viral

A student posted a video of a dog doing a back flip into a pile of laundry, and the meme went viral! Each person that sees the meme falls in love with it, and shares it with 10 new friends.

6) Is the function increasing or decreasing?_____

7) When the student posts it (x=0), how many total times has it been shared?_____

8) How many times will it have been shared after...

(the next person shares) x=1	(their friends share) x=2	x=3	x=4

9) What is the **form** of this function? linear \Box quadratic \Box exponential \Box

10) If it's exponential,

Fill in the blanks to write a function that shows how many times the meme has been shared after a given number of "share cycles":

horizontal asymptote k

f(x) = _____ x ___ x ___ represented for the second se

Is it exponential growth \square or decay \square

What Kind of Model? (Definitions)

Decide whether each situation is best described by a linear, quadratic, or exponential function. If the function is exponential: Identify the growth factor and the initial value.

$f(x) = 6x^2 - 5$	$ ext{miles(hours)} = rac{22 imes ext{hours} + 14}{12-9}$
1) Linear Quadratic Exponential How did you know?	2) Linear Quadratic Exponential How did you know?
If it's exponential, what's the <u>growth factor</u> ??	If it's exponential, what's the?
$\operatorname{cost}(\mathit{w}) = 5(1.2^{\scriptscriptstyle w}) + 16$	$t(g) = 42 - 2g^2$
3) Linear Quadratic Exponential How did you know?	4) Linear Quadratic Exponential How did you know?
If it's exponential, what's the?	If it's exponential, what's the ?
$\operatorname{price}(d) = d^2 + 6d$	$j(x)=\frac{1}{2}^x+22$
$price(d) = d^2 + 6d$ 5) Linear Quadratic Exponential How did you know?	$j(x)=rac{1}{2}^x+22$ 6) Linear Quadratic Exponential How did you know?
$price(d) = d^{2} + 6d$ 5) Linear Quadratic Exponential How did you know?	$j(x) = \frac{1}{2}^{x} + 22$ 6) Linear Quadratic Exponential How did you know? If it's exponential, what's the?
price(d) = $d^2 + 6d$ 5) Linear Quadratic Exponential How did you know? If it's exponential, what's the growth factor initial value? $f(a) = 20000 - 4.1^a$	$j(x) = \frac{1}{2}^{x} + 22$ 6) Linear Quadratic Exponential How did you know? If it's exponential, what's the? $g(x) = 8(3^{-4x})$
price(d) = $d^2 + 6d$ 5) Linear Quadratic Exponential How did you know? If it's exponential, what's the? f(a) = 20000 - 4.1 ^a 7) Linear Quadratic Exponential How did you know?	$j(x) = \frac{1}{2}^{x} + 22$ 6) Linear Quadratic Exponential How did you know? If it's exponential, what's the? $g(x) = 8(3^{-4x})$ 8) Linear Quadratic Exponential How did you know?

Exponential Models of Covid in Massachusetts

You'll need to have Slide 4: Exponential Model for MA of Modeling Covid Spread (Desmos) open on your computer.

Change the values of k, then a, then b to find three promising exponential models. Graph each one and label your values on the graphs below. **growth rate** (r) can be calculated from the base: base = (1 + rate)

Exponential Model 1



Exponential Model 2



1) What is the initial value (# infections as of June 9th) of this model?

3) What is the y-intercept (and horizontal asymptote) of this model?

4) For what range of dates is this model *most* accurate? ______

5) For what range of dates is this model *least* accurate?

2) What is the **growth rate** of this model?

Exponential Model 3



11) What is the initial value (# infections as of June 9th) of this model?
12) What is the growth rate of this model?
13) What is the y-intercept (and horizontal asymptote) of this model?
14) For what range of dates is this model <i>most</i> accurate?
15) For what range of dates is this model <i>least</i> accurate?
12) What is the growth rate of this model?

Selecting a Model

16) Choose the model that you think best fits the data, and write it in function notation below:

exponential(x) =_____

 $a(b)^{x} + k$

Limits of Computational Modeling

Make sure you are working in your copy of the <u>Covid Spread Starter File</u>.

Benefits and Downsides of working with Approximations

1) What are some possible **benefits** to approximating large and small numbers when doing computations?

2) What are some possible downsides to approximating large and small numbers when doing computations?

Exponentiation and "RoughNums" in Pyret

3) Write each of the expressions below in Pyret, then evaluate them and write down the answer. The first one has been started for you!

	Pyret Code	Evaluates to		Pyret Code	Evaluates to
10 ²	expt(10, 2)	100	1/3		
2 ^{1/2}			3 ³		
27 ^{1/3}			31/3		

4) When do you think Pyret switches to RoughNums, instead of Numbers?

5) In Pyret, evaluate the following expressions:	1 ==	1,	~1	==	~1
Why do you think Pyret gives an error when comparing	identica	I Rough	lums	?	

6) The pros of using ~1 involve speed. What are the potential downsides of using ~1 to speed up a calculation?

7) How likely are these downsides to apply to our Bootstrap work in <u>code.pyret.org (CPO)</u>?

Predicting Exponential Growth

Estimating with our Model

1) In the space below, define exponential for the best model from Exponential Models of Covid in Massachusetts.

<pre>fun exponential(x): (</pre>		* expt(, (~	1 * x))) +		end
	а		h		k	

2) Fill in the table below, to show how many positive cases you **estimate** based on your model after X days. **DO NOT** use a calculator, keyboard or mouse! Instead, use **only your eyes** to look at the formula above or the Desmos graph on your screen.

After	ESTIMATED model prediction	How confident are you? (1=Very confident, 5=Not at all)
50 days		
150 days		
250 days		
350 days		
450 days		

Fitting Exponential Models in Pyret

3) Define exponential in <u>Covid Spread Starter File</u>. Click "Run" and use fit-model to determine how closely exponential fits the MA-table. *Hint:* If you forgot the contract for fit-model, look it up in the <u>Contracts Pages</u>!

y-	in MA. This number increased by a fact	tor of or % every day. Growth Factor: b Growth Rate: (b - 1) × 100
The error i	n the model is described by an S-value of ~S	units
that this m	odel is a good fit considering that	in this dataset range from to lowest y-value
Jse Pyret to co	ompute your model's predictions after each number	of days and fill in the table below.
After	ACTUAL model prediction	How close were you? (1=Very close, 5=Very far)
50 days		
150 days		
250 days		
350 days		

Campus Housing Data

A college is looking at enrollment and housing data for students who've decided what their major will be, vs. those who are undecided:

	# On Campus	# Off Campus	% On Campus
Undecided	120	80	120/200 = 60%
Decided	80	100	80/180 = 44%

1) According to the table, how many Undecided Majors live off-campus? ______

2) According to the table, how many Decided Majors live *on* -campus?

3) Who is more likely to live on campus: Decided or Undecided Majors?

4) Do you think there is a relationship between deciding on a major and living on or off campus? If so, why?

Models for Vermont

For this page, you'll need to have the <u>Covid Spread Starter File</u> open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.

Find the definition of is-MA in your starter file. The code is shown here:

```
# is-MA :: Row -> Boolean
# consumes a Row, and checks if state == "MA"
fun is-MA(r): r["state"] == "MA" end
```

1) Under the definition of is-MA, define a *new* function called is-VT, which tests to see if the state value is equal to "VT." Click run and try it out!

Find the definition of MA-table in your starter file. The code is shown here:

2) Under the definition of MA-table, define a *new* grouped sample called VT-table containing all the rows for the state of Vermont. Click run and try it out!

3) Use lr-plot to obtain the best-possible linear model for the relationship between day and positive in the VT-table, then fill in the blanks below:

The optimized linear model for this dataset predicts an _	increase/decrease	of about	y-variable	per x-variable
The error in the model is described by an <i>S-value</i> of about	ıts	units .	strongly agree, agree, disa	agree, strongly disagree
that this model is a good fit considering that	in this data	aset range from _	to	highest y-value

Exponential Model for Vermont (VT)

For this section, in addition to Pyret, open Slide 5: Exponential Model for VT of Modeling Covid Spread (Desmos).

4) Turn to **Slide 5: Exponential Model for VT** of **Modeling Covid Spread (Desmos)** and adjust the sliders until you've come up with the best exponential model you can for the Vermont dataset. Record your model below:

5) Return to <u>Covid Spread Starter File</u>. At the bottom of the Definitions Area, define exponential-VT to be the model you just found.

Click "Run" to load your definition, then fit the model using VT-table.

According to this model, on June 9, 2020 there	were about+		in VT, for a total of about
day zero	а	k y-units	
a+k This number grew exponential	lly, increasing by a factor of	, or Growth Factor: b	Growth Rate: (b - 1) × 100 % every day.
The error in the model is described by an S-value	e of aboutS	units	
Ithat_thatttattattattattattattattattattattatta	t this model is a good fit con	sidering that	in this dataset range from
lowest y-value highest y-value			

Do People in Rich Countries Live Longer?

- Per-capita GDP is a measure of what a "typical income" is for people in each country (US Dollars).
- Median lifespan is a measure of what a "typical lifespan" is for people in each country

A point at (75, 62) would represent a country where the average GDP is \$75,000/year and the median lifespan is 62 years old. The table below shows the per-capita GDP and median life expectancy for three different countries.

Country	Per-Capita GDP (\$)	Median Lifespan (years)
Mozambique	\$1,348	54.1
Pakistan	\$5,085	68.4
Poland	\$29,176	77.9
Luxembourg	\$103,511	82.4

1) Plot these four countries as points on the graph below.

2) Use your best guess to draw a curve that fits all four points.

3) Plot the points on your curve corresponding to the four per-capita GDPs below. What does your curve predict the median lifespan to be, at each of these points?

Country	Per-Capita GDP (\$)	Median Lifespan (years)
Country A	\$5,000	
Country B	\$25,000	
Country C	\$125,000	
Country D	\$150,000	



4) What do you Notice about the shape of the curve? Does it look like one of the functions we've seen before?

Exploring the Countries Dataset

For this section, you'll need the <u>Countries of the World Starter File</u> open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. The columns in this dataset are described in the left column below:

median-lifespan, and sketch it below.

- **country** name of the country
- **gdp** total Gross Domestic Product of the country. GDP is often used to measure the economic health of a country.
- **population** number of people in the country
- **pc-gdp** the average GDP *per-person*, in US Dollars
- has-univ-healthcare indicates if the country has universal healthcare
- median-lifespan the median life expectancy of people in the country



In Pyret, make a scatter plot showing the relationship between pc-gdp and

1) What do you **Notice** about the point cloud?

2) What do you **Wonder** about the point cloud?

3) Do you see a relationship? If so, describe it (e.g. - linear or nonlinear? Strong or weak?)

4) Find the point for Equatorial Guinea, an outlier with pc-gdp = \$38,058 and median-life-expectancy = 65. Why might this point be so far from the rest?

5) Find the point for Japan, another outlier with pc-gdp = \$43,030 and median-life-expectancy = 85.5. Why might this point be so far from the rest?

6) Are there any countries that stand out? Why or why not? _____

7) Suppose a wealthy country is suffering heavy casualties in a war. Draw a star on the scatter plot, showing where you might expect it to be.

Fitting Models for the Countries Dataset

For this page you will be working with both the Countries of the World Starter File and the Desmos file Fitting Wealth-v-Health and Exploring

Logarithmic Models. 1) Find the optimized linear model for this data using lr-plot. Then update the definition for linear on line 29 of your starter file. $linear(x) = \underbrace{x + \underbrace{y - intercept / vertical shift}_{y - intercept / vertical shift}$ S-value The optimized linear model for this dataset predicts that a ________ in ______ in ______ in ______ in ______ er-capita gdp ______ will increase by ______. The error in the model is described by an S-value of about ______, y-units in this dataset range from to lowest y-value to highest y-value which is 2) Find the best quadratic model you can, using the first slide (Wealth-v-Health Quadratic) in the Desmos activity. Then return to pyret. Update the quadratic function defined on line 35. Click "Run" and use fit-model to calculate the S-value. $quadratic(x) = \underbrace{(x - borizontal shift(h))^2}_{quadratic coefficient (a)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizontal shift(h))^2}_{quadratic (x)} (x - borizontal shift(h))^2 + \underbrace{(x - borizont$ S-value The vertex of the parabola drawn by my model is a ______at about (______). Before this point, as ______ increases, ______ increases, ______ increases or decreases? The error in the model is described by an S-value of about _____, which is _____, which is ______, insignificant/reasonable/significant/extreme v-units in this dataset range from to highest y-value highest y-value. considering _____ 3) Find the best exponential model you can, using the second slide (Wealth-v-Health Exponential) in the Desmos activity. Then return to pyret. Update the exponential function defined on line 39. Click "Run" and use fit-model to calculate the S-value. S-value According to this exponential model, a country with a ______ of zero ______ would have a ______ of ______, for a total of about ______. This number grows exponentially, increasing by a factor of ______ or _____ frow th Factor: b or ______ Grow th Rate: (b - 1) × 100 % with every ______ increase in ______ increase in ______ x-variable The error in the model is described by an S-value of about ______, which is _____, which is ______, insignificant/reasonable/significant/extreme considering ______ in this dataset range from ______ to _____. 4) Are any of these models a good fit for this data? Why or why not?

Swapping the Axes: Notice and Wonder

The scatter plots below were made with the same data... we just swapped the x- and y-axes.

- The first scatter plot shows an exponential relationship: the number of Covid Infections as a function of Days
- The second scatter plot shows a logarithmic relationship: Days as a function of Infections



What do you Notice?	What do you Wonder?



3) This third scatter plot is our current dataset. Which of the scatter plots above does it resemble? Days \rightarrow Infections Infections \rightarrow Days

What Kind of Model? (Graphs & Scatter Plots)

- Decide whether each representation is best described by a quadratic, exponential, or logarithmic function.
- If you think it's exponential OR logarithmic: Draw a diagonal line for y = x, and then sketch the reflection of the curve.



What Kind of Model? (Tables)

Decide whether each representation is best described by a quadratic, exponential, or logarithmic function.

- If the function is exponential, find the base (also called the growth factor): How much does y increase (2x? 10x?) for a single increase in x?
- If the function is logarithmic, find the base: How much does x need to increase (2x? 10x?) just to get a single increase in y?

HINT: Can you draw the arrows to calculate the first difference? The second? What does it mean if neither one is constant?

x	у	x	У
1	0	0	1
10	1	1	10
100	2	2	100
1000	3	3	1000
10000	4	4	10000
100000	5	5	100000
1000000	6	6	1000000
1) Quadratic Exponential ba	Logarithmic base	2) Quadratic Exponentia	Logarithmic base
x	У	x	У
70	-169	5	1
71	-126	10	2
72	-81	20	3
73	-34	40	4
74	15	80	5
75	66	160	6
76	119	320	7
3) Quadratic Exponential ba	Logarithmic <u>base</u>	4) Quadratic Exponentia	Logarithmic base
x	у	x	у
-3	36	1	0
-2	16	6	1
-1	4	36	2
0	0	216	3
1	4	1296	4
2	16	7776	5
3	36	466656	6
5) Quadratic Exponential	Logarithmic base	6) Quadratic Exponentia	Il Logarithmic base

Graphing Logarithmic Models: $f(x) = a \log_b x + k$

You'll need to have **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)** open to **Slide 4: Exploring Logarithmic Functions**. The curve you'll see is the graph of r(x), an logarithmic function. Another, **identical** curve s(x) is hiding behind it.

1) Use the starting values of a , b , and k to complete	e the following equation: r(.	$f(x) = s(x) = f(x) = \frac{1}{\log x}$	$\frac{\log \log (x)}{\log \log (x)} + \frac{\log (x)}{\log (x)}$	vertical shift
Base <i>b</i>				
Keep \mathbf{k} at 0 and \mathbf{a} at 1. Change the value of \mathbf{b} as in	dicated on each grid below.			
2) Sketch each graph and label the coordinates whe	re $x = 1$, $y = 1$, $y = 2$ and $y = 2$	= 3.		
> $b=3$	>	<i>b</i> = 5	>	<i>b</i> = 10
×		x		х
1				
3) How does the value of b impact the shape of a log	arithmic function?			
4) What connections can you draw between the value	e of b and exponents?			
Vertical Shift k	acted an acale swidth alour			
Set a to 1 and b to 2. Change the value of k as indic	catea on each gria below.			
5) Sketch each graph and label the coordinate wher	e x = 1.	1. 0		1 5
\rightarrow $k = -5$	>	$\kappa = 0$	>	K = 5
х		x		x
4) How does the value of this most the change of a loss	arithmic function?			
6) How does the value of k impact the shape of a log				
7) Why does $y = k$ when $x = 1$?				
Logarithmic Coefficient <i>a</i>				
Set k to 0 and b to 10 , then zoom out out so you can	see as far as $x = 1,000$.			
Change $s(x)$ to $s(x) = 1 \log_{10} (x) + 0$ so that the blue	curve lands on top of the red	d curve.		
8) In each graph, label the coordinates where $x = 10$	and $x = 100$ and $x = 1000$.			
\rightarrow $a = -2$	>	a = 0	>	<i>a</i> = 2
x		x		x
9) What is the value of x when $1\log_2(x) = 4$?	What about when 2	$\log_4(x) = 4$?	When $3 \log_8(x)$	=4?
★ How are a and b related?				

What Kind of Model? (Descriptions)

Decide whether each situation describes a quadratic, exponential, or logarithmic function. HINT: draw a table and plug in some points!

1) Earthquakes release enormous amounts of energy, which we can compare to the energy released by blowing up pounds of dynamite. e.g. The force of blowing up 12,000 pounds of dynamite produces a 4.0 on the Richter scale! \rightarrow richter(12,000) = 4.0 richter(400,000) = 5.0 richter(12,540,000) = 6.0 richter(398,000,000) = 7.0

Quadratic Exponential Logarithmic 2) A car accelerates at a constant rate of 5 mph/s. After 1 second, distance(1) = 2.5 miles . distance(2) = 10distance(3) = 22.5distance(4) = 40Quadratic Exponential Logarithmic 3) Moore's law says that the number of transistors in a microprocessor will double roughly every 1.5 years. Starting with 16 transistors, how many years will it take to reach 4,294,967,296 transistors? Quadratic Exponential Logarithmic 4) The population of a colony of bacteria can double every 20 minutes, as long as there is enough space and food. Starting with 1 bacteria... f(20) = 2f(40) = 4f(60) = 8f(80) = 16Quadratic Exponential Logarithmic 5) Sequan puts \$100 in a savings account, earning 4% interest. After a year... savings(1) = \$104savings(2) = \$108.16savings(2) = \$112.49Quadratic Exponential Logarithmic 6) If the width and length of a rectangle doubles, how much does the area change? Quadratic Exponential Logarithmic

Balancing Function Growth and Axis Growth (Linear)

 Make sure you're on Slide 5: Changing the Scale (i Both the x- and y-axis are labeled with a sec 	Linear) of Fitting Wealth-v-Health quence of numbers <i>(e.g. 0, 2, 4, 6</i>	and Exploring L .).	ogarithmic Mo	odels.	
1) What kind of growth do the sequences on th	ese axes show? (circle one)	Linear	Quadratic	Exponential	Logarithmic
2) The function f is	, and its slope is	Which two po	ints are plotte	d on <i>f</i> ?	
Faking a Change in Slope					
 Click on the wrench button () in the top-r Change the scale of the y-axis so that - 2 ≤ 	ight corner of the Desmos graph t $y \le 2$, making sure you can see be	to open the "Gra oth labeled poin ⁻	p h Settings " ts	window.	
3) What kind of growth does the sequence of n	umbers on the y-axis show <i>now</i> ?	Linear	Quadratic	Exponential	Logarithmic
4) Did our two points on the line change?	Did the slope of <i>f</i> change?	How did the	e graph chango	e?	
5) What is going on here?					
6) Without changing the y-axis, change the <i>x-</i> o	axis to put the line roughly back w	/here it was. Wh	at scale did yc	ou use?	_≤ x ≤
Faking a Change to Logarithmic					
 Open the "Graph Settings" window, and exp Change the y-axis scale from Linear to Lo Drag the graph to make sure you can see bo 	pand the "More Options" section ogarithmic. (The y-axis labels shoth labeled points	n by clicking the n nould change to se	triangle (▶). omething like C	.1, 1, 10, 100)	
7) What kind of growth does the sequence of n	umbers on the y-axis show <i>now</i> ?	Linear	Quadratic	Exponential	Logarithmic
8) Did our two points on the line change?	Did the slope of <i>f</i> change?	How did the	e graph change	e?	
9) What is going on here?					
Faking a Change to Exponential					
 Change the y-axis scale back to Linear, and 	d the x-axis scale to Logarit	hmic.			
• Drag the graph to make sure you can see bo	oth labeled points				
10) What kind of growth does the sequence of	numbers on the x-axis show <i>now</i>	? Linear	Quadratic	Exponential	Logarithmic
11) Did our two points on the line change?	. Did the slope of <i>f</i> change?	How did t	he graph char	ge?	
12) What is going on here?					
★ Make a prediction: What do you think will h	appen if <i>both axes</i> are switched t	oLogarithmi	c?		
13) Change both scales to Logarithmic. Was	s your prediction accurate?	What happen	ed, and why?		

Balancing Function Growth and Axis Growth (Logs and Exponents)

Make sure you're on Slide 6: Changing Scale (Exponential and Logarithmic) of Fitting Wealth-v-Health and Exploring Logarithmic Models.

Balancing Logarithmic Growth
• The folder for a logarithmic function g is shown in red, and the folder for h is "turned off". Do not turn it on yet!
• Both the x- and y-axis are be labeled with a sequence of numbers (e.g. 0, 2, 4, 6).
1) What kind of growth do the sequences on these axes show? (circle one) Linear Quadratic Exponential Logarithmic
2) The function g is, and its base is Which two points are plotted on g?
• Click on the wrench button (🖋) in the top-right corner of the Desmos graph to open the "Graph Settings" window.
3) The x-axis goes from $\leq x \leq $. Can you change these numbers to make g look linear? Why?Why?
 Click on the wrench button (>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
4) Did our two points on the line change? How did the graph change?
5) What is going on here?
6) Make a prediction: What would the graph of g look like with a Linear x-axis a Logarithmic y-axis?
7) Try it out! Was your prediction accurate? What happened, and why?
Balancing Exponential Growth
Set the scale for both axes back to Linear.
• "Turn off" the folder for g , and "turn on" the folder for our exponential function h .
8) The function <i>h</i> is, and its base is Which two points are plotted on <i>h</i> ?
9) Without switching either axis to Logarithmic, can we change the minimum and maximum x or y to make h look Linear?
10) Make a prediction: Which axis should we switch to Logarithmic, in order to make h look Linear?
11) Try it out! Was your prediction accurate? What happened, and why?
 Change the y-axis back to Linear, and change the x-axis to Logarithmic
12) What happened to the graph of <i>h</i> , and why?
★ Desmos has two choices for scale:
 Linear (each interval is the same size as the one before it) Logarithmic (each interval is 10x larger than the one before it)
If we wanted to make h annear linear using the x-axis what kind of scale would we need?

Fitting Logarithmic Models

To complete this page you will need Desmos and the <u>Countries of the World Starter File</u> open on your computer.
Fitting a Logarithmic Model $f(x) = a \log_b x + k$
You should be on Slide 7: Wealth-v-Health (Logarithmic) of Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos).
• The x-axis should labeled with a sequence of numbers that looks something like this: 20000, 40000, 60000, 80000, 10000, 120000
1) What kind of growth does the sequence on the x-axis show? (circle one) Linear Quadratic Exponential Logarithmic
2) Use the sliders for a and k to make the best-fitting logarithmic model you can find. Write it below. (Note: Pyret's log always uses $b = 10$)
$logarithmic(x) = _logcoefficient(a) = log_{10}(x) + _vertical shift(k) = fun logarithmic(x): (_ * log(x)) + _ end$
3) Define $logarithmic(x)$ in the <u>Countries of the World Starter File</u> to be this model, and fit it using fit-model.
The error in the model is described by an S-value of about I I agree or disagree - not strong that this
model is a good fit considering that in this dataset ranges from to
Scaling the x-Axis
 Click on the wrench button (>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
 Expand the "More Options" section by clicking the triangle ().
• Change the x-axis scale from Linear to Logarithmic. (The x-axis labels should change to something like 100, 1000, 10^4 , 10^5)
4) What kind of growth does the sequence on the x-axis show? (circle one) Linear Quadratic Exponential Logarithmic
5) What is the shape of the point cloud, after changing the x scale to Logarithmic? Linear Quadratic Exponential Logarithmic
6) Adjust the sliders for a and k to improve the model. Toggle back and forth between logarithmic and linear x-axis scales as you work.
When you are satisfied with your model, record both forms of the definition below.
$logarithmic2(x) = _logcoefficient(a) = log_{10}(x) + _vertical shift(k) = fun logarithmic2(x): (_x + log(x)) + _end(x) = logcoefficient(a) = log$
7) In Pyret, define logarithmic2(x) to match this model. Then use the fit-model function to find its S-value :
8) How much error do we expect in predictions made using logarithmic2 than with the logarithmic model?
$Percent Change = \frac{\text{Difference between the S-values}}{\text{S-value for logarithmic model}} \times 100 =$
We expect percent error from predictions made with logarithmic2 than with the logarithmic model!
9) Do we know for sure that either of these models is optimal? Explain.
10) Why does transforming the x-axis makes our data look linear?

Transforming the Data

For this page, you'll need Slide 8: Wealth-v-Health (Transformed) of Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos).

1) Compare the two tables below from the Desmos file we are about to work with.

Wealth vs.	Health	Log(Wealth)	vs. Health	What do you Notice or Wonder?
x_1	$ \bigcirc y_1 $	$g(x_1)$	$\bigvee y_2$	
1.99051	52.1	3.2989644	52.1	
		4.0706137	78.6	
11.76559	78.6	4.1816421	77.2	
15.19295	77.2	3.7971962	60.6	
		4.3972056	76.9	
6.26897	60.6	4.313631	77.5	
24.95776	76.9	3.9688606	75.1	

Fitting a Model

This slide contains two tables, a list and a function:

- Wealth vs. Health the same table we've seen before.
- y_2 a copy of the table's y_1 column
- g(x) a function that takes in a number x, and produces $log_{10}(x)$
- Log(Wealth) vs. Health a new table!
 - x_2 uses g(x) on the x_1 column in the first table.
 - y_2 is our copy of the y_1 column in the first table.

2) Notice how the red dots (representing the data points from the original table) are dispersed on the graph.

- Hide these data points from the graph by clicking on the "points" circle (:•) at the top of the y, column.
- Then click on the triangle (▶) in front of the folder name to hide the table.
- Turn ON the points for the Log(Wealth) vs. Health table by clicking on the "points" circle (:) at the top of the y, column

How does the new graph look different from the original graph?

3) Scroll to the bottom of the Log(Wealth) vs. Health table and click the ${f e}$ to rescale the graph.								
4) Looking at the point cloud, the best model for this dataset would be (circle one): linear quadratic exponential								
5) Why do you think transforming the x-values makes our data look linear?								
6) Move the sliders for m and b at the bottom left to find the best-fitting lin $f(x) = \underline{\qquad} x + \underline{\qquad}$	ear model you can: y-intercept / vertical shift	-						
Let's compare the model settings from you	ur linear and logarithmic	c models.						
Linear (From above)	slope (m)	y-int	ercept / vertical shift					
Logarithmic (From <u>Fitting Logarithmic Models</u>)	log coefficient (a)		vertical shift (k)					

Logarithmic Models

Open your copy of the Countries of the World Starter File and click "Run".

Transforming: From Logarithmic Plots to Linear Ones

1) Find the definition of g(r). What does this function do?

2) Find the Contract for build-column on the Contracts Page.

What is its Range? _____ What is its Domain? _____

3) At the end of the program, you'll find this code:

countries-transformed =	= buil	d-column	<pre>(countries-table,</pre>	"log(pc-gdp)"	, g)

What do you think it does?

4) Click "Run", and evaluate countries-transformed in the Interactions Area on the right to test it out!

a. What is different about this Table? Hint: Find the last column!

b. Where did the column on the right come from?

5) Use this new table to make an lr-plot comparing log(pc-gdp) and median-lifespan, with country as the label. Record the regression line and S value below:

y = _____ slope x+ ______vertical shift *S*:

Inverting: From Linear *Models* to Logarithmic Ones

6) Use the model settings of the *linear* model you just made to complete the *logarithmic* model below:

logarithmic3(x) =	$\log_{10}(x) + \log_{10}(x)$	- fun	<pre>logarithmic3(x): (</pre>	(* log(x))	+ end
1	ug cuenncient (a)	VELLICAL STILL (K)			

7) Let's interpret this model:

A country where the		istime	es higher than another is also
	explanatory variable (x)	base (b)	
predicted to have a	that	t is	longer.
	response variable (y)	log coefficient (a)	y-axis units

8) Add the definition of logarithmic3 to your starter file. Use fit-model to calculate the S-value and complete the table below:

	Linear	Quadratic	Exponential	Logarithmic
S-value	~5.926626			

 $\frac{\text{Difference between the S-values}}{\text{S-value for second best model}} \times 100 = ----$ 9) Compare the two smallest S values using percent change. -

Predictions made with the logarithmic model are expected to have ______ percent ______ error than predictions made with the

model!

Exploring Periodic Data

time (minutes)	altitude(feet)	20	0			
0	5.0					
5	55.0	17	75			
10	154.9	15	io			
15	205.0					
20	155.2	12	25			
25	55.2	۲۵ التياريم	0			
30	5.0					
35	54.7	7	5			
40	154.6	5	io —			
45	205.0		-			
50	155.5	2	.5			
55	55.5		0	20	40	60
60	5.0		0	20	time	00
2) What do you Wonde	er?					
3) The ride goes from _	feet to	feet in	minutes.	und from bink un	intto high point ou	leur asiat to lour asiat)
 F) Distance does a fuil of the second sec			ner be measu		int to high-point of i	
5) Plot each of the poin	ts in the table (left) on tr	ne coordinate plane (right	t) to create a	scatter plot.		
6) Working from left to	right, connect the dots c	one pair at a time using st	raight lines. ⁻	This will create a	a data visualization	known as a line-grap
7) Draw a dotted horize	ontal line on the graph, p	precisely halfway betwee	n the highest	and lowest poi	nt. What is its altitu	ude?fee
8) Describe the relation	nship you see between t	ime and altitude.(Is	t linear, quad	dratic, exponent	ial, etc.?)	
9) What kind of ride do	you think the teacher w	vas on, and why?				

Reasoning about Unit Clocks

A unit clock (shown below) has a radius of 1, and is centered at the origin (0,0). As time passes, the point (A, B) rotates around the circle.

A and B, around the clock

1) The line connecting (A,B) to the origin is the hypotenuse of a right triangle. How long is this line, no matter what time it is?

The tables below show the values of A (left table) and B (right table) at different times.

2) The values for 12, 3, and 6 o'clock are already shown in the tables below. Fill in the values of A and B at 9 o'clock.





Time	В
12:00	1
1:30	
3:00	0
4:30	
6:00	-1
7:30	
9:00	
10:30	
12:00	1

3) On the unit clock above (and the right triangle to the right) the hand is pointing to (A, B) at **1:30**, when A = B. Calculate the lengths of A and B in the space below. Then label them on the right triangle diagram.

 $A^2 + B^2 = 1$ and A = B, so...



4) Fill in the rest of the table with values of A and B at 4:30, 7:30, and 10:30.

A and B, over time

5) In the graph below, draw a dot for the coordinates (time, A) in each row of the table. Connect them from left-to-right, to form a curve.

6) In the graph below, draw a star for the coordinates (time, B) in each row of the table. Connect them from left-to-right, to form a curve.

1												
ength												
4											Time	
C	1	2	3	4 :	5 0	5	7 8	3 9	€ 1	0 1	1 .	2
_1												

Unit Clock Starter File

Open the Unit Clock Starter File. The questions below refer to the animation you'll see when you click "Run"

7) The green curve measures...

8) The **blue** curve measures...

Converting Between Angles

1) In the circle below, fill in the blanks to label the number of degrees and radians at each point.



2) Use Pyret's **sin** and **cos** functions to complete the table below. Note: These function use radians not degrees.

Angle (θ)	sin(θ)	cos(θ)
0°/0π		
45°/ 0.25π	sin(0.25 * PI) =	
90° / 0.50π		
135°/0.75π		
180°/1.00π		
225°/1.25π		
270°/1.50π		
315° / 1.75π		

3) Which function computes the **horizontal** leg of the right triangle (*A*)?

4) Which function computes the **vertical** leg of the right triangle (*B*)?

Graphing Periodic Models: Amplitude (a)

The standard form of periodic models is $f(x) = a \sin(b \cdot (x - h)) + k$. Let's explore the role of **amplitude** a in periodic functions! Open the Desmos File **Exploring Periodic Functions** to Slide 1: **Modeling the Ferris Wheel Dataset (sine)**. You should see four sliders for a, b, h, and k.

1) Adjust the sliders to fit the data as best you can, then record your model settings: _____, ____, ____, ____, ____, and _____.

2) Changing **ONLY** the slider for *a*, experiment with values at 100, 50, -50, and 0, graphing each curve below. For each curve, label the coordinates at time= 15, 30, and 45.



3) What does *a* tell us about a periodic function?

The distance between two adjacent peaks or valleys is called the period: the interval over which the pattern repeats itself.

4) What effect does changing *a* have on the **period** of a periodic function?

Graphing Periodic Models: Frequency (b)

The standard form of a periodic model is $f(x) = a \sin(b \cdot (x - h)) + k$. On this page, we'll explore the role of **amplitude** a in periodic functions. Open the Desmos File **Exploring Periodic Functions**. You should be on **Slide 1: Modeling the Ferris Wheel Dataset (sine)** and see four sliders for a, b, h, and k.

1) Adjust the sliders to fit the data as best you can, then record your model settings: ______, ____, ____, ____, ____, and _____.

2) Click on one of the *peaks* (highest-points) on the graph of your periodic function. Desmos will add a gray dot to *all* of the peaks.

3) Leaving the other sliders where they best fit the data, change **ONLY** the slider for *b*, experimenting with values at 0.2, 0.1, 0.05, and 0, graphing each curve below.

For each curve, label two adjacent peaks.



Graphing Periodic Models: Horizontal/Phase Shift (h)

The standard form of a periodic model is $f(x) = a \sin(b \cdot (x - h)) + k$. On this page, we'll explore the role of **amplitude** a in periodic functions. Open the Desmos File **Exploring Periodic Functions**. You should be on **Slide 1: Modeling the Ferris Wheel Dataset (sine)** and see four sliders for a, b, h, and k.



Describing Periodic Functions



Based on what you can learn from the diagram, describe each of the terms below in your own words.

Peaks -			
Valleys -			
Period -			
Midline -			
Amplitude -			

Matching Periodic Descriptions



Modeling the Ferris Wheel Data

Modeling with sin

For this section, use **Slide 1: "Modeling the Ferris Wheel Dataset (sine)"** of the **Exploring Periodic Functions** Desmos File. You'll find **the data from the Ferris wheel plotted in blue**, along with a basic periodic model of the form $f(x) = a \sin(b(x - h)) + k$.

1) Use the sliders to estimate the best periodic fit.

2) The peaks	are at	, valleys are at	feet	_, midline is at	and the amp	itude is feet
3) The perio a	of the data is	 minutes	Ifperiod =	$\frac{2\pi}{\text{frequency}}$, what is the	e frequency ?	r minute
4) Adjust the	slider for horizor	ital shift to find the bes	t fit, then write th	ne model below in Func	tion and Pyret notation.	Write <i>h</i> in terms of <i>pi</i> .
Function Notation	f(x) =	× sin	(frequency	(x))+vertical shift	-
Pyret Notation	fun f(x): (* sin	(* (x)))) +	end

Translating from sin to cos

For this section, advance to **Slide 2: "Translating from sine to cosine"** of the **Exploring Periodic Functions** Desmos File. You'll see a function f(x) defined here graphed in blue, which uses *cos* instead of *sin*.

5) Adjust the sliders so that the function q perfectly overlaps the function p. What is the value of a? ______ b? ______ k? _____

6) What was the value of h, expressed as a decimal? What was the value of h, expressed a fraction of pi?

7) In row 1 below, write the definition of q using the values of a, b, and k that you found earlier. Change the definition of p and write it in the second row, then show how q would need to change to overlap it.

Function using sin	Function using cos	Vertical Shift k
p(x) = 10 sin(1 imes (x - 0)) + 2	q(x) =	
p(x) =	q(x) =	

8) Do you think that all basic cosine functions can be expressed as sine functions? Why or why not?

Modeling with cos

For this section, advance to Slide 3: "Modeling the Ferris Wheel Dataset (cosine)" of the Exploring Periodic Functions Desmos File.

9) Translate your sin -based model to a cos -based one. Express the horizontal shift in terms of pi.

Function Notation	g(x) =	× cos	((x))+ horizontal shift vertical shif	ft
Pyret Notation	fun g(x): (* cos	* (x	-))) +	⊦ end
Make Your Own Ferris Wheel!

Match each upgrade on the left to the property or properties that it will change on the	e right	t.
1) The wheel is being raised <i>higher</i>	A	midline
	В	vertical shift
2) The wheel is being made to spin <i>faster</i>	с	frequency
	D	amplitude
3) The wheel is being made <i>larger</i>		
	Е	period

Design a New Wheel

The Ferris wheel is being upgraded!

4) Design your own Ferris wheel! Fill in the table below, then trade papers with someone else.

Radius	Altitude of Center	Speed

5) Based on the table above, what function will model the height of the wheel over time?

Function Notation	$f(x) = _$ amplitude	_ × sin ((x))+))+	vertical shift
Pyret Notation	fun f(x): (_ * sin(* (x))) +	end

Modeling Recent Carbon Dioxide Levels

The data below was generated from the <u>Carbon Dioxide Starter File</u>. The **dark dots** show the amount of CO_2 in the atmosphere (in parts per million) recorded between December 2022 to November 2023. NOTE: the date column is the **decimal year** (so "June 15th, 2023" would be 2023.5).



1) Connect the dark dots on the scatter plot from left to right to form a line-graph of the data in recent-table.

2) The amount of CO_2 varies from the lowest valley to the highest peak.
3) This represents a change in atmospheric CO_2 of parts per million. difference between highest and lowest
4) Find the <i>amplitude</i> (<i>a</i>) by cutting the vertical distance you calculated in half: CO ₂ in the atmosphere in parts per million
5) Draw the midline on the graph. (It should be a horizontal line passing in the middle between the lowest valley and the highest peak .)
6) The midline makes the vertical shift (k) visible. What is the vertical shift (k) of the model? HINT: The vertical shift can also be calculated by adding the amplitude(<i>a</i>) to the valley.
7) The phase shift (h) is the decimal year when the data first crosses the midline. Estimate The phase shift (h)
8) Calculate the <i>period</i> between the valleys: <u>2023.708</u> date for the lowest values in 2023 - <u>2022.792</u> date for the lowest values in 2022 = <u>years (round to the nearest full year)</u>
9) If $\mathbf{period} = \frac{2\pi}{\text{frequency}}$ then $\mathbf{frequency} = \frac{2\pi}{\text{period}}$. How do you know that this statement is true?

Modeling Recent Carbon Dioxide Levels (continued)

This page relies on the Carbon Dioxide Starter File. Make sure you have it open on your computer!

Define Your Periodic Model

1) Define a periodic model using the values you computed for a, k, h and b on Modeling Recent Carbon Dioxide Levels.

Function Notation	$periodic - sin(x) = (____ amplitude(a)]$	$\times sin($ (x	;))) +	vertical shift (k)	
Pyret Notation	fun periodic-sin(x): (* sin(* (x	_))) +	end

2) Then update the periodic-sin function definition in your starter file to match what you've just written.

★ Try translating the definition you wrote for periodic so that it uses cosine instead of sine :

Function Notation	$periodic - cos(x) = (_$	amplitude (a)	× cos (frequency (b)	_ (<i>x</i> -	horizontal shift (h)	_))) + _	vertical shift (k)	_	
Pyret Notation	fun periodic-cos(x): (* c	os(* (x		_))) +		end

Fit Your Periodic Model to the Data

3) Use fit-model to fit your periodic model to the data in the recent-table. What S-value do you get?



4) On the scatter plot below, make a sketch of what you see in pyret showing how the periodic model fits the data.

5) Then fill in the blanks below to tell us what the model means. Between the end of 2022 and 2023, the amount of CO_2 in the air fluctuated between _and _ parts-per-million. highest lowest This pattern appears to be **periodic**, with an amplitude of _ amplitude _____. We expect this rising and falling around a **midline** of ____ ___. The error in the model is pattern to repeat every _ x-units period described by an **S-value** of about y-units that this model is a good fit strongly agree, agree, disagree, strongly disagree considering that in this dataset ranges from about y-variable to lowest y-value highest y-value

	Deconstructing a Model			
1)]	The four functions below are defined in t	the <u>Hybrid CO2 Models Starter File</u> . Use fit-model to	ofit periodic-cos and periodic-cos2	2 tothe recent-table data.
	<pre>fun periodic-cos(x): (4 fun wave-cos(x): (4 fun midline-cos(x): fun periodic-cos2(x):</pre>	.13 * cos(6.28 * (x - 2023.35)) + 419.8 .13 * cos(6.28 * (x - 2023.35))) 419.8 wave-cos(x) + midline-cos(x)	87 end end 87 end ×) end	
2) F	Read these 4 functions carefully, and exp	olain why periodic-cos2 will produce the <i>same grap</i>	1 as periodic-cos.	
ι.	Seview: Other Models We've S	Seen		
	Linear	Quadratic	Exponential	Logarithmic
73	Sketch			
	Key Characteristics			
-	What kind of model?			
200	2000 2012 2016 2015 2020 2025	 3) What do you Notice about this scatter plot? What do 4) Which of the other models you described above would 	you Wonder? d make the most sense to fit to this scatter pl	lot?Why?

Modeling Modern Carbon Dioxide Levels

This page focuses on the modern-table of the Hybrid CO2 Models Starter File, which tracks atmospheric CO2 (parts per million) from 2010-2023.

Decomposing Your Periodic Model

Towards the bottom of the Definitions Area, find the section of the starter file where you're asked to "Define your periodic-sin functions..."

1) Define periodic-sin to be the periodic model you found earlier, for CO₂ levels from 2022-2023.

- You should already have defined it in Carbon Dioxide Starter File.
- You can also look at Modeling Recent Carbon Dioxide Levels (continued), the workbook page from the previous lesson.

2) Using the deconstruction of periodic-cos as your model, change the other three functions in this section to show how to separate the wave and midline of your periodic-sin model and define periodic-sin2 using function composition.

Fitting the Optimal Linear Model

3) Use lr-plot to find the best linear model for the modern-table, and record the function below:

fun linear-modern(x): (_____ * x) + _____ end

4) Change the linear-modern function in the starter file to match the function above. Then use fit-model to fit it to the modern-table.

The **S-value** is:

5) Sketch the model on the graph to the right.

Fitting your Periodic Model

6) Use fit-model to fit periodic-sin to the data in the modern-table.

The **S-value** is:

7) Sketch the model on the graph to the right. What would need to change about your

model, to fit this data?



2,017.5

date

2,020.0

2,022.5

420

415 410

405

400

Imagining the **Best Possible** Model

8) Sketch the *best possible model* you can imagine for this data on the graph to the right, and describe it. Do parts of it look linear? Quadratic? Exponential? Logarithmic? Periodic?



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Building a Hybrid Model

Open your copy of <u>Hybrid CO2 Models Starter File</u> and click "Run".

Building a hybrid model for the modern-table

Both periodic-sin and periodic-cos models are built to follow a *horizontal* midline with the equation f(x) = 419.87.

1) What line do we wish the model would follow instead?

2) Find hybrid-modern in the starter file and define it using function composition.
Hint: Like periodic-sin2 this function will use wave-sin. What will it use instead of midline-sin?

3) Fit hybrid-modern to the modern-table, and describe what you see:

Comparing Models		
4) What S-value describes the expected error in our hybrid-modern model for this data?		
	5 -value	y-units
5) What S-value describes the expected error in our Linear-modern model for this data?	S -value	y-units
6) How much error do we expect in predictions made using hybrid-modern than with t	helinear-moderr	n model?
$Percent Change = \frac{Difference between the S-values}{S-value for linear model} \times 100 =$		
We expect percent error from predictions made with hybrid-modern	than with linear-	nodern model!
Interpreting the Hybrid Model		
This model predicts that:		
The overall trend will be a(n) of of	every	nits
• Within this trend, the data fluctuates following a periodic pattern that rises and falls around this midline .		
The wave has an amplitude of and repeats every		
The error in the model is described by an S-value of about I I	e, agree, disagree, strongly	disagree that this
model is a good fit considering that in this dataset ranges from about	to highest y-value	<u>.</u>

More Models

Open your copy of <u>Hybrid CO2 Models Starter File</u> and click "Run".

Building a hybrid model for the entire co2-table

1) How well would you expect our hybrid-modern model to fit the data in the full co2-table, with data covering a span of 50 years?

2) Let's test it out. What **S-value** do you get?

At the bottom of the Definitions Area, find the section titled "HYBRID MODEL for the full co2-table".

3) Define hybrid-all as a model that fits the full dataset, referring back to hybrid-modern to help you think through which functions you will need to define in order to be able to compose your function definition.

4) Use fit-model to fit your new hybrid-all model to the co2-table. What is the S-value of hybrid-all with this data?

5) Compute the change in *S-values* between hybrid-all and hybrid-modern, when used with the co2-table:

 $Percent Change = \frac{Difference between the S-values}{S-value for linear model} \times 100 = -----$

"For co2-table, we expect ______ percent ______ error from predictions made with hybrid-all than with hybrid-modern."

Comparing S-values doesn't always make sense

You've had a lot of practice comparing the **S-values** of two different models on the same dataset (including what you just did on this page!), to quantify the error between them. **But can we compare the S-values for one model fit to two different datasets**?

6) In the first row of the table below, we've fit the periodic-cos model to all three datasets.

	recent-table	modern-table	co2-table
S-value of periodic-cos	~1.2ppm	~17.76ppm	~55.89ppm
lowest y-value (CO_2 in ppm)	415.74ppm	387.03ppm	327.28ppm
highest y-value (CO_2 in ppm)	424ppm	424ppm	424ppm

The periodic-cos model was built from recent-table, so there's no surprise it was a good fit for recent-table! But for datasets that reach farther and farther back in time, that model fits worse and worse.

7) It's tempting to compare the S-values in this table, to quantify how much more error we'd expect from the periodic-sin model for the co2-table. Why shouldn't we?

★ Just as there's nothing to say that midlines have to be horizontal, there's also nothing to say that midlines need to be straight! If you look closely, the co2-table's midline almost looks quadratic... can you come up with a hybrid-curve model for the co2-table that can beat the **S-value** you just found?

Linear Models

Slope-Intercept Form:
$$f(x) = m(x) + k$$



Quadratic Models

Factored Form:	$f(x) = a(x - r_1)(x - r_2)$
Standard Form:	$f(x) = a(x)^2 + b(x) + c$

Exponential & Logarithmic Models

$$f(x) = \underset{\text{initial value}}{a} \left(\begin{array}{c} b \\ \hline base \end{array} \right)^x + \underset{\text{vertical shift}}{k}$$

Logarithmic Models

$$f(x) = \underset{\text{log coefficient}}{\mathsf{a}} \log_{\frac{b}{\mathsf{base}}}(x) + \underset{\text{vertical shift}}{\mathsf{k}}$$

Periodic Models

Sine

$$f(x) = a_{\text{amplitude}} \sin(b_{\text{frequency}}(x - h_{\text{horizontal shift}})) + k_{\text{vertical shift}}$$

Cosine

.

$$f(x) = a_{\text{amplitude}} \cos(b_{\text{frequency}}(x - h_{\text{horizontal shift}})) + k_{\text{vertical shift}}$$

Contracts for Algebra 2

Contracts tell us how to use a function, by telling us three important things:

- 1. The Name
- 2. The **Domain** of the function what kinds of inputs do we need to give the function, and how many?
- 3. The Range of the function what kind of output will the function give us back?

For example: The contract triangle :: (Number, String, String) -> Image tells us that the name of the function is triangle, it needs three inputs (a Number and two Strings), and it produces an Image. With these three pieces of information, we know that typing triangle(20, "solid", "green") will evaluate to an Image.

Name	Domain		Range
<pre># build-column ::</pre>	(<u>Table</u> , <u>String</u> , <u>(Row</u> -> <u>Value</u>))	->	Table
build-column(animals-tab	le, "kilos", kilograms)		
# circle ::	(<u>Number</u> , <u>String</u> , <u>String</u>) radius (<u>Ill-style</u>)	->	Image
circle(50, "solid", "pur	ple")		
<pre># ellipse</pre> ::	(<u>Number</u> , <u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
ellipse(100, 50, "outlin	e", "orange")		
# filter ::	(<u>Table</u> , (<u>Row</u> -> <u>Boolean</u>))	->	Table
filter(animals-table, is	-dog)		
<pre># fit-model ::</pre>	(<u>Table</u> , <u>String</u> , <u>String</u> , <u>String</u> , (<u>Num</u> -> <u>Num</u>)) table-name labels xs ys model-function	->	Image
fit-model(animals-table,	"name", "pounds","weeks", f)		
<pre># isosceles-triangle ::</pre>	(<u>Number</u> , <u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
isosceles-triangle(50, 2	0, "solid", "grey")		
# log ::	(<u>Number</u>)	->	Number
log(4)			
<pre># log-base ::</pre>	(<u>Number</u> , <u>Number</u>)	->	Number
log-base(2, 4)			
# overlay ::	(<u>Image</u> , <u>Image</u>)	->	Image
overlay(circle(10, "soli	d", "black"), square(50, "solid", "red"))		
<pre># radial-star ::</pre>	(<u>Num</u> , <u>Num</u> , <u>Num</u> , <u>Str</u> , <u>Str</u>)	->	Image
radial–star(6, 20, 50, "	solid", "red")		
<pre># rectangle ::</pre>	(<u>Number</u> , <u>Number</u> , <u>String</u> , <u>String</u>) width height fill-style	->	Image
rectangle(100, 50, "outl	ine", "green")		
<pre># regular-polygon ::</pre>	(<u>Number</u> , <u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
regular-polygon(25,5, "s	olid", "purple")		

Name	Domain		Range
# rhombus ::	(<u>Number</u> , <u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
rhombus(100, 45, "outline", "pink")			
<pre># right-triangle ::</pre>	(<u>Number</u> , <u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
right-triangle(50, 60, "	putline", "blue")		
# rotate ::	(<u>Number</u> , <u>Image</u>)	->	Image
rotate(45, star(50, "sol.	id", "dark-blue"))		
# S ::	(<u>Table</u> , <u>String</u> , <u>String</u> , (<u>Num</u> -> <u>Num</u>)) table-name xs ys model-function	->	Number
S(animals-table, "name",	"pounds","weeks", f)		
<pre># scatter-plot ::</pre>	(<u>Table</u> , <u>String</u> , <u>String</u> , <u>String</u>)	->	Image
<pre>scatter-plot(animals-table, "name", "pounds","weeks")</pre>			
# sqr ::	(<u>Number</u>)	->	Number
sqr(4)			
# sqrt ::	(<u>Number</u>)	->	Number
sqrt(4)			
# square ::	(<u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
<pre>square(50, "solid", "red</pre>	')		
# star ::	(<u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
<pre>star(50, "solid", "red")</pre>			
<pre># star-polygon ::</pre>	(<u>Number</u> , <u>Number</u> , <u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
star-polygon(100, 10, 3	,"outline", "red")		
<pre># string-contains ::</pre>	(<u>String</u> , <u>String</u>)	->	Boolean
<pre>string-contains("hotdog", "dog")</pre>			
<pre># string-length ::</pre>	(<u>String</u>)	->	Number
<pre>string-length("rainbow")</pre>			
# text ::	(<u>String</u> , <u>Number</u> , <u>String</u>)	->	Image
text("Zari", 85, "orange")			
<pre># triangle ::</pre>	(<u>Number</u> , <u>String</u> , <u>String</u>)	->	Image
<pre>triangle(50, "solid", "fuchsia")</pre>			
<pre># triangle-asa ::</pre>	(<u>Number</u> , <u>Number</u> , <u>Number</u> , <u>String</u> , <u>String</u>) left-side <u>left-side</u> <u>color</u>	->	Image
triangle-asa(90, 200, 10,	, "solid", "purple")		
<pre># triangle-sas ::</pre>	(<u>Number</u> , <u>Number</u> , <u>Number</u> , <u>String</u>) bottom-R-side, top-R-angle, top-side, <u>fill-style</u> , <u>color</u>)	->	Image
triangle-sas(50, 20, 70,	"outline", "dark-green")		



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