Name: ____

Algebra 2

Fall 2025 Student Workbook - Pyret Edition

Workbook v1.0

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Introduction to Programming in a Nutshell

The **Editor** is a software program we use to write Code. Our Editor allows us to experiment with Code on the right-hand side, in the **Interactions Area**. For Code that we want to *keep*, we can put it on the left-hand side in the **Definitions Area**. Clicking the "Run" button causes the computer to re-read everything in the Definitions Area and erase anything that was typed into the Interactions Area.

Data Types

Programming languages involve different ***data types***, such as Numbers, Strings, Booleans, and even Images.

- Numbers are values like `1`, `0.4`, `1/3`, and `-8261.003`.
  - Numbers are *usually* used for quantitative data and other values are *usually* used as categorical data.
  - In Pyret, decimals *must* start with a zero. For example, `0.22` is valid, but `.22` is not.
- Strings are values like `"Emma"`, `"Rosanna"`, `"Jen and Ed"`, or even `"08/28/1980"`.
  - All strings *must* be surrounded by quotation marks.
- Booleans are either `true` or `false`.

All values evaluate to themselves. The program `42` will evaluate to `42`, the String `"Hello"` will evaluate to `"Hello"`, and the Boolean `false` will evaluate to `false`.

Operators

Operators (like `+`, `-`, `*`, `<`, etc.) work the same way in Pyret that they do in math.

- Operators are written between values, for example: `4 + 2`.
- In Pyret, operators must always have spaces around them. `4 + 2` is valid, but `4+2` is not.
- If an expression has different operators, parentheses must be used to show order of operations. `4 + 2 + 6` and `4 + (2 * 6)` are valid, but `4 + 2 * 6` is not.

Applying Functions

Functions work much the way they do in math. Every function has a name, takes some inputs, and produces some output. The function name is written first, followed by a list of ***arguments*** in parentheses.

- In math this could look like $f(5)$ or $g(10, 4)$.
- In Pyret, these examples would be written as `f(5)` and `g(10, 4)`.
- Applying a function to make images would look like `star(50, "solid", "red")`.
- There are many other functions in Pyret, for example `sqr`, `sqrt`, `triangle`, `square`, `string-repeat`, etc.

Functions have ***contracts***, which help explain how a function should be used. Every Contract has three parts:

- The *Name* of the function - literally, what it's called.
- The *Domain* of the function - what *type(s) of value(s)* the function consumes, and in what order.
- The *Range* of the function - what *type of value* the function produces.

Strings and Numbers

Make sure you've loaded [code.pyret.org (CPO)](code.pyret.org), clicked "Run", and are working in the ***Interactions Area*** *on the right. Hit Enter/return to evaluate expressions you test out.*

Strings

String values are always in quotes.

- Try typing your name *(in quotes!)*.
- Try typing a sentence like "I'm excited to learn to code!" *(in quotes!)*.
- Try typing your name with the opening quote, but *without the closing quote.* Read the error message!
- Now try typing your name *without any quotes.* Read the error message!

1) Explain what you understand about how strings work in this programming language. ______________________________

____

Numbers

2) Try typing `42` into the Interactions Area and hitting "Enter". Is `42` the same as `"42"`? Why or why not?

____

3) What is the largest number the editor can handle?

____

4) Try typing `0.5`. Then try typing `.5`. Then try clicking on the answer. Experiment with other decimals.

Explain what you understand about how decimals work in this programming language. ______________________________

____

5) What happens if you try a fraction like `1/3`? ______________________________

____

6) Try writing **negative** integers, fractions and decimals. What do you learn? ______________________________

____

____

Operators

7) Just like math, Pyret has ***operators*** like `+`, `-`, `*` and `/`.
Try typing in `4 + 2` and then `4+2` (without the spaces). What can you conclude from this?

____

8) Type in the following expressions, **one at a time**: `4 + 2 * 6`   `(4 + 2) * 6`   `4 + (2 * 6)`   What do you notice?

____

9) Try typing in `4 + "cat"`, and then `"dog" + "cat"`. What can you conclude from this?

____

____

Booleans

Boolean-producing expressions are yes-or-no questions, and will always evaluate to either `true` ("yes") or `false` ("no").
What will the expressions below evaluate to? *Write down your prediction, then type the code into the Interactions Area to see what it returns.*

	Prediction	Result		Prediction	Result
1) `3 <= 4`			2) `"a" > "b"`		
3) `3 == 2`			4) `"a" < "b"`		
5) `2 < 4`			6) `"a" == "b"`		
7) `5 >= 5`			8) `"a" <> "a"`		
9) `4 >= 6`			10) `"a" >= "a"`		
11) `3 <> 3`			12) `"a" <> "b"`		
13) `4 <> 3`			14) `"a" >= "b"`		

15) In your own words, describe what `<` does. ______________________________

____

16) In your own words, describe what `>=` does. ______________________________

____

17) In your own words, describe what `<>` does. ______________________________

____

	Prediction:	Result:
18) `string-contains("catnap", "cat")`		
19) `string-contains("cat", "catnap")`		

20) In your own words, describe what `string-contains` does. Can you generate another expression using `string-contains` that returns true?

____

★ There are infinite string values ("a", "aa", "aaa"...) and infinite number values out there (...-2,-1,0,-1,2...). But how many different *Boolean* values are there? ______________________________

Applying Functions

Open [code.pyret.org (CPO)] and click "Run". We will be working in the Interactions Area on the right.

Test out these two expressions and record what you learn below:

- `regular-polygon(40, 6, "solid", "green")`
- `regular-polygon(80, 5, "outline", "dark-green")`

1) You've seen data types like Numbers, Strings, and Booleans. What data type did the `regular-polygon` function produce? ____________

2) How would you describe what a regular polygon is? ____

____

3) The `regular-polygon` function takes in four pieces of information (called arguments). Record what you know about them below.

	Data Type	Information it Contains
Argument 1		
Argument 2		
Argument 3		
Argument 4		

There are many other functions available to us in Pyret. We can describe them using ***contracts***. The Contract for `regular-polygon` is:

```
# regular-polygon :: Number, Number, String, String -> Image
```

- Each Contract begins with the function name: *in this case* `regular-polygon`
- Lists the data types required to satisfy its Domain: *in this case* Number, Number, String, String
- And then declares the data type of the Range it will return: *in this case* Image

Contracts can also be written with more detail, by annotating the Domain with *variable names*:

```
# regular-polygon :: ( Number , Number , String , String ) -> Image
                       size     number-of-sides  fill-style  color
```

4) We know that a square is a regular polygon because ____

____

5) What code would you write to make a big, blue square using the `regular-polygon` function?

______________ (__________ , __________ , __________ , __________)
function-name — size :: Number — number-of-sides :: Number — fill-style :: String — color :: String

6) Pyret also has a `square` function whose contract is:

```
# square :: ( Number , String , String ) -> Image
              size     fill-style  color
```

What code would you write to make a big blue square using the `square` function?

______________ (__________ , __________ , __________)
function-name — size :: Number — fill-style :: String — color :: String

7) Why does `square` need fewer arguments to make a square than `regular-polygon`? ______________________

____

★ Where else have you heard the word ***contract*** used before?

Practicing Contracts: Domain & Range

Note: The contracts on this page are not defined in Pyret and cannot be tested in the editor.

is-beach-weather

Consider the following Contract:
`# is-beach-weather :: Number, String -> Boolean`

1) What is the **Name** of this function? ______________________

2) How many arguments are in this function's **Domain**? ______________________

3) What is the **Type** of this function's **first argument**? ______________________

4) What is the **Type** of this function's **second argument**? ______________________

5) What is the **Range** of this function? ______________________

6) Circle the expression below that shows the correct application of this function, based on its Contract.

A. `is-beach-weather(70, 90)`
B. `is-beach-weather(80, 100, "cloudy")`
C. `is-beach-weather("sunny", 90)`
D. `is-beach-weather(90, "stormy weather")`

cylinder

Consider the following Contract:
`# cylinder :: Number, Number, String -> Image`

7) What is the **Name** of this function? ______________________

8) How many arguments are in this function's **Domain**? ______________________

9) What is the **Type** of this function's **first argument**? ______________________

10) What is the **Type** of this function's **second argument**? ______________________

11) What is the **Type** of this function's **third argument**? ______________________

12) What is the **Range** of this function? ______________________

13) Circle the expression below that shows the correct application of this function, based on its Contract.

A. `cylinder("red", 10, 60)`
B. `cylinder(30, "green")`
C. `cylinder(10, 25, "blue")`
D. `cylinder(14, "orange", 25)`

Matching Expressions and Contracts

Match the Contract (left) with the expression that uses it correctly (right).
Note: The contracts on this page are not defined in Pyret and cannot be tested in the editor.

Contract			Expression
`# make-id :: String, Number -> Image`	1	A	`make-id("Savannah", "Lopez", 32)`
`# make-id :: String, Number, String -> Image`	2	B	`make-id("Pilar", 17)`
`# make-id :: String -> Image`	3	C	`make-id("Akemi", 39, "red")`
`# make-id :: String, String -> Image`	4	D	`make-id("Raïssa", "McCracken")`
`# make-id :: String, String, Number -> Image`	5	E	`make-id("von Einsiedel")`

Contract			Expression
`# is-capital :: String, String -> Boolean`	6	A	`show-pop("Juneau", "AK", 31848)`
`# is-capital :: String, String, String -> Boolean`	7	B	`show-pop("San Juan", 395426)`
`# show-pop :: String, Number -> Image`	8	C	`is-capital("Accra", "Ghana")`
`# show-pop :: String, String, Number -> Image`	9	D	`show-pop(3751351, "Oklahoma")`
`# show-pop :: Number, String -> Number`	10	E	`is-capital("Albany", "NY", "USA")`

Contracts for Image-Producing Functions

Log into [code.pyret.org (CPO)](https://code.pyret.org) and click "Run". Experiment with each of the functions listed below in the interactions area. Try to find an expression that produces an image. Record the contract and example code for each function you are able to use!

Name		Domain		Range
# triangle	::	Number, String, String	->	Image
triangle(80, "solid", "darkgreen")				
# star	::		->	
# circle	::		->	
# rectangle	::		->	
# text	::		->	
# square	::		->	
# rhombus	::		->	
# ellipse	::		->	
# regular-polygon	::		->	
# right-triangle	::		->	
# isosceles-triangle	::		->	
# radial-star	::		->	
# star-polygon	::		->	
# triangle-sas	::		->	
# triangle-asa	::		->	

Catching Bugs when Making Triangles

Learning about a Function through Error Messages

1) Type `triangle` into the Interactions Area of code.pyret.org (CPO) and hit "Enter". What do you learn? ______________________

2) We know that all functions will need an open parenthesis and at least one input! Type `triangle(80)` in the Interactions Area and hit Enter/return. Read the error message. What hint does it give us about how to use this function?

____

3) Using the hint from the error message, experiment until you can make a triangle. What is the contract for `triangle`?

____

4) Read the explanation below. Then explain the difference in your own words.

syntax errors - when the computer cannot make sense of the code because of unclosed strings, missing commas or parentheses, etc.
contract errors - when the function isn't given what it needs (the wrong type or number of arguments are used)

The difference between **syntax errors** and **contract errors** is: ____

____

Finding Mistakes with Error Messages

The following lines of code are all BUGGY! Read the code and the error messages below. See if you can find the mistake WITHOUT typing it into Pyret.

5) `triangle(20, "solid" "red")`

```
Pyret didn't understand your program around
triangle(20, "solid" "red")
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

6) `triangle(20, "solid")`

```
This application expression errored:
triangle( 20 , "solid")
2 arguments were passed to the operator. The operator evaluated to a function accepting 3 parameters. An application expression expects the number of parameters and arguments to be the same.
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

7) `triangle(20, 10, "solid", "red")`

```
This application expression errored:
triangle(20, 10, "solid", "red")
4 arguments were passed to the operator. The operator evaluated to a function accepting 3 parameters. An application expression expects the number of parameters and arguments to be the same.
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

8) `triangle (20, "solid", "red")`

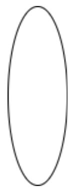
```
Pyret thinks this code is probably a function call:
triangle (20, "solid", "red")
Function calls must not have space between the function expression and the arguments.
```

This is a ____________ error. The problem is that ______________________________
contract / syntax

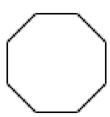
Using Contracts

For questions 1,2,4,5,8 & 9, use the contracts provided to find expressions that will generate images similar to the ones pictured.
Test your code in [code.pyret.org (CPO)](code.pyret.org) before recording it.

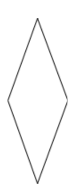
```
# ellipse :: ( Number , Number , String , String ) -> Image
              width      height     fill-style  color
```

1)		
2)		
3)	Write an expression using `ellipse` to produce a circle.	

```
# regular-polygon :: ( Number , Number , String , String ) -> Image
                      side-length  number-of-sides  fill-style  color
```

4)		
5)		
6)	Use `regular-polygon` to write an expression for a square!	
7)	How would you describe a **regular polygon** to a friend?	

```
# rhombus :: ( Number , Number , String , String ) -> Image
              size      top-angle  fill-style  color
```

8)		
9)		
10)	Write an expression to generate a `rhombus` that is a square!	

Triangle Contracts

Respond to the questions. Go to [code.pyret.org (CPO)](https://code.pyret.org) to test your code.

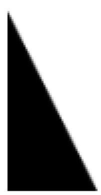
1) What kind of triangle does the `triangle` function produce?

There are lots of other kinds of triangles! And Pyret has lots of other functions that make triangles!

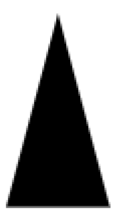
```
# triangle :: (Number, String, String) -> Image
               size    fill-style color
# right-triangle :: (Number, Number, String, String) -> Image
                     base    height  fill-style color
# isosceles-triangle :: (Number, Number, String, String) -> Image
                         leg     angle   fill-style color
```

2) Why do you think `triangle` only needs one number, while `right-triangle` and `isosceles-triangle` need two numbers?

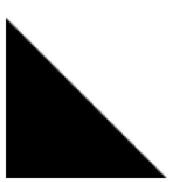
3) Write `right-triangle` expressions for the images below using `100` as one argument for each.



4) Write `isosceles-triangle` expressions for the images below using `100` as one argument for each.



5) Write 2 expressions that would build **right-isosceles** triangles. Use `right-triangle` for one expression and `isosceles-triangle` for the other expression.



6) Which do you like better? Why?

Composing with Circles of Evaluation

Notice and Wonder

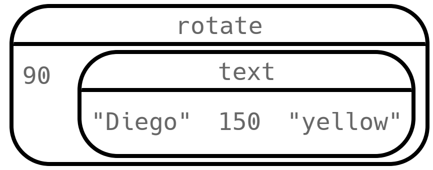
Suppose we want to see the `text` *"Diego" written vertically in yellow letters of size 150. Let's use Circles of Evaluation to look at the structure:*

We can start by generating the Diego image.

`text("Diego", 150, "yellow")`

→

And then use the `rotate` function to rotate it 90 degrees.



`rotate(90, text("Diego", 150, "yellow"))`

1) What do you Notice? ____

____

2) What do you Wonder? ____

____

Let's Rotate an Image of Your Name!

Suppose you wanted the computer to show your name in your favorite color and rotate it so that it's diagonal…

Write your name (any size), in your favorite color

3) Draw the circle of evaluation:

5) Convert the Circle of Evaluation to code:

`rotate` **the image so that it's diagonal**

4) Draw the circle of evaluation:

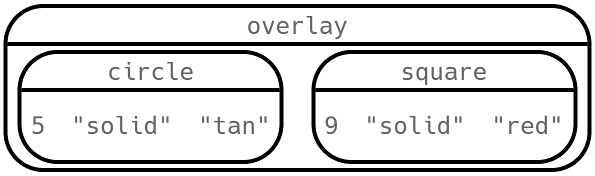
6) Convert the Circle of Evaluation to code:

Circle of Evaluation to Code (Scaffolded)

Complete the Code by Filling in the Blanks!

Finish the Code by filling in the blanks.

1)

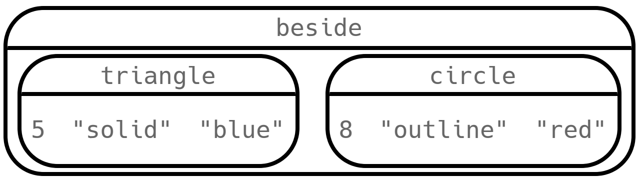



```
overlay(circle(____, "solid", ____________________), ____________________(9, ____________________, "red"))
```

Complete the Code by adding Parentheses

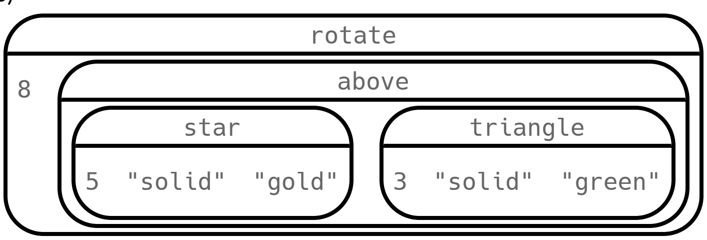
For each Circle of Evaluation, finish the Code by adding parentheses and commas.

2)



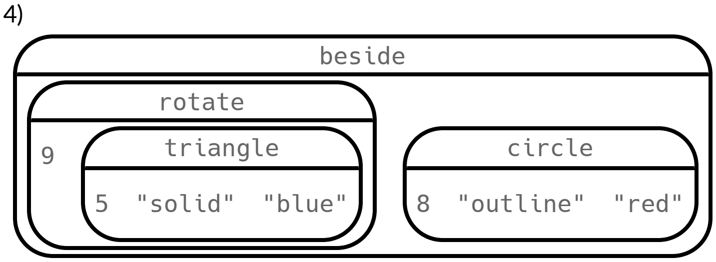

```
beside    triangle    5    "solid"    "blue"    circle    8    "outline"  "red"
```

3)




```
rotate    8    above    star    5    "solid"    "gold"    triangle    3    "solid"    "green"
```

4)




```
beside    rotate    9    triangle    5    "solid"    "blue"    circle    8    "outline"    "red"
```

Frayer Model: Domain and Range

My Definition

Facts and Characteristics

Domain

Examples

Non-Examples

My Definition

Facts and Characteristics

Range

Examples

Non-Examples

(optional)

Frayer Model: Function and Variable

My Definition

Facts and Characteristics

Function

Examples

Non-Examples

My Definition

Facts and Characteristics

Variable

Examples

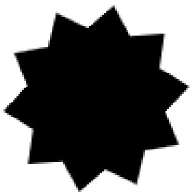
Non-Examples

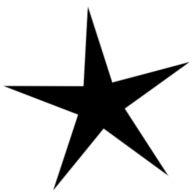
(optional)

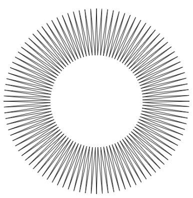
Radial Star

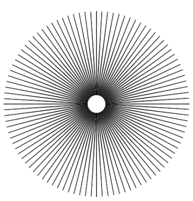
```
# radial-star :: (___Number___, ___Number___, ___Number___, ___String___, ___String___) -> Image
                     points      outer-radius  inner-radius   fill-style     color
```

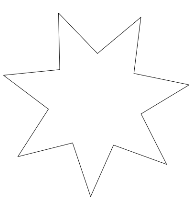
Using the Contract above, match the images on the left to the expressions on the right. Test the code at [code.pyret.org (CPO)](https://code.pyret.org).

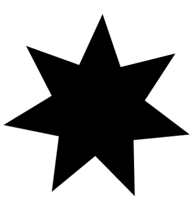

1 A `radial-star(5, 200, 50, "solid", "black")`

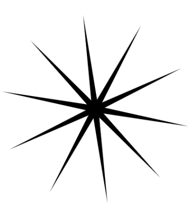

2 B `radial-star(7, 200, 100, "solid", "black")`


3 C `radial-star(7, 200, 100, "outline", "black")`


4 D `radial-star(10, 200, 150, "solid", "black")`


5 E `radial-star(10, 200, 20, "solid", "black")`


6 F `radial-star(100, 200, 20, "outline", "black")`


7 G `radial-star(100, 200, 100, "outline", "black")`

(optional)

Triangle Contracts (SAS & ASA)

Type each expression (left) below into the code.pyret.org (CPO) and match it to the image it creates (right).

Expression			Image
`triangle-sas(120, 45, 70, "solid", "black")`	1	A	
`triangle-sas(120, 90, 70, "solid", "black")`	2	B	
`triangle-sas(120, 135, 70, "solid", "black")`	3	C	
`triangle-sas(70, 135, 120, "solid", "black")`	4	D	

Contracts

Think about how you would describe each `triangle-sas` argument to someone who'd never used the function before.

5) Annotate the Contract below using descriptive variable names.

`triangle-sas` :: (____Number____, ____Number____, ____Number____, ____String____, ____String____) `-> Image`

If you have a printed workbook, add examples of each of the triangle functions we've explored to your contracts pages.

★ If you have time, experiment with the `triangle-asa` function.

```
# triangle-asa :: (__Number__, __Number__, __Number__, __String__, __String__) -> Image
```

top-left-angle, left-side, bottom-angle, fill-style, color

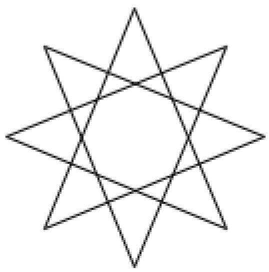
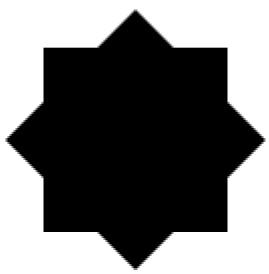
★ Why did these two functions need to take in one more Number than `right-triangle` did?

Star Polygon

```
# star-polygon :: (__Number__, __Number__, __Number__, __String__, __String__) -> Image
```

side-length, points-on-polygon, points-to-skip-for-star, fill-style, color

1. Using the Contract above, write expressions to create images like those pictured below.
2. Go to [code.pyret.org (CPO)](code.pyret.org) to test your code.
3. Then write expressions to generate two more star polygons of your choosing.
   Sketch them and record your working code.

1		______________________
2		______________________
3		______________________
4		______________________

(optional)

Modeling Data

A Quick Review…

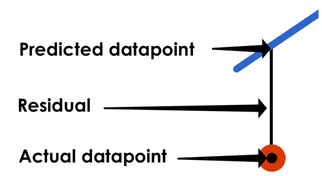
When viewing a cloud of points on a scatter plot, sometimes we can see a pattern in the data.

- If the points cluster around a straight line, it might mean there's a ***linear relationship*** between the ***explanatory variable*** (x) and ***response variable*** (y).
- The line can slope up or down, indicating a *positive* relationship (where the two variables increase together) or *negative* relationship (where the response variable decreases as the explanatory variable increases).

These lines are known as ***models*** for the data. The straight-line function describing a linear relationship is called a *linear model* .

- With a good model, the point cloud will hug the line tightly. A poor model will have lots of points that stray far from the line.
- Models *summarize* the data. For most datasets that means there will be data points that are not *exactly* on the line! And sometimes the line of best fit won't even pass through a single point in the dataset.
- We can use **linear regression** (`lr-plot` in Pyret) to compute the *best possible linear model* for a dataset, known as the **line of best fit**.

S : Measuring Error in a Model



Differences between the predicted y-value and actual y-value for each x-value are called ***residuals***.
A residual tells us "how wrong" the model was at that particular point.

$$\text{Data} = \text{Model} + \text{Error}$$

We can summarize the error for *all* the points in a dataset using the ***Standard Deviation of the Residuals*** - known as ***S*** - to get a sense of how much to trust the predictions made by a model.

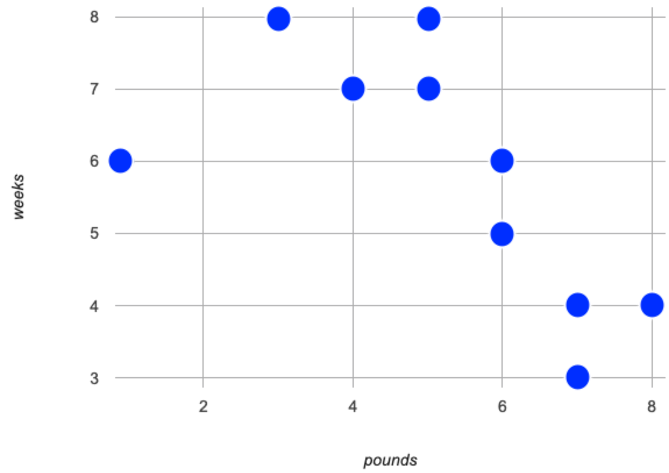
- ***S*** is expressed in terms of *units of the response variable* (y) and tells us how much error we expect in predictions made from the model. *(e.g. up to $8000, 5 years, 11 inches, etc.)*
- The closer the data points are to the model, the smaller the residuals are, and the smaller ***S*** will be.
- If the ***S-value*** for a model is zero, *it fits the data perfectly!*
- When we compare two models for the same dataset, the one with the lower ***S-value*** fits better.
- We have no way of knowing whether or not ***S-values*** represent a small or large amount of error until we consider them in relation to the range of the dataset! *(e.g. errors of $20,000 are huge in the context of median salary, but small in the context of national budgets.)*

How could we Measure Whether a Model is a Good Fit? (Lizards)

Summarize the Relationship You See

Below is a sample of lizards from the Animals Dataset.

name	pounds	weeks
Amy	5	7
Aries	4	7
Boss	6	5
Brittany	3	8
Buck	7	4
Butterscotch	1	6
Chico	8	4
Coconut	6	6
Dodger	5	8
Dylan	7	3



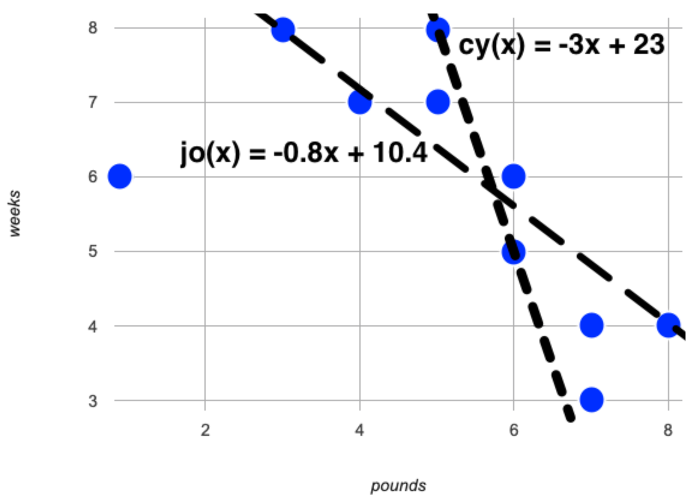
1) Use a straightedge to draw the ***line of best fit*** that best summarizes the relationship you see in the data on the scatter plot.

2) Describe how you decided where to draw the line. ______________________________

____

Comparing Models

3) Cy and Jo drew the two lines below. Do you think `cy(x)` or `jo(x)` is a better model for this data? Why?



____

____

____

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____

____

4) What could we measure, to calculate *how much better of a model* it is? ______________________________

____

____

5) *Neither of these models is the best possible model!* What would have to be true of a third model, for us to know that it was a better fit than these two? ______________________________

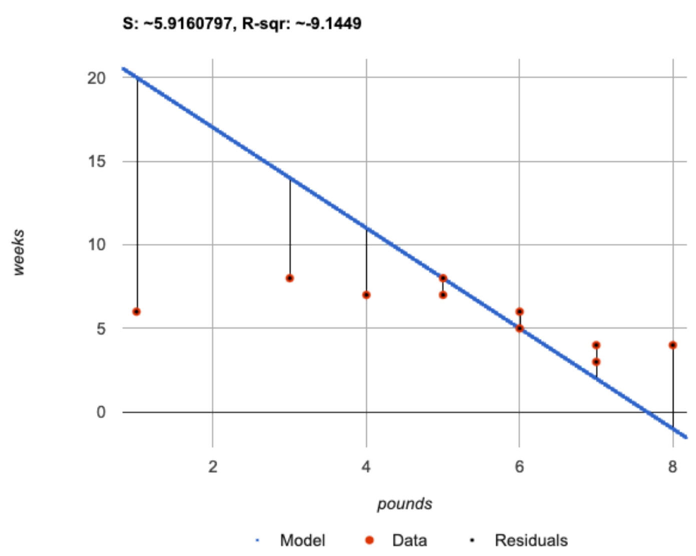
____

____

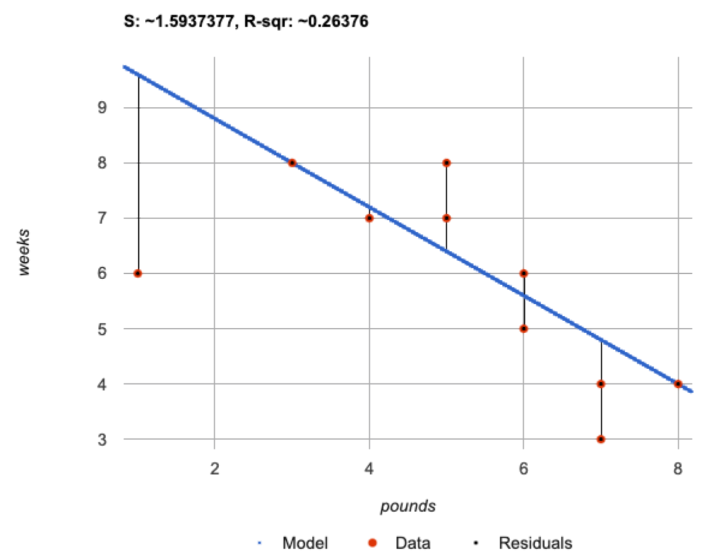
Introducing fit-model

These data visualizations were generated using the [Lizard Sample Starter File](). They can be viewed as interactive charts by uncommenting the final lines in the Definitions Area and clicking "Run".

```
fun cy(x): (-3 * x) + 23 end
fit-model(
 lizard-sample, "name", "pounds", "weeks", cy)
```




```
fun jo(x): (-0.8 * x) + 10.4 end
fit-model(
 lizard-sample, "name", "pounds", "weeks", jo)
```



What do you Notice?	What do you Wonder?

1) How is the `fit-model` plot for cy's model **similar** to the `fit-model` plot for jo's model? ______________________________

____

____

2) How is the `fit-model` plot for cy's model **different** from the `fit-model` plot for jo's model? ______________________________

____

____

3) What do you think the three terms in the legend refer to?

- Model: ____
- Data: ____
- Residuals: ____

Considering *S* in Context

For each model below, decide whether you agree that the model is a good fit. *Then* rank the models from 1 (best fit) to 8 (worst fit).

How good is the model?	Ranking
1) A data scientist is working with data from animals at a shelter. • The range of days to adoption in this dataset are from 0 to 400. • An S value of 300 means predicted adoption times could be off by 300 days. I ______________________ that this model is a good fit. strongly agree, agree, disagree, strongly disagree	
2) A student is exploring a dataset on climate change. • The range of Arctic Sea Ice is from 3,920,000 to 7,670,000 square kilometers • An S value of 300 means predicted Arctic Sea Ice coverage could be off by 300 square kilometers. I ______________________ that this model is a good fit. strongly agree, agree, disagree, strongly disagree	
3) A data scientist is working with data from US public schools. • The range of graduates per school per year is 2 to 2003. • An S value of 300 means predicted graduate values could be off by 300 students. I ______________________ that this model is a good fit. strongly agree, agree, disagree, strongly disagree	
4) A student is exploring a dataset on earthquakes. • The range of earthquake depths in this dataset are from 4200m to 664000m. • An S value of 300 means predicted earthquake depths could be off by 300 meters. I ______________________ that this model is a good fit. strongly agree, agree, disagree, strongly disagree	
5) A student is exploring a dataset on arrests in Los Angeles. • The age range in this dataset is from 0 to 92. • An S value of 1 means predicted ages could be off by 1 year. I ______________________ that this model is a good fit. strongly agree, agree, disagree, strongly disagree	
6) A data scientist is working with data about snowflakes. • The range of snowflake weights is from 0.001 grams to 0.02 grams. • An S value of 1 means predicted values could be off by 1 gram. I ______________________ that this model is a good fit. strongly agree, agree, disagree, strongly disagree	
7) A data scientist is working with data from animals at a shelter. • The range of ages is from 0.5 years to 16 years. • An S value of 1 means predicted ages could be off by 1 year. I ______________________ that this model is a good fit. strongly agree, agree, disagree, strongly disagree	
8) A student is working with a dataset of adult blue whales. • The range of weights is 200,000 to 330,000 pounds. • An S value of 1 means predicted weights could be off by 1 pound. I ______________________ that this model is a good fit strongly agree, agree, disagree, strongly disagree	

Interpreting our Models

Cy's Model: $cy(x) = -3x + 23$

This model predicts that lizards weighing 0 pounds will be adopted in ________ (y-intercept) ________ (y-units) and that,

for every additional pound a lizard weighs (x-variable units), ________ (y-variable) will ________ (increase/decrease) by ________ (rate of change) ________ (y-units).

The error in the model is described by an ***S-value*** of about ________ (S) ________ (y-units). I ________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ________ (y-variable units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

Jo's Model: $jo(x) = -0.8x + 10.4$

This model predicts that lizards weighing 0 pounds will be adopted in ________ (y-intercept) ________ (y-units) and that,

for every additional ________ (x-variable units), ________ (y-variable) will ________ (increase/decrease) by ________ (rate of change) ________ (y-units).

The error in the model is described by an ***S-value*** of about ________ (S) ________ (y-units). I ________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ________ (y-variable units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

Comparing Models

1) Is Jo's model better or worse than Cy's model? ________________

2) How much ________ (more / less) error do we expect in predictions made with Jo's model than predictions made with Cy's model?

$$\text{Percent Change} = \frac{\text{Difference between the S-values}}{\text{S-value for Cy's model}} \times 100 = \_\_\_$$

We expect predictions made with Jo's model to have ________ percent ________ (more / less) error than predictions made with Cy's model!

My Model

If your teacher had you complete [From Lines to Functions], write the function you defined for your model on the line below and then complete the interpretation. If your class did not define models for the lines you drew, you can skip this section.

$\text{my-model}(x) =$ ________________

This model predicts that lizards weighing 0 pounds will be adopted in ________ (y-intercept) ________ (y-units), and that,

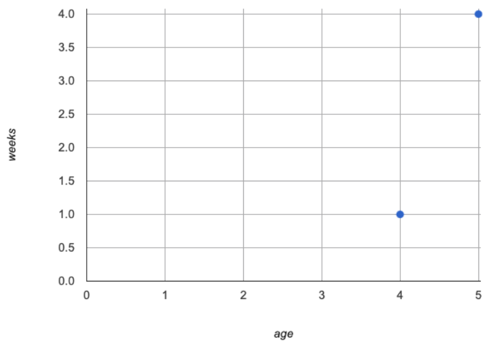
for every additional ________ (x-variable units), ________ (x-variable) will ________ (increase/decrease) by ________ (rate of change) ________ (x-units).

The error in the model is described by an ***S-value*** of about ________ (S) ________ (units). I ________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ________ (y-variable units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

From Lines to Functions

Defining a Function from Two Points

1) The scatter plot displays the relationship between clownfish age and adoption time in weeks. Identify the coordinates of the data points.



1st point: (________ , ________)
x_1 y_1

2nd point: (________ , ________)
x_2 y_2

2) We want to understand how change (Δ) in the ___age___ (x-value) of the clownfish relates to the change (Δ) in their ___adoption time in `weeks`___ (y-value).

Compute the ***slope*** (rate of change) between the points: $\dfrac{\Delta \text{y}}{\Delta \text{x}} = \dfrac{y_2 - y_1}{x_2 - x_1} = \underline{\quad\quad}$

Based solely on data from these two clownfish, an increase in age of ________ (Δ year(s)) *predicts an increase of* ________ (Δ weeks) *in adoption time.*

3) Now, use ***slope-intercept form*** ($y = mx + b$) to calculate the ***y-intercept*** (vertical shift) of the line passing through the two points.

Hint: Fill in the blanks for y *and* x *below with the coordinates of the first point. Then use the slope we just calculate for* m. *Finally, solve for* b.

________ (y) $=$ ________ (slope (m)) $\times$ ________ (x) $+$ b (y-intercept / vertical shift)

4) Use the slope and y-intercept you calculated to write the complete model below (in both Function and Pyret notation):

$clownfish(x) =$ ________ (slope (m)) x + ________ (y-intercept / vertical shift)

`fun clownfish(x): (____ * x) + __ end`

Define a Function for Your Lizard Line

5) Refer to the line you drew on [How could we Measure Whether a Model is a Good Fit? (Lizards)] to show the relationship you saw between the lizards' weight in `pounds` and adoption time in `weeks`. Identify two points that could be used to define the line (the points do not have to be dots from the scatter plot itself).

First point: (________ , ________) Second point: (________ , ________)
x_1 y_1 x_2 y_2

6) Compute the ***slope*** (rate of change) between the points: $\dfrac{\Delta \text{y}}{\Delta \text{x}} = \dfrac{y_2 - y_1}{x_2 - x_1} = \underline{\quad\quad}$

Based solely on these 2 points, increasing a lizard's weight by 1 pound, predicts a(n) ________ (increase or decrease?) *of* ________ (Δ weeks) *week(s) in adoption time .*

7) Now, use ***slope-intercept form*** ($y = mx + b$) to calculate the ***y-intercept*** (vertical shift) of the line passing through the two points.

________ (y) $=$ ________ (slope (m)) $\times$ ________ (x) $+$ b (y-intercept / vertical shift)

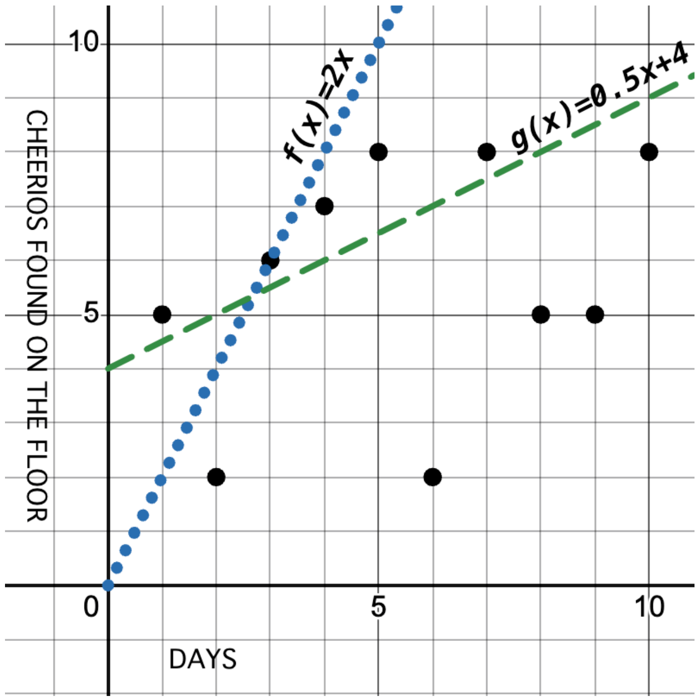
8) Use the slope and y-intercept you calculated to write the complete model below (in both Function and Pyret notation):

$lizard(x) =$ ________ (slope (m)) x + ________ (y-intercept / vertical shift)

`fun lizard(x): (__ * x) + __ end`

(optional)

How could we Measure Whether a Model is a Good Fit? (Cheerios)



id	Days	Cheerios found on the floor
a	1	5
b	2	2
c	3	6
d	4	7
e	5	8
f	6	2
g	7	8
h	8	5
i	9	5
j	10	8

1) Do you think f(x) or g(x) is a better model for this data? ______________________________

2) What makes you think that? ______________________________

____

____

____

3) What could we measure, to calculate *how much better of a model* it is? ______________________________

____

____

____

4) *Neither of these models is the best possible model!* What would have to be true of a third model, for us to know that it was a better fit than these two? ______________________________

____

____

____

Exploring the States Dataset

Open the [State Demographics Starter File](#).
Then, click "Run" and type `states-table` into the Interactions Area on the right to see the dataset.

What do you Notice about this dataset?	What do you Wonder about this dataset?

Describing Income

1) Find the two states with the lowest **median** income and complete the table below with their information.

State	Median Household Income	Per-capita Income

2) Would a model built from two states with low `median-income` be likely to fit the rest of the data well? Why or Why not?

____

____

3) Why do you think states typically have higher `median-income` than `per-capita income`? ____________________

____

____

____

Definitions in the Code

The two lines of code under `# Define some rows` *extract rows 0 and 1 from the table, and define them as* `alabama` *and* `alaska`.

4) Type `alabama` into the Interactions Area. What do you get back? ______________________________

5) Underneath the definition of those rows, **add new definitions** for `california`, `michigan`, and one other state. Click "Run", so that Pyret reads your new definitions and test them out in the Interactions Area to make sure you defined them correctly!

★ Add any additional Notices or Wonderings you have about this dataset to the table at the top.

Looking for Patterns

Open the State Demographics Starter File.

What columns do you think might have a relationship?
(e.g. is the number of veterans related to the land-area? Is the population in 2010 related to the population in 2020?)

1) I think that ______________________________

might be related to ______________________________

because ______________________________

____

2) I think that ______________________________

might be related to ______________________________

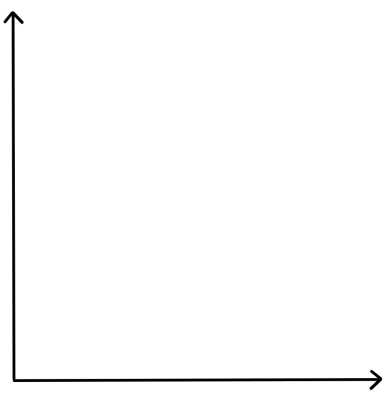
because ______________________________

____

What do you predict the relationships will look like? (*e.g. Negative? Positive? Curves?*)

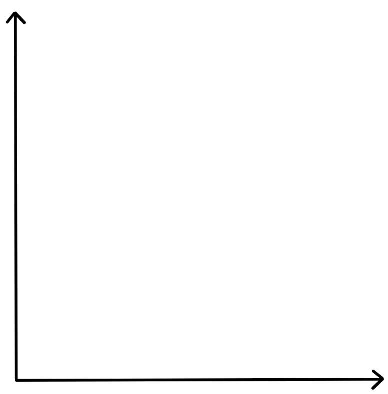
Label the axes on the graphs below with your variables from 1 and 2. Then sketch how you imagine they might be related.

3)



How confident are you in your prediction?
not at all *somewhat* *very*

4)



How confident are you in your prediction?
not at all *somewhat* *very*

```
# scatter-plot :: (Table, String , String , String ) -> Image
                          labels   explanatory  response
```

Use the Contract above to make a scatter plot for each relationship.
(Use `"state"` as the label, so that clicking on a point will show you which state you're looking at.)

If there's a pattern, describe it so that someone else could sketch it. What does it mean? If there isn't a pattern, what does *that* mean?

5) ______________________________

____

6) ______________________________

____

More Relationships

Pyret allows us to generate many scatter plots, quite quickly! Take advantage of that to identify two relationships that appear to be strong.
Hint: If you're working in the Interactions Area of Pyret, you can hit the up arrow to bring back previously used lines of code.

7) There appears to be a ______________ (negative / positive) relationship between ______________________ (explanatory variable (x-axis)) and ______________________ (response variable (y-axis))

8) There appears to be a ______________ (negative / positive) relationship between ______________________ (explanatory variable (x-axis)) and ______________________ (response variable (y-axis))

★ What have you learned about our state and the others we decided to focus on?

Build a Model from Samples: College Degrees v. Income

The Alabama - Alaska Model

Open the State Demographics Starter File.

1) Record the `pct-college-or-higher` and `median-income` values for the `alabama` and `alaska` rows, as (x,y) pairs below:

(________________ , ________________)
AL pct-college-or-higher, AL median-income

(________________ , ________________)
AK pct-college-or-higher, AK median-income

2) Compute the equation of the line passing between these two points using the space below. **This line will be your linear model** (also known as the "predictor function", or "line of best fit"), which predicts `median-income` as a function of `pct-college-or-higher`.

3) Write the complete model below (in both Function and Pyret notation):

$\text{al-ak}(x) =$ ____________ x + ____________
slope(m), y-intercept / vertical shift

```
fun al-ak(x): (__________ * x) + __________ end
```

4) What median income does this model expect a state without ANY college graduates (0%) to earn? ________________

5) What does this model predict?

A model built from Alaska and Alabama expects that median income for a state without ANY college graduates (0%) will be

____________ and predicts that a 1 percent increase in college degrees (x-variable units) is associated with a

____________ (y-units) ____________ (increase / decrease) in ____________ (y-variable units).

Return to your copy of the starter file and define `al-ak` on line 44 of the Definitions Area. Then Click "Run".
(If there are any errors or warnings, fix them and click "Run" again.)

6) In the Interactions Area, try plugging in the `pct-college-or-higher` value for Alabama by typing `al-ak(22.6)`.

- How well does it predict the correct median income for Alabama? ________________
- What would you type to predict median income for Alaska? ________________
- How well does it predict the correct median income for Alaska? ________________
  Consider: If it doesn't predict it perfectly, why might that be?

Try different `pct-college-or-higher` values from *other* states, to see how well our Alabama-Alaska model fits the rest of the country.

7) Identify a state for which this model works well: ________________

8) Identify a state for which this model works poorly: ________________

Another Model

9) Look at the scatter plot. Imagine you were going to define another model. Identify two other states that would likely generate a better fit.

________________ and ________________

Defining a Linear Function from Two Points

Define the linear function through (-2,5) and (3,-10).

Start by assigning values for x_1, y_1, x_2, y_2 *from the coordinates above:* -2 (x_1) ____ (y_1) ____ (x_2) ____ (y_2)

Step 1: Calculate the slope of the line by replacing the variables in the equation below with their corresponding coordinates.

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\quad - \quad}{\quad - \quad} = \_\_\_\_$$

Step 2: Use the slope intercept form of the line to calculate the y-intercept.

- replace m with the slope we just calculated
- replace x and y with the values from the first point: $(-2, 5)$
- solve for b

Slope-intercept form of the line: $y = mx + b$ ________ $=$ ________ ________ $+ b$

________ $=$ b

Note: *We could also have done* ***Step 2*** *using the second point:* $(3, -10)$. *Let's do that now to make sure we get the same result!*

________ $=$ ________ ________ $+ b$

________ $=$ b

Step 3: Use the ***slope*** and ***y-intercept*** we calculated to write our function definition!

$f(x) =$ ________ x + ________

Define the linear function through (-5,2) and (3,6).

Start by assigning values for x_1, y_1, x_2, y_2 *from the coordinates above:* ____ (x_1) ____ (y_1) ____ (x_2) ____ (y_2)

Step 1: Calculate slope.

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\quad - \quad}{\quad - \quad} = \_\_\_\_$$

Step 2: Calculate the y-intercept.

Hint: You can use either point. Which would be simpler?

________ $= b$

Step 3: Write the function definition!

$f(x) =$ ________ x + ________

(optional)

Build a Model from Samples: College Degrees v. Income (Scaffolded)

Open the State Demographics Starter File.

The Alabama - Alaska Model

1) Record the `pct-college-or-higher` and `median-income` values for the `alabama` and `alaska` rows, as (x,y) pairs below:

(________________ , ________________) (________________ , ________________)

AL pct-college-or-higher, AL median-income — AK pct-college-or-higher, AK median-income

2) How does change (Δ) in the percent of the population that attended college relates to change (Δ) in the median income?

Compute Δ median-income $=$ ________________ (AK median-income) - ________________ (AL median-income) = ________________

Compute Δ pct-college-or-higher $=$ ________________ (AK pct-college-or-higher) - ________________ (AL pct-college-or-higher) = ________________

Compute the slope/rate of change between AL and AK: $\frac{\Delta\text{median-income}}{\Delta\text{pct-college-or-higher}} = \frac{\qquad}{\qquad} =$

Based solely on data from Alabama and Alaska, we are seeing that a ________________ (Δ pct-college-or-higher) *percent increase in college graduates among the population translates to a* ________________ (Δ median-income) *dollar increase in median income.*

3) Let's use the slope-intercept form of the line to calculate the y-intercept / vertical shift of the line passing through AK and AL.

____________ (y) $=$ ____________ (slope (m)) $\times$ ____________ (x) $+$ b (y-intercept / vertical shift)

- Find the x and y values from the AK row: (____________ (x :: AK pct-college-or-higher), ____________ (y :: AK median-income))
- Now, replace y, m, and x in the equation above with values from the AK row and the slope we just calculated in question 2.
- Then solve the equation above for $b =$ ____________

4) Use the slope and y-intercept you calculated to write the complete model below (in both Function and Pyret notation):

$al\text{-}ak(x) =$ ____________ (slope (m)) x + ____________ (y-intercept / vertical shift)

```
fun al-ak(x): (____________ * x) + ____________ end
```

Return to your copy of the starter file and add the code you just wrote to line 44 of the Definitions Area. Then Click "Run".

5) In the Interactions Area, try plugging in the `pct-college-or-higher` value for Alabama by typing `al-ak(22.6)`.

- How well does it predict the correct median income for Alabama? ________________
- What expression would predict median income for Alaska? ________________
- How well does it predict the correct median income for Alaska? ________________

Try `pct-college-or-higher` values from other states, to see how well our Alabama-Alaska model fits the rest of the country.

6) Identify a state for which this model works well: ________________

7) Identify a state for which this model works poorly: ________________

8) What median income does this model expect a state with zero college graduates to earn? ________________

Another Model

9) Look at the scatter plot. Imagine you were going to define another model. Identify two other states that would likely generate a better fit.

________________ and ________________

(optional)

Exploring Horizontal Shift in Linear Models

This page is designed to accompany ***Exploring Horizontal Shift in Linear Models (Desmos)****. You will need a link from your teacher.*

Exploring $z(x) = m(x - 0) + k$

These questions are for slide 1.

1) When $m = 0.5$ and $h = 4.6$, what value for k will land the blue line on top of the black line? ______________________

Move the sliders to new values for $m =$ ________ and $h =$ ________.

2) What value for k will land the blue line on top of the black line now? ______________________

Continue experimenting with the sliders.

3) Can you find any values for m and h for which you can't land the blue line on top of the black line by changing k?

____

____

What pattern do you notice about the relationship between m, h and k?

4) If you knew the m and h values, how could you find the value of k? ______________________

____

____

____

5) If you knew the m and k values, how could you find the value of h? ______________________

____

____

____

6) What value of h do you think would be comparable to a vertical shift of 6 for lines with a slope of 2? ______________________

Explain. ______________________

____

____

Advance to slide 2 and confirm that your answer for the last question was correct.

7) What directions would you give to someone about how to predict the h-value of a horizontal shift that will translate to a vertical shift?

____

____

____

Fitting Models: College Degrees v. Income

Open your copy of the State Demographics Starter File.

The `al-ak` Model

Type `fit-model(states-table, "state", "pct-college-or-higher", "median-income", al-ak)` in the Interactions Area, then find the points representing AL and AK along the predictor line. *Hint: You know their coordinates and they will help you know where to look!*

1) What do you Notice? ______________________________

____

2) What do you Wonder? ______________________________

____

3) Find S in the upper left corner. What is the S value (the number after S)? ______________________________

4) With median income ranging from ______________ (lowest median income) to ______________ (highest median income), what does the S-value of the `al-ak` model tell us?

____

____

Comparing Models

5) Use `fit-model` to find the S-value for the `mi-ca` model. ______________________________

6) Is the `mi-ca` model better or worse than the `al-ak` model? ______________________________

7) How much __________ (more / less) error do we expect in predictions made with the `mi-ca` model than predictions made with the `al-ak` model?

Percent Change $= \dfrac{\text{Difference between the S-values}}{\text{S-value for al-ak model}} \times 100 =$ ____ `mi-ca` model predictions are expected to have

________ percent ______________ (more / less) error than `al-ak` model predictions!

A Model of Your Own

8) Identify two other states that you think would make a better model: ______________________ and ______________________.

Add two new definitions for these states to your State Demographics Starter File.

9) Record the `college-or-higher` and `median-income` values for these states, as (x,y) pairs below:

(______________ (college-or-higher), ______________ (median-income))   (______________ (college-or-higher), ______________ (median-income))

10) Derive your model and write it below (in both Function and Pyret notation), then fit the model and record the S-value:

$\text{other}(x) =$ ______________ (slope (m)) x + ______________ (y-intercept / vertical shift)

`fun other(x): (` ______________ `* x) +` ______________ `end` S: __________

11) Adjust the slope and y-intercept of your model to get the **smallest S possible.** Write the best model you find (and corresponding S) below:

`fun best(x): (` ______________ `* x) +` ______________ `end` S: __________

12) How much ________ (more / less) error do we expect in predictions made with your model than predictions made with the `mi-ca` model? ________ %

Optimizing and Interpreting Linear Models

Open your copy of the State Demographics Starter File and click "Run".

Build a Model Computationally

1) Evaluate `lr-plot(states-table, "state", "pct-college-or-higher", "median-income")`. What is S? ____________

2) On the line below, write the optimal linear model that was computed through linear regression:

$optimal(x) =$ ______________ x + ______________________ `fun optimal(x): (`____________ `* x) +` ____________ `end`

slope (m) — y-intercept / vertical shift

Interpret the `al-ak` Model

We started with a model based on Alabama and Alaska (`fun al-ak(x): (5613.67 * x) + -83616.02 end`), and got an S-value of `~36164.68`. We can interpret this as follows:

The ___*Alabama-Alaska*___ (sensible name) *model predicts that a 1* ___*percent*___ (x-axis units) *increase in* ___*percent college degrees*___ (explanatory variable (x)) *is associated with a* ___*~5614 dollar*___ (rate of change in y-units) ___*increase*___ (increase / decrease) *in* ___*median household income*___ (response variable (y)). *With* ___*median household income*___ (response variable (y)) *ranging from* ___*$39,031*___ (lowest y-value) *to* ___*$73,538*___ (highest y-value) *and an* S *-value of* ___*~36,164.68*___ (S -value) ___*dollars*___ (y-axis units), *I* ___*strongly disagree*___ (strongly agree, agree, disagree, strongly disagree) *that this model is a good fit.*

3) What does the **slope (m)** of this linear model tell us? ______________________________

____

4) How would we use this model predict to predict the median income for a state where 30% of the population has attended college?

____

Interpreting the Optimal Model

The ___linear-regression___ (sensible name) model predicts that a 1 ____________ (x-axis units) increase in ____________ (explanatory variable (x)) is associated with a ____________ (rate of change in y-units) ____________ (increase / decrease) in ____________ (response variable (y)). With ____________ (response variable (y)) ranging from ____________ (lowest y-value) to ____________ (highest y-value) and an S -value of ____________ (S -value) ____________ (y-axis units), I ____________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit.

5) What median income does this model predict when 30% of a state's population has attended college? ____________

6) Why doesn't it make sense to use this model to predict median income for a state with no college attendees (0%)? ____________

____

____

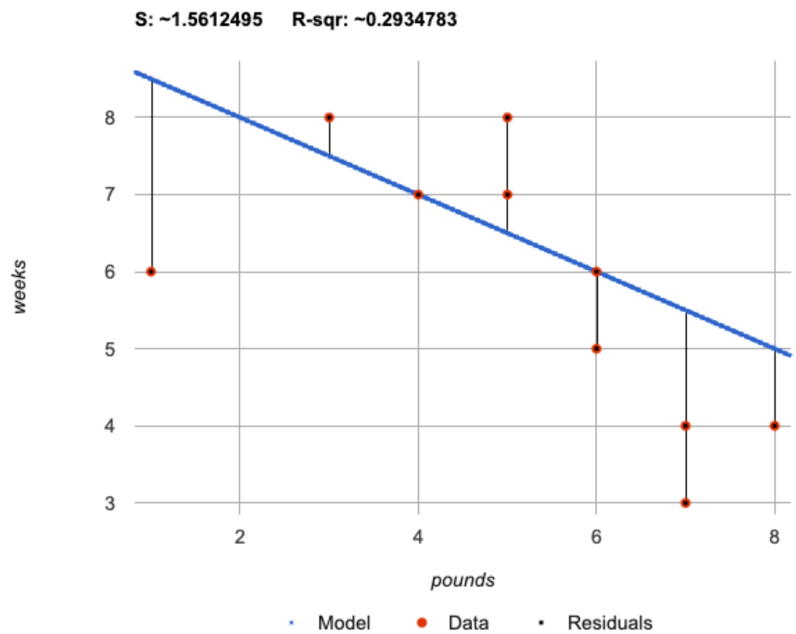
7) If a state's college graduation increases from 10% to 11%, what change in median income does this model predict? ____________

- What if it goes from 50% to 51%? ____________
- What if it goes from 90% to 91%? ____________
- Does this model predict the same increase in income for *every* additional 1% `college-or-higher`? Why or why not? ____________

____

Review: What can we learn from Residuals and S-values?

1) Below we see the model `lizard-adopt-time(pounds) = (-0.5 * pounds) + 9` fit to the lizard sample.



This model predicts that lizards weighing 0 pounds will be adopted in ________ (y-intercept) ________________ (y-units), and that,

for every additional ______________________ (x-variable units), ________________ (x-variable) will ____________ (increase/decrease) by ________ (rate of change) ________ (x-units).

The error in the model is described by an ***S-value*** of about ________ (S) ____________ (units). I ______________________ (strongly agree, agree, disagree, strongly disagree) that

this model is a good fit considering that ______________ (y-variable units) in this dataset range from __________ (lowest y-value) to __________ (highest y-value).

2) When we mouse over a `fit-model` visualization, one of the popup boxes we'll see is titled "Residuals".

Residuals:
x: 34.5
y: 25000
y_1: 45000

What can we learn from this set of values?

____

____

____

3) A government office reported that the error in a model they made is described by an ***S-value*** of 3000. Is the model a good or bad fit? Explain.

____

____

____

4) In order to interpet an ***S-value*** we need to know:

- ____
- ____

More Models: College Degrees v. Income

Open your copy of the [State Demographics Starter File](#).

Adjusting Models through Trial and Error

In the Definitions Area, find the section titled ***Define some other models by modifying mi-ca****.*
For now, all three definitions in this section are exactly the same as `mi-ca`*. You will be changing them according to the directions below.*

1) `less-steep`

- If you wanted the model to be *less steep*, what slope could you use? ______________________
  Change the definition for `less-steep` to use the slope you wrote above and click "Run" to load your new definition.
- Type `fit-model(states-table, "state", "pct-college-or-higher", "median-income", less-steep)`
  - What is the S value of `less-steep`? ______________________
- Identify a y-intercept that would make the model fit the data better: ______________________
  - Adjust the definition to use the new y-intercept, click "Run" and refit the model.
    Hint: You can click in the Interactions Area and hit the up arrow to bring back the last line of code you used.
  - What is the S value of `less-steep` now? ______________________

2) `negative`

- If wanted your model to have a *negative slope*, what slope could you use? ______________________
  Change the definition of `negative` to use the slope you wrote above and click "Run" to load your new definition. Then fit the model.
- What is the S value of `negative`? ______________________

3) `horizontal`

- If you wanted your model to be horizontal, what would you have to change? ______________________
  Change the definition of `horizontal` so that it draws a horizontal model. Click "Run" and fit this model.
  - What is the S value of `horizontal`? ______________________
- Change the y-intercept so that the horizontal line passes through more of the data points. Click "Run" and fit this model.
  - What y-intercept did you use? ____________ What is the S value of `horizontal` now? ____________

The `ma-nv` Model

In the Definitions Area, find the section titled ***Define some rows****.*

4) Add two new definitions for `MA` (row 21) and `NV` (row 28), using the definitions for `alaska` and `alabama` as a model.

5) Record the `college-or-higher` and `median-income` values for `MA` and `NV`, as (x,y) pairs below:

(____________ , ____________)　　　　(____________ , ____________)
MA college-or-higher, MA median-income　　　　NV college-or-higher, NV median-income

6) Derive the `MA-NV model` (using the same steps you followed to derive the `AL-AK model` on [Fitting Models: College Degrees v. Income](#)) and write it below (in both Function and Pyret notation), then fit the model and record the S-value:

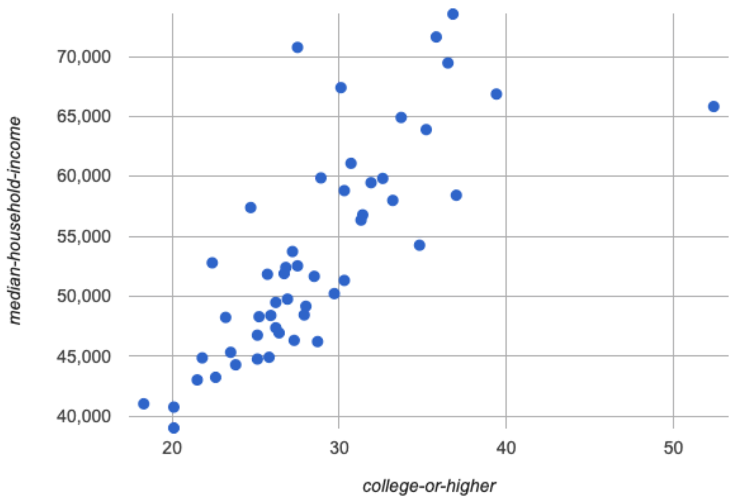
$\text{ma-nv}(x) =$ ____________ x + ____________　`fun ma-nv(x): (` ________ `* x) +` ________ `end`　S: ________
slope (m)　　y-intercept / vertical shift

(optional)

Graphing Linear Models

Sketch three of your linear models from [Build a Model from Samples: College Degrees v. Income](), [Fitting Models: College Degrees v. Income](), and *Optional:* [More Models: College Degrees v. Income]() on the scatter plots below.
Then label the slope, y-intercept, and S value of each model!

1)

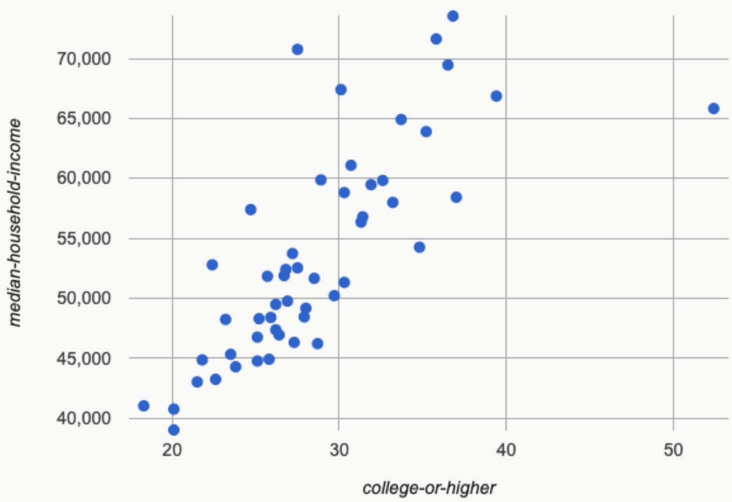


Slope

y-Intercept

S

2)

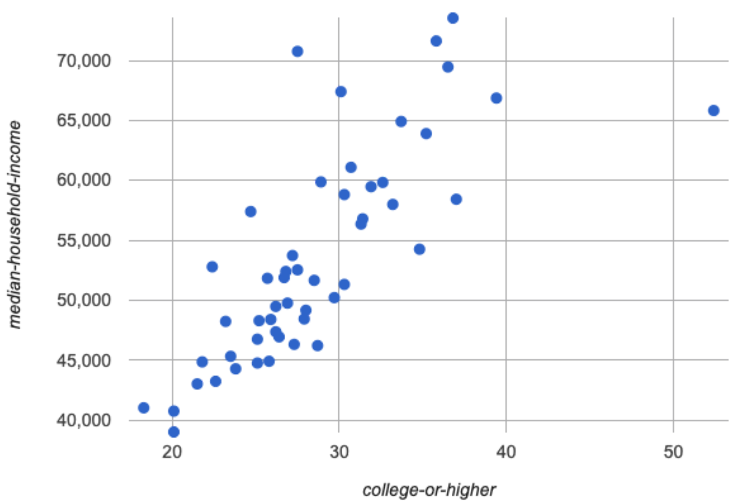


Slope

y-Intercept

S

3)



Slope

y-Intercept

S

(optional)

Building Models for Another Relationship in the Data

Open your copy of [State Demographics Starter File](). If you haven't already, ***Save a Copy*** *now.*
We've already built models for `pct-college-or-higher` *and* `median-income`*, but that's not the only relationship in this dataset!*

1) What other relationship are you curious about?

____________________ and ____________________
explanatory variable (x) | response variable (y)

2) Fill in the code to make a scatter plot exploring the relationship between those columns:

```
scatter-plot(states-table, "state", ____________________, ____________________)
```

3) Pick two states that you think would generate a good first model:

____________________ and ____________________

4) Calculate the slope and y-intercept for a line passing through these two point and use them to define your model. Then type this model into Pyret, and fit it to your data use `fit-model`.

```
fun my-new(x): (__________ * x) + __________ end
```

S -value ____________________

5) Use `lr-plot` to build the optimal linear model for this relationship.

```
fun best-new(x): (__________ * x) + __________ end
```

S -value ____________________

6) Based on these S -values, we would expect predictions made with this best possible model to have __________ percent less error than predictions made with the linear model I generated from two points.

7) **What does this model actually mean? Try completing the sentences below:**

This model predicts that a 1 ___percent___ (x-axis units) *increase in* ____________________ (explanatory variable (x)) *is associated with a*

____________________ (rate of change in y-units) ___increase___ (increase / decrease) *in* ____________________ (response variable (y)).

Based on the S *of* __________ (S) *and* ____________________ (response variable (y)) *ranging from* __________ (lowest y-value) *to* __________ (highest y-value),

I ____________________ (strongly agree, agree, disagree, strongly disagree) *that this model is a good fit.*

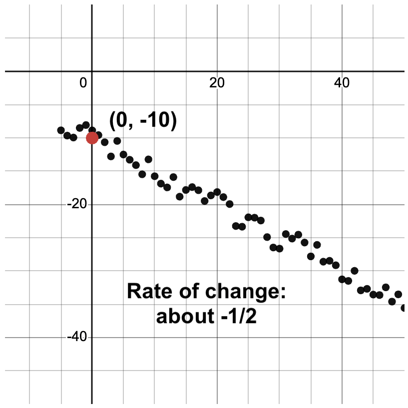
(optional)

Which Form is Best?

For each set of data provided below,

- Decide which form of the line would be the easiest to build from the available information.
- Write a definition of the linear model in that form.
- Translate the definition into Pyret notation.

1

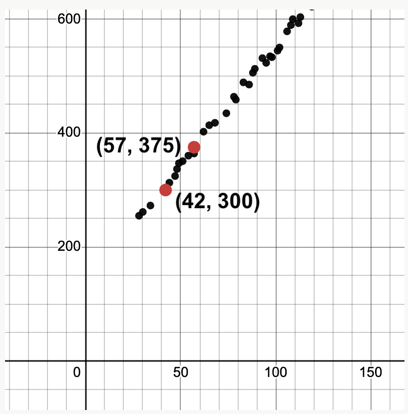


Which Form is easiest? ______________________________
slope-intercept, point-slope, or standard form?

Equation: ______________________________

`fun f(x) :` ______________________________ `end`

2

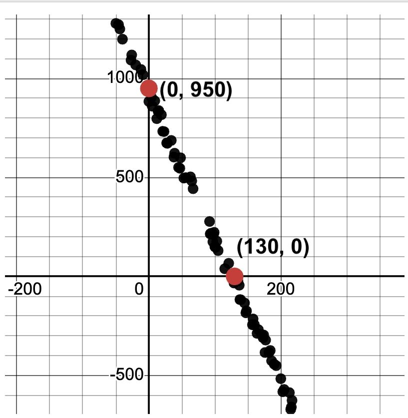


Which Form is easiest? ______________________________
slope-intercept, point-slope, or standard form?

Equation: ______________________________

`fun f(x) :` ______________________________ `end`

3



Which Form is easiest? ______________________________
slope-intercept, point-slope, or standard form?

Equation: ______________________________

`fun f(x) :` ______________________________ `end`

Other Forms of Linear Models

Depending on what you want to *do* with a model, it can be more convenient to use one form instead of another!

Slope-Intercept Form	Standard Form	Point-Slope Form
$y = mx + b$	$Ax + By = C$	$y - y_1 = m(x - x_1)$

Optimal Linear Model

The **Slope-Intercept Form** of the optimal linear model for predicting median income from pct-college-or-higher is :

$y =$ 1142 (slope) x + 20868.14 (y-intercept)

1) Write the Standard Form and Point-Slope Form of the **optimal** linear model below.

Standard Form	Point-Slope Form
______ x + ______ y = ______ (A, B, C)	y - ______ = ______ (x - ______) (y1, m, x1)

The `al-ak` model

The **Slope-Intercept Form** of the `al-ak` model for predicting median income from pct-college-or-higher is :

$y =$ 5614 (slope) x + 83616 (y-intercept)

2) Write the Standard Form and Point-Slope Form of the `al-ak` linear model below.

Standard Form	Point-Slope Form
______ x + ______ y = ______ (A, B, C)	y - 83616 (y1) = ______ (m) (x - 0 (x1))

Your Choice

Choose **another** linear model you came up with.

3) The **Slope-Intercept Form** of my ____________ model is: $y =$ ______ (slope) x + ______ (y-intercept)

4) Write the Standard Form and Point-Slope Form of that linear model below.

Standard Form	Point-Slope Form
______ x + ______ y = ______ (A, B, C)	y - ______ = ______ (x - ______) (y1, m, x1)

Comparing Forms

5) Which form is most useful to YOU, and why? ____

____

____

____

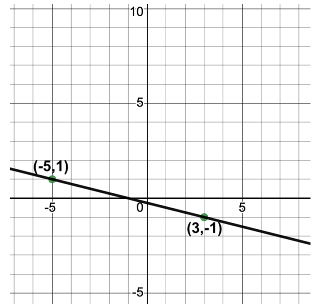
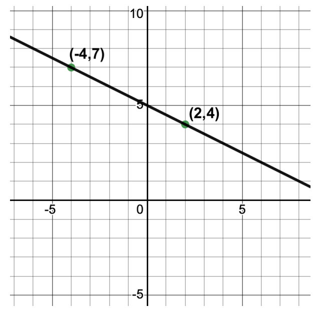
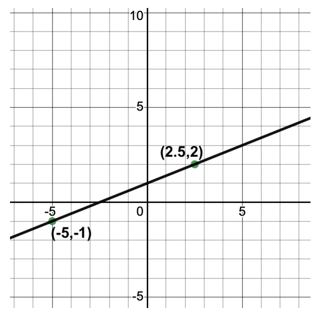
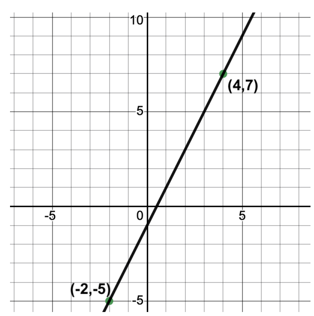
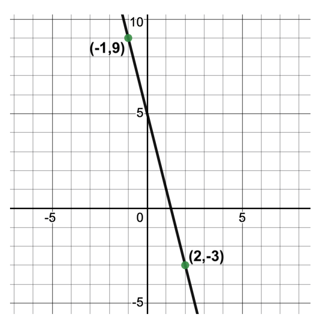
____

(optional)

Matching Point-Slope Form to Graphs

Point-Slope Form: $y - y_1 = m(x - x_1)$ m: *slope* y_1: *y-coordinate of a point* x_1: *x-coordinate of the same point*

Each of the graphs below represents a line of best fit derived from some data. **Match each definition below to the linear model it describes.**

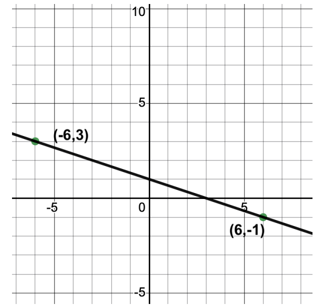
$y - 7 = -.5(x + 4)$	**1**	**A**	
$y + 3 = -4(x - 2)$	**2**	**B**	
$y + 5 = -0.25(x - 1)$	**3**	**C**	
$y - 7 = 2(x - 4)$	**4**	**D**	
$y + 5 = 0.4(x + 1)$	**5**	**E**	

(optional)

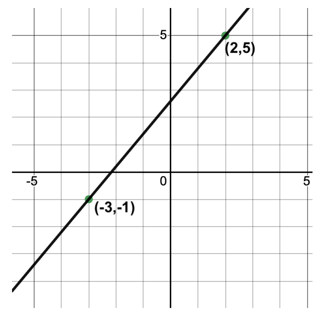
Matching Standard Form to Graphs

Each of the graphs below represents a line of best fit derived from some data. **Match each definition below to the linear model it describes.**

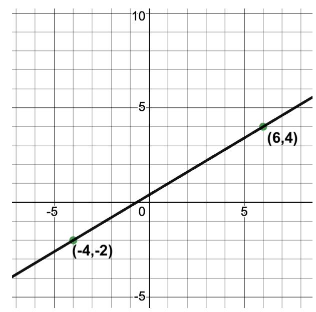
$2x + 5y = 28$ 1 A



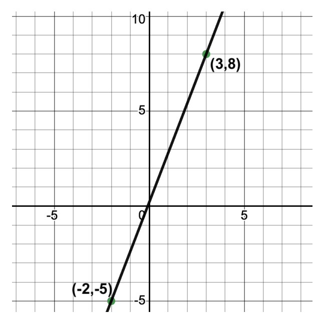
$x + 3y = 3$ 2 B



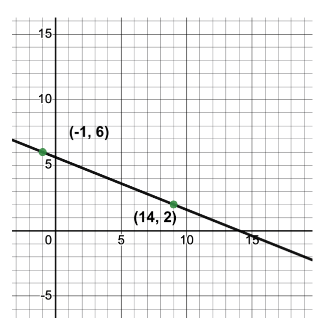
$-13x + 5y = 1$ 3 C



$-9x + 15y = 6$ 4 D



$-6x + 5y = 13$ 5 E

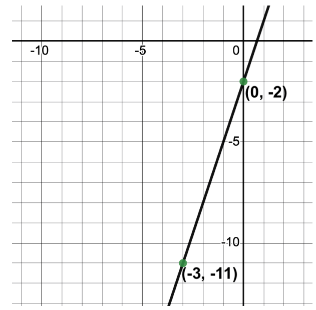
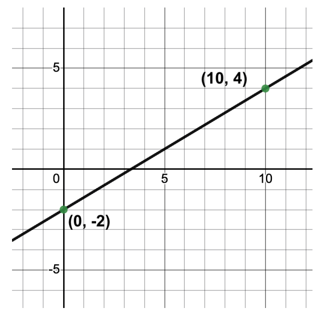
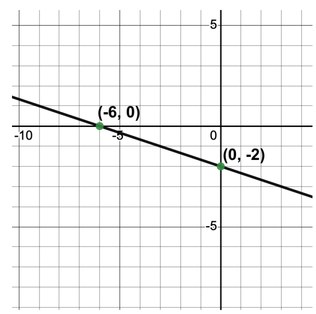
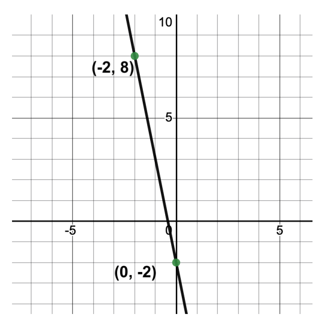
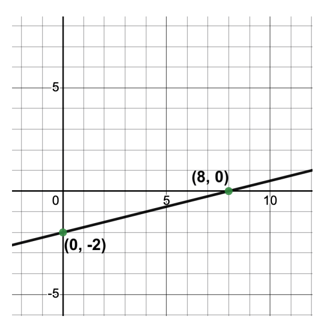


(optional)

Matching Slope-Intercept Form to Graphs

Slope-intercept form: $y = mx + b$ m: *slope* b: *y-intercept*

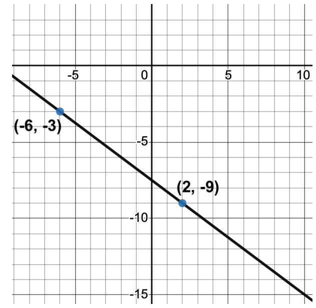
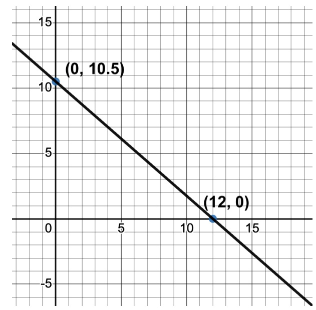
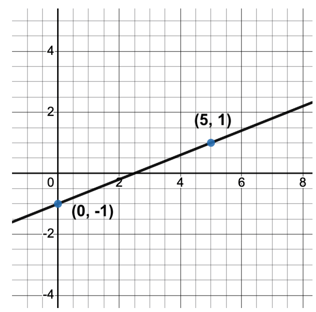
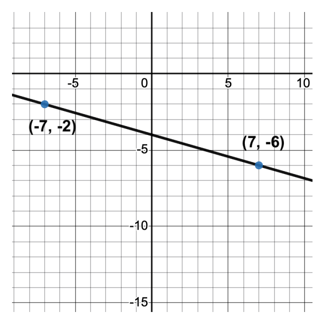
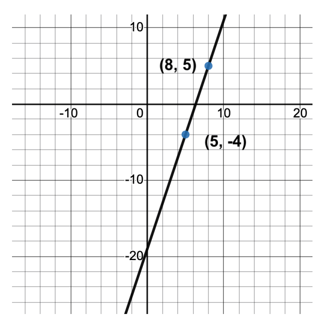
Each of the graphs below represents a line of best fit derived from some data. **Match each definition below to the linear model describes.**

$f(x) = -5x - 2$	1	A	
$g(x) = \frac{1}{4}x - 2$	2	B	
$h(x) = 3x - 2$	3	C	
$j(x) = -\frac{1}{3}x - 2$	4	D	
$k(x) = 0.6x - 2$	5	E	

(optional)

Mixed Practice: Matching Graphs to their Function Definitions

Each of the graphs below represents a line of best fit derived from some data. **Match each equation on the left to its graphical representation on the right.**

$7x + 8y = 84$	1	A	
$y = \frac{2}{5}x - 1$	2	B	
$y - 5 = 3(x - 8)$	3	C	
$f(x) = \frac{-2}{7}x - 4$	4	D	
$y + 9 = -\frac{3}{4}(x - 2)$	5	E	

(optional)

Exploring the Aaron Judge Dataset

*For this page, you'll need to open the [Aaron Judge Starter File] on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. **Read the comments at the top of the file**, which describe what each column in the dataset means.*

1) What is the name of the table being loaded in this file? ______________________

Looking for Possible Relationships

We can look for potential relationships between quantitative columns by building scatter plots. Let's see what they reveal.

2) Make a scatter plot comparing ___pitch speed___ (explanatory variable) and ___hit speed___ (response variable).

- Does there appear to be a relationship between `pitch-speed` and `hit-speed`? If so, describe it. ______________________
- Based on the scatter plot, what hit speed would you predict for a ball pitched at 95mph? __________
  - Why? ______________________
- How confident are you in your prediction? *not at all* 1 2 3 4 5 *very*

3) Make a scatter plot comparing ___pitch speed___ (explanatory variable) and ___hit distance___ (response variable).

- Does there appear to be a relationship between `pitch-speed` and `hit-distance`? If so, describe it. ______________________
- Based on the scatter plot, what hit distance would you predict for a ball pitched at 95mph? __________
  - Why? ______________________
- How confident are you in your prediction? *not at all* 1 2 3 4 5 *very*

4) Make a scatter plot comparing ___bat angle___ (explanatory variable) and ___hit distance___ (response variable).

- Does there appear to be a relationship between `bat-angle` and `hit-distance`? If so, describe it. ______________________
- Based on the scatter plot, what distance would you predict for a ball hit at an angle of 60 degrees? __________
  - Why? ______________________
- How confident are you in your prediction? *not at all* 1 2 3 4 5 *very*

5) Make a scatter plot comparing ___bat angle___ (explanatory variable) and ___hit speed___ (response variable).

- Does there appear to be a relationship between `bat-angle` and `hit-speed`? If so, describe it. ______________________
- Based on the scatter plot, what speed would you predict for a ball hit at an angle of 20 degrees? __________
  - Why? ______________________
- How confident are you in your prediction? *not at all* 1 2 3 4 5 *very*

6) What is happening on lines 21 - 36 of the starter file? ______________________

What Shape of model would Fit the Data?

*For this page, you'll need to open the [Aaron Judge Starter File] on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

Note: For this page we will be focusing on just the curve balls, so make sure you are writing code using `curve-table`

Fitting Linear Models

1) Use `lr-plot(curve-table, "id", "bat-angle", "hit-distance")` to find the optimal **linear** model.

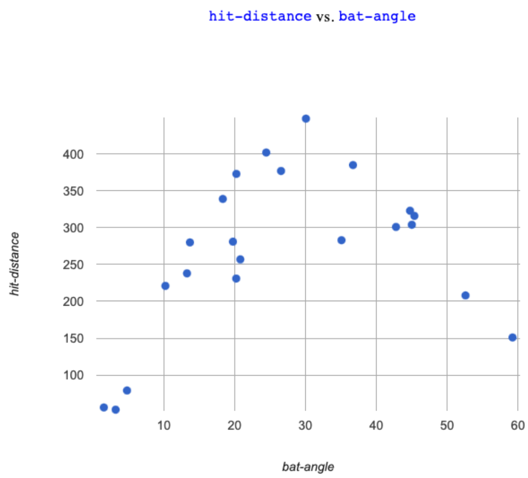
What is S for this model? ______________________________

Write the model below, in both math and Pyret notation.

$y =$ ______________ x + ______________
slope / y-intercept / vertical shift

`fun y(x): (_____ * x) + _____ end`

2) Sketch the linear model from your `lr-plot` below!



3) What do you **Notice?** ______________________________

____

____

4) What do you **Wonder?** ______________________________

____

____

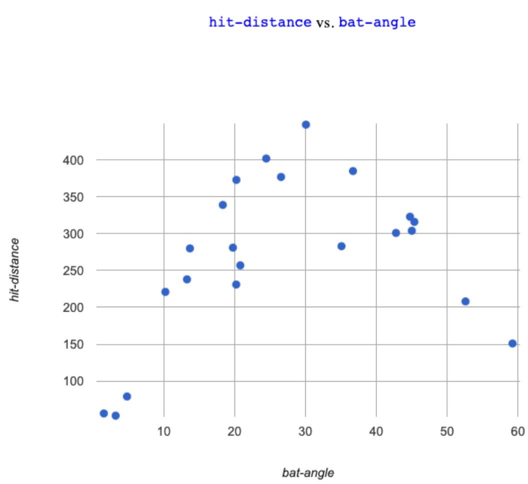
5) Is the best-possible linear model a good fit? Why or why not?

____

____

Fitting Curves

6) Draw a **curve** on the scatter-plot, which shows the overall shape in the data.



7) How would you describe the pattern? ______________________________

____

____

____

8) At what `bat-angle` is the curve's "peak"? ______________

9) Based on your best-guess curve, predict the `hit-distance` for new hits at each `bat-angle` below.

- 15 degrees: __________
- 35 degrees: __________
- 50 degrees: __________

What Kind of Model? (Descriptions)

Decide whether each situation sounds like it would be better modeled by a linear or quadratic function, and circle your answer.

1) A car is 50 miles away, traveling at 65mph. How far away is the car after each hour?

Linear Quadratic

---

2) The data plan for a cell phone bill costs $5/gb, plus $15/mo. How much is the bill for a given month, after x number of gigabytes?

Linear Quadratic

---

3) A cannonball is fired from the deck of the S.S. Parabola, and arcs through the sky before hitting its target, 17 miles away.

Linear Quadratic

---

4) The **area** of a circle, as its radius increases.

Linear Quadratic

---

5) The **circumference** of a circle, as its radius increases.

Linear Quadratic

---

6) A ball is dropped from the top of the Empire State Building, and starts falling faster and faster. **How far has it dropped** after x seconds?

Linear Quadratic

★ A ball is dropped from the top of the Empire State Building, and starts falling faster and faster. **How fast is it moving** after x seconds?

Linear Quadratic

Exploring the Fuel Efficiency Dataset

*For this page, you'll need to open the Fuel Efficiency Starter File on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. **Read the comments at the top of the file**, which describe what each column in the dataset means.*

Fitting Linear Models

1) Evaluate `A15`, `A45` and `A75` in the Interactions Area. What **model** of car is used in all three rows? ____________________

2) At what three **speeds** is this model being tested in these rows? ____________________

3) Does there appear to be a relationship between speed and miles-per-gallon? __________.

4) Looking at the numbers in the `mpg-table`, does it appear to be linear or nonlinear? Describe its shape. ____________________

____

____

5) Use `lr-plot(mpg-table, "model", "speed", "mpg")` to find the optimal **linear** model. What is S for this model? __________
Write the model below, in both math and Pyret notation.

$y =$ ______________ x + ______________
slope / y-intercept / vertical shift

`fun y(x): (_____ * x) + _____ end`

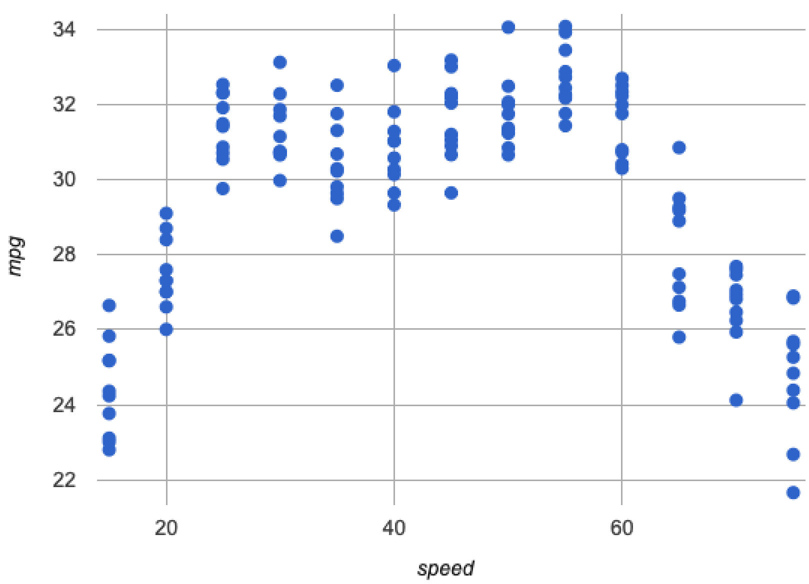
6) Is the best-possible linear model a good fit? __________. Why or why not? ____________________

____

____

Fitting Curves

7) Sketch your lr-plot in the space below, showing the relationship between `speed` and `mpg`. Be sure to label your axes, and draw the linear model!



8) What do you **Notice?** ____________________

____

____

____

9) What do you **Wonder?** ____________________

____

____

____

10) Do you think a **curve** would fit better?

____

11) Draw a **curve** on your scatter-plot, which shows the overall shape in the data. At what speed is the "peak"? ____________________

12) Based on your best-guess curve, what do you predict `mpg` would be for a new test run at **25mph** ______? **60mph** ______? **75mph** ______?

(optional)

What Kind of Model? (Tables)

Decide whether each representation is best described by a linear model, a quadratic model, some *other* model, or no model at all! Record how you decided by showing any work that you feel is useful or writing an explanation.

For Class Discussion:

1

x	0	1	2	3	4	5	6
y	5	6	9	14	21	30	41

Linear
Quadratic
Some other model/pattern
No model

2

x	0	1	2	3	4	5	6
y	0	3	6	9	12	15	18

Linear
Quadratic
Some other model/pattern
No model

For Independent Practice:

3

x	1	2	3	4	5	6	7
y	1	3	5	7	9	11	13

Linear
Quadratic
Some other model/pattern
No model

4

x	-3	-2	-1	0	1	2	3
y	-23	-38	-47	-50	-47	-38	-23

Linear
Quadratic
Some other model/pattern
No model

5

x	-3	-2	-1	0	1	2	3
y	1	2	1	2	1	1	1

Linear
Quadratic
Some other model/pattern
No model

6

x	1	2	3	4	5	6	7
y	2	5	10	17	26	37	50

Linear
Quadratic
Some other model/pattern
No model

7

x	1	2	3	4	5	6
y	10	100	1000	10000	100000	1000000

Linear
Quadratic
Some other model/pattern
No model

8

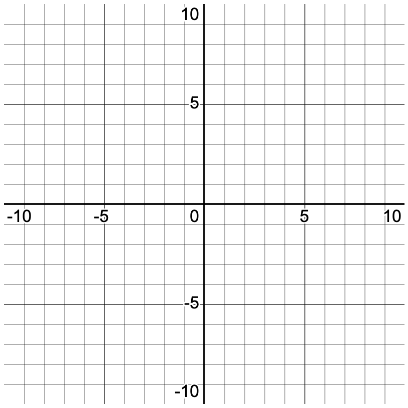
x	1	2	3	4	5	6	7
y	100	102	105	109	114	120	127

Linear
Quadratic
Some other model/pattern
No model

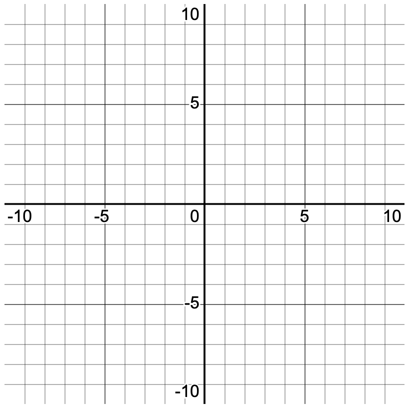
Parabolas

1) Sketch a ***parabola*** on each of the grids below that matches the description.

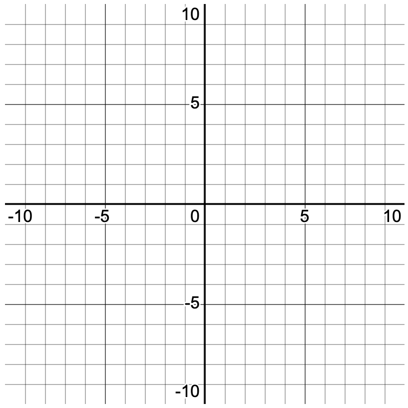
one x-intercept, opens down



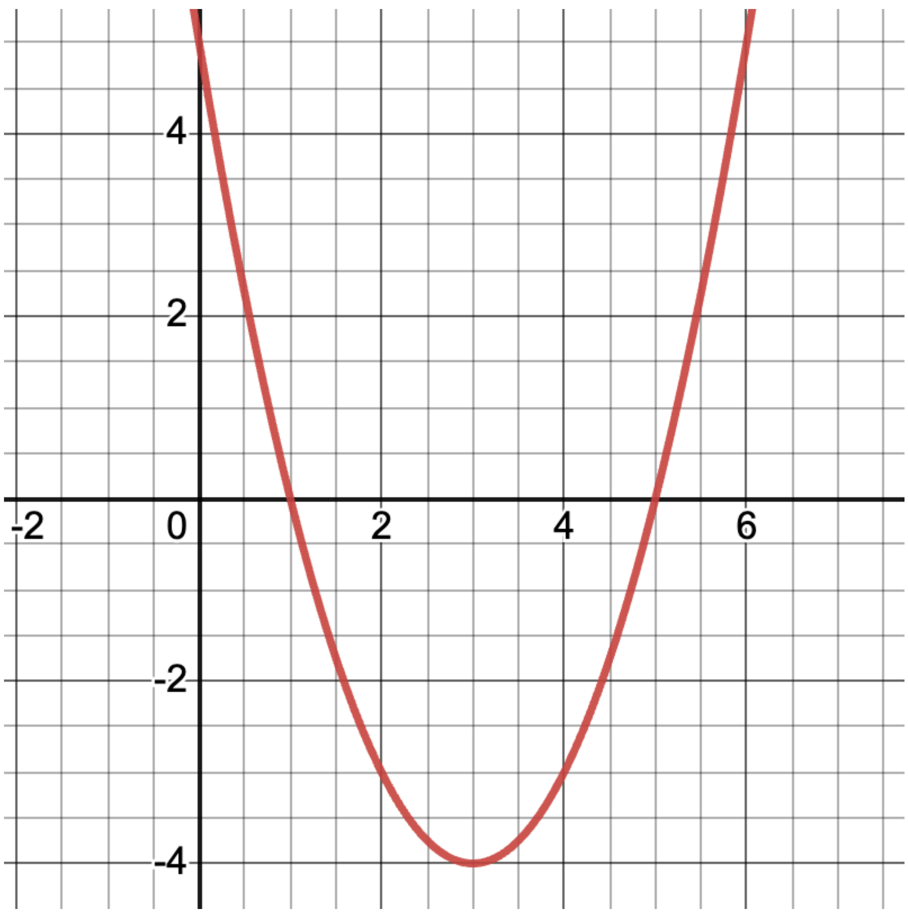
two x-intercepts, opens up



no x-intercepts



2) Label the ***vertex***, ***root(s)***, and ***y-intercept*** of the parabola below with: A) the coordinates B) the vocabulary word that describes it



3) Draw a dotted line representing the ***axis of symmetry*** and label it with the equation that defines it.

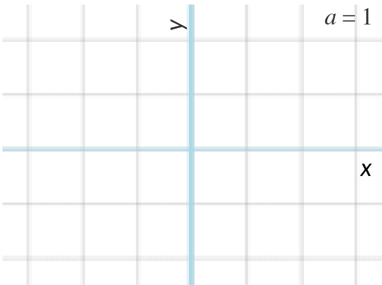
Graphing Quadratic Models

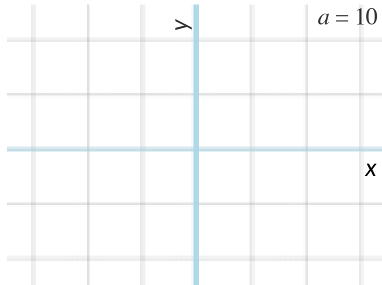
*For this page, you'll need to have **Exploring Quadratic Functions (Desmos)** open to **Slide 1: Transforming Parabolas**.*
The parabola you'll see is the graph of $g(x) = x^2$. Another, **identical** parabola is hiding behind it.
This second parabola is written in Vertex Form: $f(x) = a(x - h)^2 + k$. Each model setting starts at the value that makes $f(x)$ equivalent to $g(x)$.

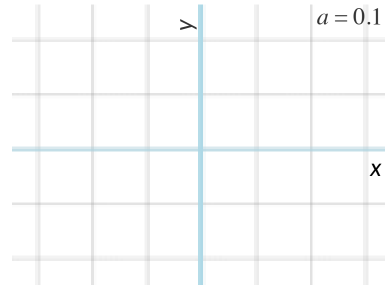
1) Use the starting values of a , h , and k you see in Desmos, to complete this equation: $g(x) = x^2 = f(x) =$ _______ $(x -$ _______ $)^2 +$ _______

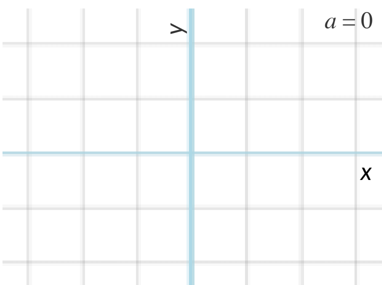
Magnitude a

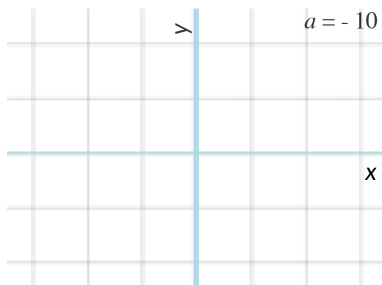
2) In the first square below, make a sketch of the original graph you see ($a = 1$, $h = 0$, $k = 0$).
Then try changing the value of a to 10, 0.1, 0, -10 and -2, graphing each parabola in the squares below. **Label the vertex "V" and any roots with "R"!**

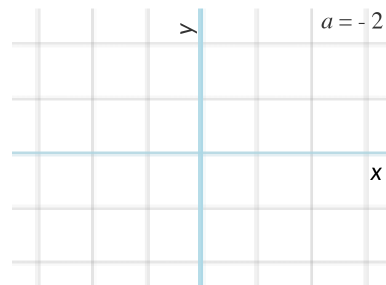












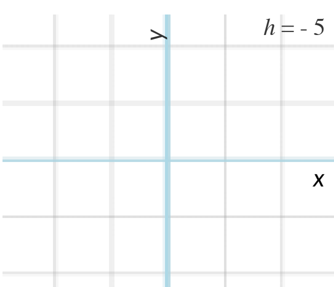
3) What does a tell us about a parabola? ____

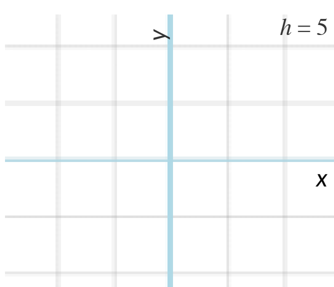
____

Translation

Horizontal Translation h

Set a back to 1. Change the value of h to -5 and 5, graphing each parabola in the squares below.



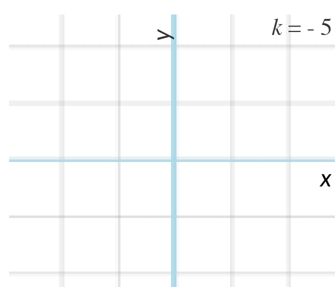


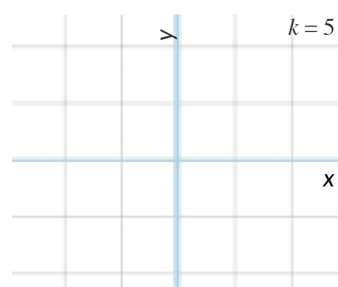
Label the vertex "V" and any roots "R"!

4 What does h tell us about a parabola? ____________________

Vertical Translation k

Set h back to 0. Change the value of k to -5 and 5, graphing each parabola in the squares below.



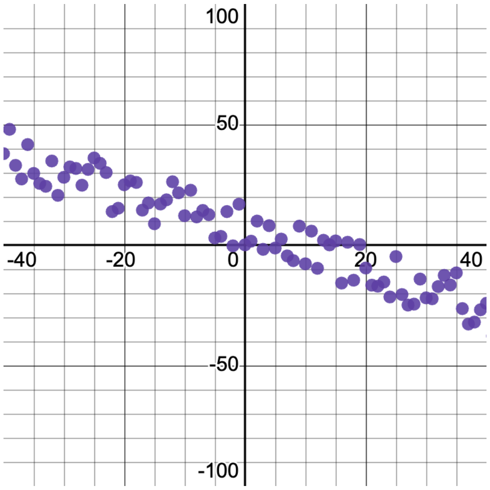


Label the vertex "V" and any roots "R"!

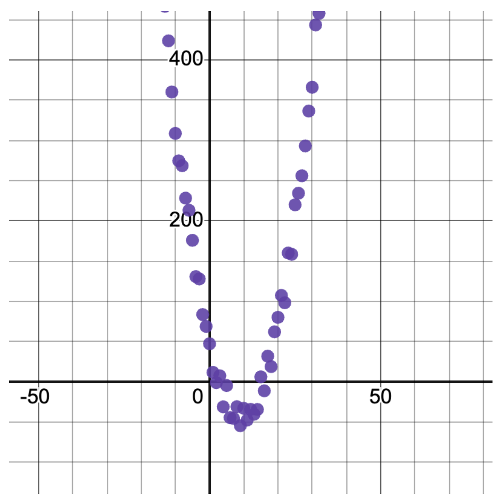
5 What does k tell us about a parabola? ____________________

Classifying Scatter Plots

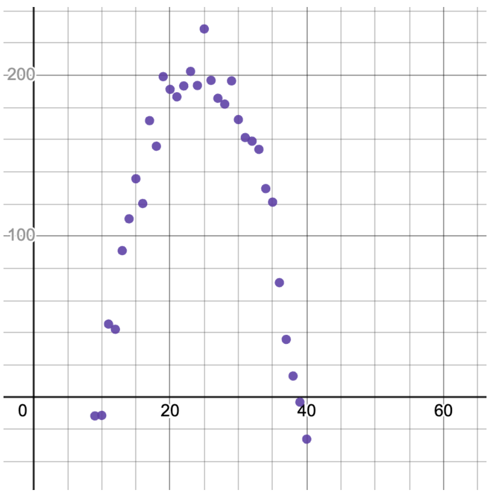
Decide whether each scatter plot would be best modeled by a linear function, a quadratic function or neither!



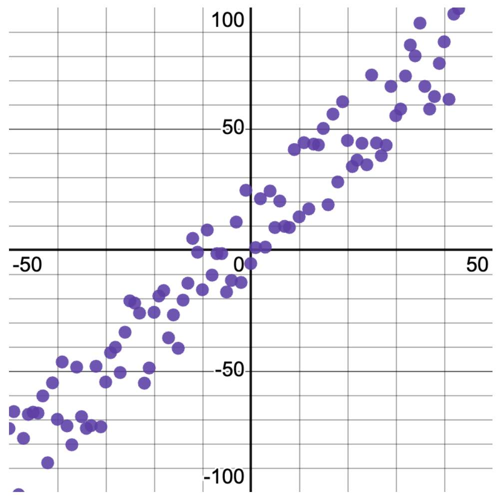
1 Linear Quadratic Neither



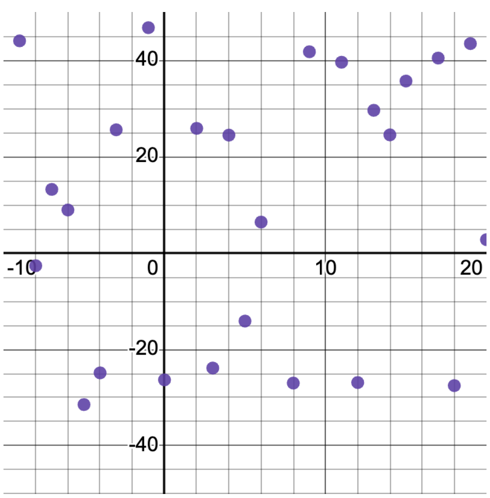
2 Linear Quadratic Neither



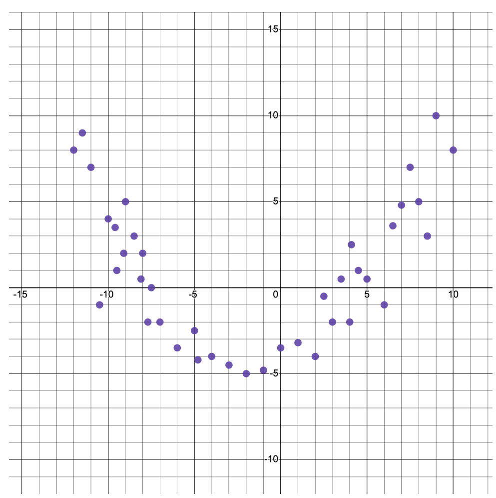
3 Linear Quadratic Neither



4 Linear Quadratic Neither



5 Linear Quadratic Neither



6 Linear Quadratic Neither

(optional)

What Kind of Model? (Descriptions)

Decide whether each situation is best described by a linear or quadratic function.

MTV Royalties

MTV paid a songwriter $1000 to use the recording on their show. After that, the songwriter gets a check for $26 in performance royalties for their songwriting every time the episode airs.

1) Do royalties increase or decrease over time? ___________________

2) When the songwriter agrees to let MTV use the recording on their show (x=0), how much money do they earn? _______________

3) How many dollars will they have after…

(first airing) x = 1	(second airing) x = 2	x = 3	x = 4

4) What is the **form** of this function? ☐ linear ☐quadratic

Tuition Savings Account

A family managed to save $18,000 by the time their child started studying to be an electrician at the community college, where tuition and fees cost $2200 per semester.

5) When the child starts college (x=0), how many dollars are in the account? _______________

6) Will the money in the account increase or decrease while they're in college? ___________________

7) How many dollars will be in the account after…

(paying for semester 1) x=1	x= 2	x = 3	x = 4

8) What is the **form** of this function? ☐ linear ☐quadratic

Stopping Time

In a certain car, from the time a driver steps on the breaks, the vehicle decelerates at a rate of 10 meters per second. We can calculate the time it takes to stop the car, by squaring the speed it's driving in meters per second and dividing by 20.

9) How long does it take to stop the car when it's driving at 0 km/h (x=0)? _______________

10) Does stop time increase or decrease as speed increases? ___________________

11) What is stopping time when it's driving at a speed of…

First, we'll need to convert driving times from kilometers/hour to meters/second.

$$\frac{10\text{ kilometers}}{1hour} \times \frac{1\text{ hour}}{3600\text{ seconds}} \times \frac{1000\text{ meters}}{1\text{ kilometer}} = \frac{10\cancel{\text{ kilometers}} \times 1\cancel{\text{ hour}} \times 1000\text{ meters}}{1\cancel{\text{ hour}} \times 3600\text{ seconds} \times 1\cancel{\text{ kilometer}}} = \frac{10000}{3600}\text{meters per second} = 2.778\text{m/s}$$

(10 km/h = 2.778 m/s) x=10	(20 km/h = 5.555 m/s) x=20	x=30	x =40

12) What is the **form** of this function? ☐linear ☐ quadratic

(optional)

Modeling Bat Angle v. Hit Distance (Curveballs)

Vertex form of a Quadratic Model

Vertex Form: $f(x) = a(x-h)^2 + k$	• a : determines whether the parabola opens up or down and how steep the curve is • h : ***horizontal shift*** (also the x-coordinate of the vertex! h is often 0) • k : ***vertical shift*** (also the y-coordinate of the vertex!)

1) We've determined that the optimum bat angle is around 30 degrees. What variable in the equation should we replace with 30? __________

2) What y-coordinate of the vertex (*vertical shift*) would best match the shape of the curve? ______________________________

3) Does it make sense for a to be negative or positive for this curve? ______________________________

Fitting the Model Visually in Desmos

*For this section, you'll need to have **Exploring Quadratic Functions(Desmos)** open to **Slide 3: Fitting a Model: Bat Angle v Hit Distance (Curveballs)**.*

4) Using your thinking about the values of a , h , and k from above, adjust the sliders to fit a quadratic model to the data. Continue adjusting the sliders until you've landed on the best model you can. Record your values for a , h , and k below.

a : ________________ h : ________________ k : ________________

5) Using the values of a , h , and k you decided on in the Desmos file, define your quadratic function below in Pyret notation.

```
fun curve(x): (_______________ * sqr(x - ____)) + _______________ end
```

Testing how Good the Model is in Pyret

Return to your copy of the Aaron Judge Starter File, adjust the definition for `curve(x)` on line 42, and click "Run".

6) Use `fit-model` to fit `curve(x)` to the `curve-table` data. What S -value did you get? ______________________________

Hint: If you forgot the contract for `fit-model`, look it up in the Contracts Pages!

7) The S -value for the optimal linear model was about 104 feet.

My quadratic model should do ______________________________ at predicting hit distances from bat angles.
a little better, much better, worse, much worse, an equally good job

What does this model actually mean?

After experimenting, I came up with a quadratic model for this dataset showing that ____________ is correlated to ________________.
x-variable y-variable

The vertex of the parabola drawn by this model is a ________________ at about ____________, which means that
minima or maxima? (x, y)

____

Before this point, as bat angle increases, hit distance ______________. After this point, as the angle increases hit distance ______________

The error in the model is described by an ***S-value*** of about ________________________. I ______________________________ that
S units strongly agree, agree, disagree, strongly disagree

this model is a good fit considering that ________________ in this dataset range from ________________ to ________________.
y-variable lowest y-value highest y-value

Modeling Bat Angle v. Hit Distance (Two-seam Fastballs)

Vertex form of a Quadratic Model: $f(x) = a(x-h)^2 + k$

1) Does it make sense for a to be negative or positive for this curve? ______________________

Why ______________________

2) Estimate h (the x-coordinate of the vertex of this dataset): ______________________

3) What y-coordinate of the vertex (k) would best match the shape of the curve? ______________________

Fitting the Model Visually in Desmos

*For this section, open **Exploring Quadratic Functions(Desmos)** to **Slide 4: Fitting a Model: Bat Angle v Hit Distance (2-seam fast balls)**.*

4) Using what you know about the values of a , h , and k , adjust the sliders to fit a quadratic model to the data. Continue adjusting the sliders until you've landed on the best model you can. Record your values for a , h , and k below.

a : ______________ h : ______________ k : ______________

5) Using the values of a , h , and k you decided on in the Desmos file, define your quadratic function below in Pyret notation.

`fun fast2seam(x) :` ______________________________________ `end`

Testing how Good the Model is in Pyret

Open your copy of the Aaron Judge Starter File. Adjust the definition for `fast2seam(x)` on line 45 and click "Run".

6) Use `fit-model` to fit your function to the data in the `fast2seam-table`. What S -value did you get? ______________

What does this model actually mean?

After experimenting, I came up with a quadratic model for this dataset showing that ______________ (x-variable) is correlated to ______________ (y-variable).

The error in the model is described by an ***S-value*** of about ______ (S) ______ (units). I ______________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ______________ (y-variable) in this dataset range from ______________ (lowest y-value) to ______________ (highest y-value).

The vertex of the parabola drawn by this model is a ______________ (minima or maxima?) at about ______ (x, y), which means that

____

Before this point, as bat angle increases, hit distance ______________. After this point, as the angle increases hit distance ______________

(optional)

Modeling Bat Angle v. Hit Distance (Sliders)

Vertex form of a Quadratic Model: $f(x) = a(x-h)^2 + k$

1) Does it make sense for a to be negative or positive for this curve? ______________________

Why ______________________

2) Estimate h (the x-coordinate of the vertex of this dataset): ______________________

3) What y-coordinate of the vertex (k) would best match the shape of the curve? ______________________

Fitting the Model Visually in Desmos

For this section, open ***Exploring Quadratic Functions(Desmos)*** *to* ***Slide 5: Fitting a Model: Bat Angle v Hit Distance (Sliders)****.*

4) Using what you know about the values of a , h , and k , adjust the sliders to fit a quadratic model to the data. Continue adjusting the sliders until you've landed on the best model you can. Record your values for a , h , and k below.

a : ______________ h : ______________ k : ______________

5) Using the values of a , h , and k you decided on in the Desmos file, define your quadratic function below in Pyret notation.

`fun sliders(x) :` ______________________________ `end`

Testing how Good the Model is in Pyret

Open your copy of the Aaron Judge Starter File. Adjust the definition for `sliders(x)` *on line 48 and click "Run".*

6) Use `fit-model` to fit your function to the data in the `sliders-table`. What S -value did you get? ______________

What does this model actually mean?

After experimenting, I came up with a quadratic model for this dataset showing that ______________ (x-variable) is correlated to ______________ (y-variable).

The error in the model is described by an ***S-value*** of about ______ (S) ______ (units). I ______________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ______________ (y-variable) in this dataset range from ______________ (lowest y-value) to ______________ (highest y-value).

The vertex of the parabola drawn by this model is a ______________ (minima or maxima?) at about ______ (x, y), which means that

____

Before this point, as bat angle increases, hit distance ______________. After this point, as the angle increases hit distance ______________

(optional)

Modeling Bat Angle v. Hit Distance (Four-seam Fastballs)

Vertex form of a Quadratic Model: $f(x) = a(x-h)^2 + k$

1) Does it make sense for a to be negative or positive for this curve? ______________________

Why ______________________

2) Estimate h (the x-coordinate of the vertex of this dataset): ______________________

3) What y-coordinate of the vertex (k) would best match the shape of the curve? ______________________

Fitting the Model Visually in Desmos

*For this section, open **Exploring Quadratic Functions(Desmos)** to **Slide 6: Fitting a Model: Bat Angle v Hit Distance (4-seam fast balls)**.*

4) Using what you know about the values of a , h , and k , adjust the sliders to fit a quadratic model to the data. Continue adjusting the sliders until you've landed on the best model you can. Record your values for a , h , and k below.

a : ____________ h : ____________ k : ____________

5) Using the values of a , h , and k you decided on in the Desmos file, define your quadratic function below in Pyret notation.

`fun fast4seam(x) :` ______________________________________ `end`

Testing how Good the Model is in Pyret

Open your copy of the [Aaron Judge Starter File](). Adjust the definition for `fast4seam(x)` on line 51 and click "Run".

6) Use `fit-model` to fit your function to the data in the `fast4seam-table`. What S -value did you get? ____________

What does this model actually mean?

After experimenting, I came up with a quadratic model for this dataset showing that ____________ (x-variable) is correlated to ____________ (y-variable).

The error in the model is described by an ***S-value*** of about ____________ (S) ____________ (units). I ______________________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ____________ (y-variable) in this dataset range from ____________ (lowest y-value) to ____________ (highest y-value).

The vertex of the parabola drawn by this model is a ____________ (minima or maxima?) at about ____________ ((x, y)), which means that

____

Before this point, as bat angle increases, hit distance ____________. After this point, as the angle increases hit distance ____________

(optional)

Modeling Bat Angle v. Hit Distance (Full Data)

Vertex form of a Quadratic Model: $f(x) = a(x - h)^2 + k$

1) Does it make sense for a to be negative or positive for this curve? ______________________

Why ______________________

2) Estimate h (the x-coordinate of the vertex of this dataset): ______________________

3) What y-coordinate of the vertex (k) would best match the shape of the curve? ______________________

Fitting the Model Visually in Desmos

*For this section, open **Exploring Quadratic Functions(Desmos)** to **Slide 8: Fitting a Model: Bat Angle v Hit Distance (Full Data)**.*

4) Using what you know about the values of a , h , and k , adjust the sliders to fit a quadratic model to the data. Continue adjusting the sliders until you've landed on the best model you can. Record your values for a , h , and k below.

a : ______________ h : ______________ k : ______________

5) Using the values of a , h , and k you decided on in the Desmos file, define your quadratic function below in Pyret notation.

`fun judge(x) :` ______________________________________ **`end`**

6) Which subset of the data appears to be visually the most similar to the full dataset? ______________________

7) What does that mean ______________________

Testing how Good the Model is in Pyret

Open your copy of the [Aaron Judge Starter File](). Adjust the definition for `judge(x)` on line 54 and click "Run".

8) Use `fit-model` to fit your function to the data in the `judge-table`. What S -value did you get? ______________________

What does this model actually mean?

After experimenting, I came up with a quadratic model for this dataset showing that ______________ (x-variable) is correlated to ______________ (y-variable).

The error in the model is described by an ***S-value*** of about ______________ (S units). I ______________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ______________ (y-variable) in this dataset range from ______________ (lowest y-value) to ______________ (highest y-value).

The vertex of the parabola drawn by this model is a ______________ (minima or maxima?) at about ______________ ((x, y)), which means that

____

Before this point, as bat angle increases, hit distance ______________. After this point, as the angle increases hit distance ______________

(optional)

Modeling Fuel Efficiency v. Speed

Open your copy of the Fuel Efficiency Starter File and click "Run".

sqr

Before we try to model our fuel-efficiency data, we need to learn a new Pyret function!

1) Can you predict what the output of the `sqr` expressions below will be?

Test them out in the Interactions Area, and record the results. `sqr(4)` ______________ `sqr(6 - 2)` ______________

2) What is the Contract for `sqr`? ____

3) What does `sqr` do? ____

Interpreting a Quadratic Model

In the Definitions Area of your Fuel Efficiency Starter File, you'll find the definition of a quadratic model `quad1`.

4) In `quad1`, the value of a is ________________, the value of h is ________________, and the value of k is ________________

5) Fit this model to your dataset, using `fit-model`. What S-value did you get? ______________________________

Hint: If you forgot the contract for `fit-model`*, look it up in the Contracts Pages!*

6) In your own words, describe what should change about this model to fit the data. ______________________________

____

Modeling Fuel Efficiency

Vertex Form:	$f(x) = a(x - h)^2 + k$	• a : determines whether the parabola opens up or down and how steep the curve is • h : horizontal shift (also the x-coordinate of the vertex! h is often 0) • k : vertical shift (also the y-coordinate of the vertex!)

7) We've determined that peak fuel efficiency is around 45 mph. What variable in the equation should we replace with 45? ____________

Update the definition of `quad1`, click "Run" and re-fit the model. What S-value did you get? ______________________

8) What y-coordinate of the vertex (*vertical shift*) would best match the shape of the curve? __________

Update the definition of `quad1`, click "Run" and re-fit the model. What S-value did you get? ______________________

9) What value of a best matches the shape of the curve? __________

Update the definition of `quad1`, click "Run" and re-fit the model. What S-value did you get? ______________________

10) Make any small changes you'd like, trying to get S as low as you can. Write your final definition below.

fun `f(x) :` ____ **end** S: ______________________

What does this model actually mean?

After experimenting, I came up with a quadratic model for this dataset showing that __________ (x-variable) is correlated to ____________________ (y-variable). The error in the model is described by an ***S-value*** of about ________ (S) ________ (units). I ______________________________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ____________________ (y-variable) in this dataset range from __________ (lowest y-value) to __________ (highest y-value). The vertex of the parabola drawn by this model is a ______________ (minima or maxima?) at about ________ (x, y) which means that ______________________________________

Before this point, as speed increases, `mpg` ________________. *After* this point, as speed increases `mpg` ________________

(optional)

What can we learn from each form of a Quadratic Function?

Draw arrows from each form to the information which you can easily learn from it.

roots (x-intercepts)

Factored Form:

$y = -2(x-5)(x+6)$

coordinates of vertex

Standard Form:

$y = 3x^2 + 5x + 6$

the parabola opens up / down

Vertex Form:

$y = 2(x-5)^2 + 17$

y-intercept

axis of symmetry

What Kind of Model? (Definitions)

Circle whether each representation below describes a **linear** or **quadratic** function, or neither.
If the function is quadratic, fill in all the information that you can *easily read from the form*, without any calculation!

1) $f(x) = 3x^2 + 22$

Linear　　Quadratic　　Neither

If it's Quadratic ...

Form: Factored, Vertex, Standard　　Parabola opens: up/down

axis of symmetry (x=...)　　Vertex (x, y)]

Roots (write both coords)

2) $g(x) = 2(x - 11)(x - 243)$

Linear　　Quadratic　　Neither

If it's Quadratic ...

Form: Factored, Vertex, Standard　　Parabola opens: up/down

axis of symmetry (x=...)　　Vertex (x, y)]

Roots (write both coords)

3) $h(y) = 100 - 4y$

Linear　　Quadratic　　Neither

If it's Quadratic ...

Form: Factored, Vertex, Standard　　Parabola opens: up/down

axis of symmetry (x=...)　　Vertex (x, y)]

Roots (write both coords)

4) $z(x) = \frac{3}{5}x + 7$

Linear　　Quadratic　　Neither

If it's Quadratic ...

Form: Factored, Vertex, Standard　　Parabola opens: up/down

axis of symmetry (x=...)　　Vertex (x, y)]

Roots (write both coords)

5)
```
fun graph(x): 12 * x end
```

Linear　　Quadratic　　Neither

If it's Quadratic ...

Form: Factored, Vertex, Standard　　Parabola opens: up/down

axis of symmetry (x=...)　　Vertex (x, y)]

Roots (write both coords)

6)
```
fun m(p): 2 * (p - 5) * (p - 16) end
```

Linear　　Quadratic　　Neither

If it's Quadratic ...

Form: Factored, Vertex, Standard　　Parabola opens: up/down

axis of symmetry (x=...)　　Vertex (x, y)]

Roots (write both coords)

7) $r(s) = 42(s - 10)^2 - 3$

Linear　　Quadratic　　Neither

If it's Quadratic ...

Form: Factored, Vertex, Standard　　Parabola opens: up/down

axis of symmetry (x=...)　　Vertex (x, y)]

Roots (write both coords)

8)
```
fun f(x): (2 * sqr(x - 1)) + 15 end
```

Linear　　Quadratic　　Neither

If it's Quadratic ...

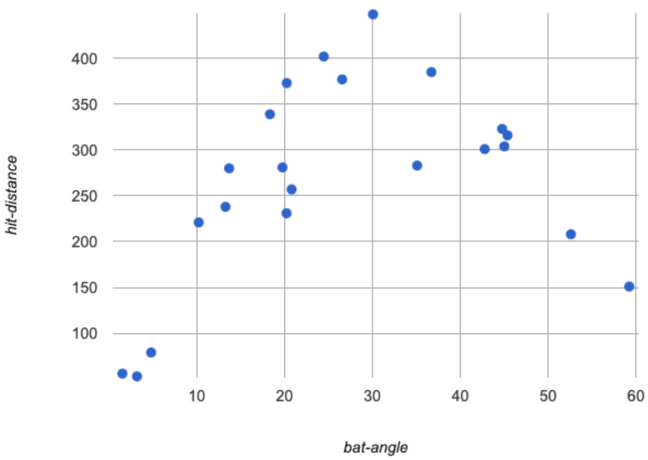
Form: Factored, Vertex, Standard　　Parabola opens: up/down

axis of symmetry (x=...)　　Vertex (x, y)]

Roots (write both coords)

Build a Model from Samples - Batting Data

We're going to compute a quadratic function from 3 representative points in the `curve-table` *we've been working with. While our model would be most accurate if we kept track of all of the decimal values, for today we are going to simplify our calculations by doing some rounding.*



1) Identify three **whole-number** coordinate pairs we can use to summarize a curve that would model the point cloud:

- a best guess for where the curve **crosses the y-axis** ____________________
- a point **from the middle** of the curve ____________________
- a point **from the right side** of the curve ____________________

2) Fill in the x and y-values in the **standard form** models below using the three points you found:

__________ = a(__________)^2 + b(__________) + c
y (mpg) x (speed) x (speed)

__________ = a(__________)^2 + b(__________) + c
y (mpg) x (speed) x (speed)

__________ = a(__________)^2 + b(__________) + c
y (mpg) x (speed) x (speed)

3) In the space below - or on another sheet of paper - solve this series of equations for a , b , and c .
As we solve the series, we'll likely get some very long decimal values. You can round these values to the thousandths place as we work.

4) Write your finished model in function and Pyret notation below.

Function Notation	Pyret Notation
$f(x) = \underset{a}{\_\_\_\_\_\_} x^2 + \underset{b}{\_\_\_\_\_\_} x + \underset{c}{\_\_\_\_\_\_}$	`fun f(x): ((________ * sqr(x)) + (________ * x)) + ________ end`

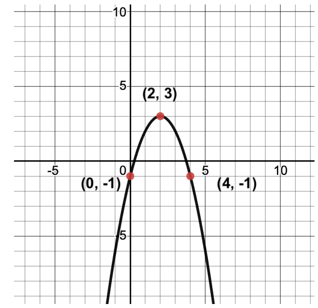
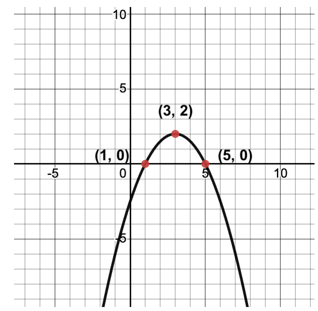
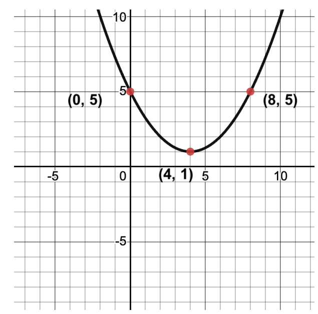
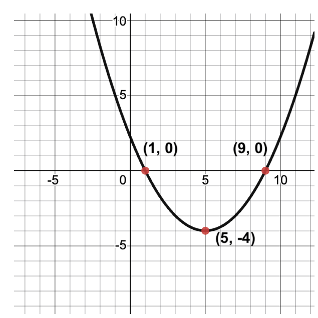
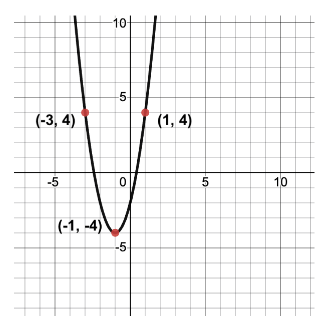
5) Update the function definition for `f` on line 57 of the [Aaron Judge Starter File] and test it out using `fit-model`!

Matching Vertex Form to Graphs

Vertex Form: $y = a(x - h)^2 + k$

- a : determines whether the parabola opens up or down and how steep the curve is
- h : x-coordinate of the vertex
- k : y-coordinate of the vertex

Match each definition below to the graph it describes.

$f(x) = -0.5(x-3)^2 + 2$	**1**	**A**	
$g(x) = 2(x+1)^2 - 4$	**2**	**B**	
$h(x) = -(x-2)^2 + 3$	**3**	**C**	
$j(x) = 0.25(x-5)^2 - 4$	**4**	**D**	
$k(x) = \frac{1}{4}(x-4)^2 + 1$	**5**	**E**	

(optional)

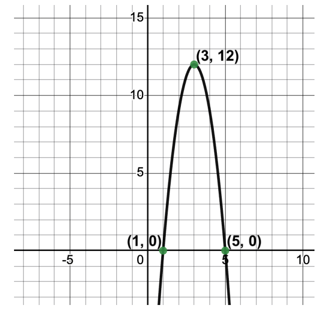
Matching Factored Form to Graphs

Factored Form: $y = a(x - r_1)(x - r_2)$

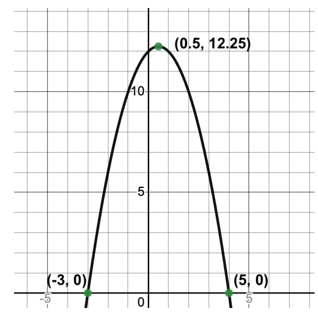
- a : determines whether the parabola opens up or down and how steep the curve is
- r_1 and r_2 : roots, x-intercepts

Match each definition below to the graph it describes.

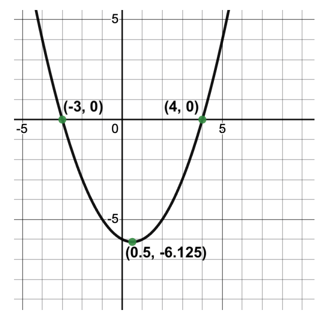
$y = 2(x - 1)(x + 5)$ **1** **A**



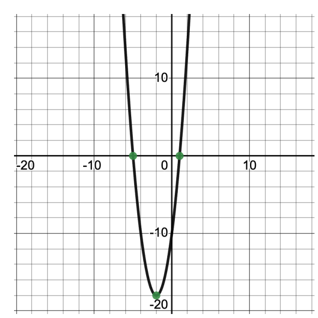
$y = (x + 3)(x + 4)$ **2** **B**



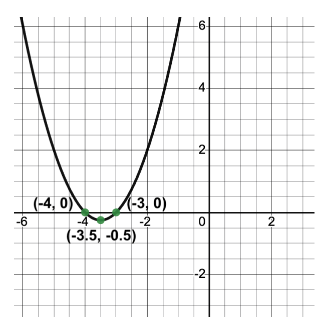
$y = -3(x - 1)(x - 5)$ **3** **C**



$y = \frac{1}{2}(x + 3)(x - 4)$ **4** **D**



$y = -(x - 5)(x + 3)$ **5** **E**



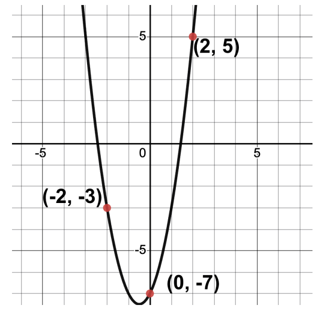
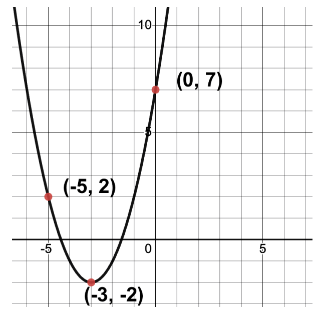
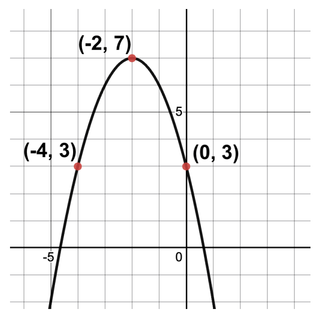
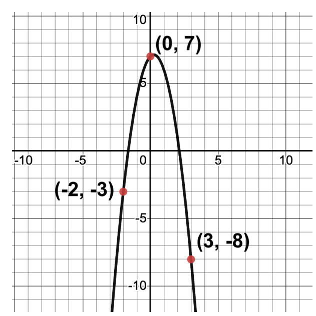
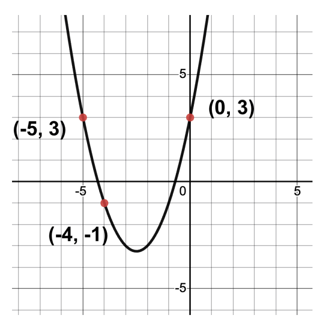
(optional)

Matching Standard Form to Parabolas

Standard Form: $y = ax^2 + bx + c$

- a : determines whether the parabola opens up or down and how steep the curve is
- c : y-intercept

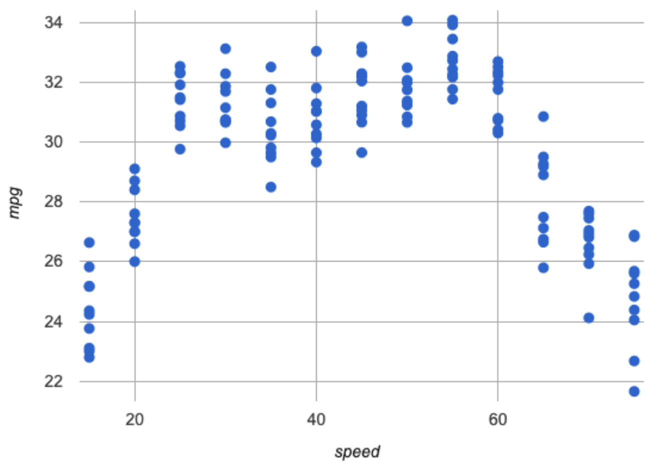
Match each definition below to the graph it describes.

$y = -x^2 - 4x + 3$	**1**	**A**	
$y = 2x^2 + 2x - 7$	**2**	**B**	
$y = x^2 + 5x + 3$	**3**	**C**	
$y = x^2 + 6x + 7$	**4**	**D**	
$y = -2x^2 + x + 7$	**5**	**E**	

(optional)

Build a Model from Samples - Fuel Efficiency

We're going to compute a quadratic function from 3 representative points in the `mpg-table` *we've been working with.*



1) Identify three **whole-number** coordinate pairs we can use to summarize a curve that would model the point cloud:

- a best guess for where the curve **crosses the y-axis** ______________________________
- a point **from the middle** of the curve ______________
- a point **from the right side** of the curve ______________

2) Fill in the x and y-values in the **standard form** models below using the three points you found:

______ = a(______)^2 + b(______) + c
y (mpg)   x (speed)   x (speed)

______ = a(______)^2 + b(______) + c
y (mpg)   x (speed)   x (speed)

______ = a(______)^2 + b(______) + c
y (mpg)   x (speed)   x (speed)

3) In the space below - or on another sheet of paper - solve this series of equations for a, b, and c.

4) Write your finished model in function and Pyret notation below, then define the function `f` in Pyret and try it out using `fit-model`!

Function Notation	Pyret Notation
$quad2(x) =$ ______ $x^2 +$ ______ $x +$ ______ (a, b, c)	`fun quad2(x): ((______ * sqr(x)) + (______ * x)) + ______ end`

5) Open the Fuel Efficiency Starter File. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. Add your function definition for `quad2` on line 30 and test it out using `fit-model`.

(optional)

Exploring the Covid Dataset

For this page, you'll need to have the [Covid Spread Starter File]() open on your computer. If you haven't already, select ***Save a Copy*** *from the "File" menu to make a copy of the file that's just for you.*

1) Take a look at the Definitions Area and find the "Notes on Columns". What is the start date for the data in this table? ______________

2) Click "Run", and evaluate `covid-table` in the Interactions Area. What do you notice? ______________

____

3) Evaluate `MA1` in the Interactions Area. What does it return? ______________

4) Evaluate `CT1`. What information do you learn? ______________

5) Evaluate `NH1`. Why is it "unbound" and how could we make it work? ______________

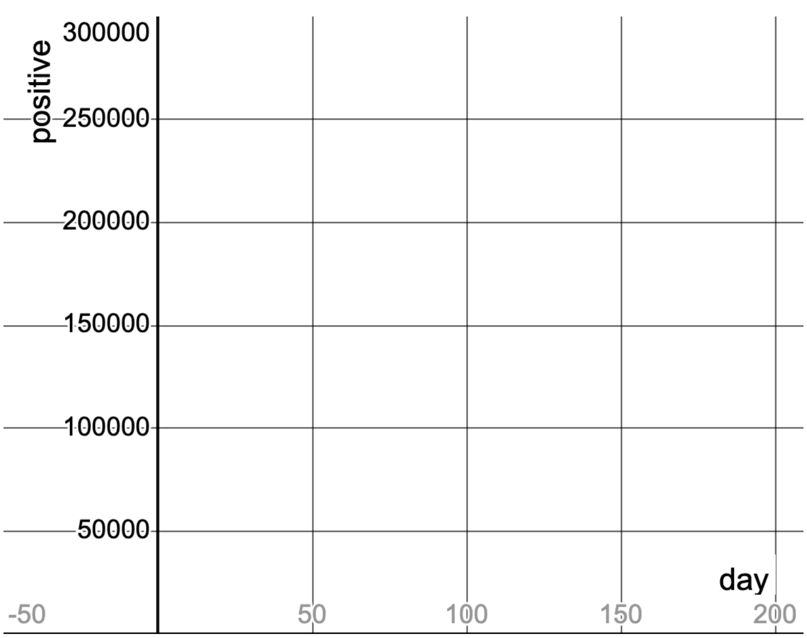
6) Define three new Rows called `NH1`, `RI1` and `VT1`, for New Hampshire, Rhode Island and Vermont. Click "Run" and test them out.

a. How many people in **Vermont** had tested positive by June 10th, 2020? ______________

b. How many people in **New Hampshire** tested positive by June 10th, 2020? ______________

c. How many people in **Rhode Island** tested positive by June 10th, 2020? ______________

7) In Pyret, make a scatter plot from `covid-table` showing the relationship between `day` and `positive`, and using `state` as your labels. Sketch the resulting scatter plot below.



8) In which state did the number of cases grow *fastest*?

____

9) In which state did the number of cases grow *slowest*?

____

10) Are these strong or weak relationship(s)?

____

11) What do you **Notice?** ______________

____

____

12) What do you **Wonder?** ______________

____

____

Linear Models for Covid in Massachusetts

*For this page, you'll need to have the [Covid Spread Starter File] open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

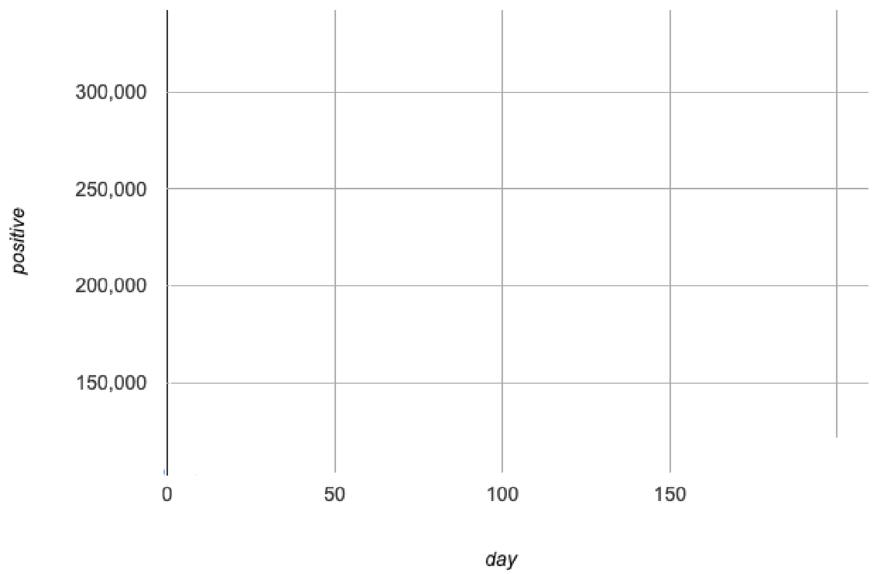
1) Evaluate `covid-table` and `MA-table` in the Interactions Area. How are these tables different? ______________________________

____

2) Discuss lines 27-34 with your partner. What do you think is going on here? ______________________________

____

3) Make a scatter plot from the `MA-table`, using `state` as the labels, and `day` and `positive` as the x- and y-axis. Sketch the plot below.



4) Use `lr-plot` to obtain the *best-possible linear model* for the MA Covid data, **graph it** on the plot above, and **define it** below:

y = ______________________________ S = ______________________________

5) The optimized linear model for this dataset predicts an ____________ (increase / decrease) of about ________ (slope) ____________ (y-variable) per ____________ (x-variable).

The error in the model is described by an ***S-value*** of about ________ (S) ________ (units), which is a ________ (poor, ok, good) fit considering that ____________ (y-variable) in this dataset range from ____________ (lowest y-value) to ____________ (highest y-value).

6) Change the definition of the `linear` function in the [Covid Spread Starter File] to match the model produced by `lr-plot` and "Save".

7) Do you think a linear function is a good fit for this dataset? Why or why not? ______________________________

____

____

____

★ What do you think the code that defines `MA-table` is actually doing? ______________________________

____

____

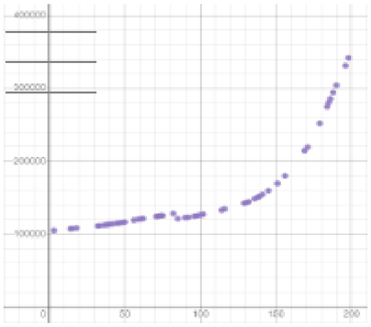
Quadratic Models for Covid in Massachusetts

Fitting the Model Visually $f(x) = a(x - h)^2 + k$

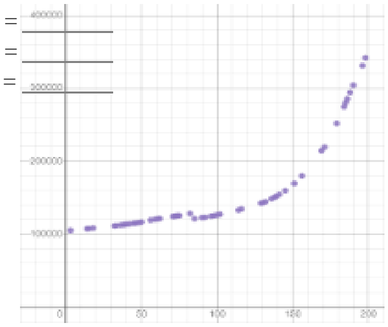
For this section, you'll need to have ***Slide 1: Quadratic Model for MA*** *of* ***Modeling Covid Spread (Desmos)*** *open on your computer.*

1) Try changing the values of a , h and k to find three promising quadratic models, graphing each one and labeling your values on the grids below.

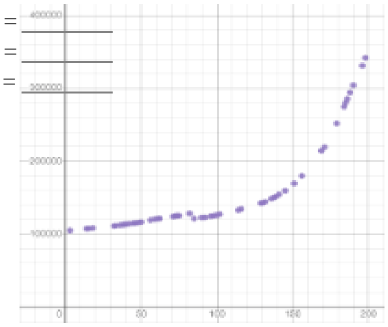
$a =$ ______
$h =$ ______
$k =$ ______



$a =$ ______
$h =$ ______
$k =$ ______



$a =$ ______
$h =$ ______
$k =$ ______



2) Do your quadratic models open up or down? __________. What does that tell us about a ? __________.

3) Describe one of your models: Where is the vertex? (________ , ________) What is the horizontal shift? ________ The vertical shift? ________
(x, y; h; k)

4) Which quadratic form would be the easiest to fit to this data? standard □ factored □ vertex□

Fitting the Model Programmatically $f(x) = a(x - h)^2 + k$

For this section, open your copy of the Covid Spread Starter File.

5) In the space below, define `quadratic1` to be the first model you fit in Desmos.

```
fun quadratic1(x): ( ______________ * (sqr( x - ______________ ))) + ______________ end
```
(a, h, k)

6) Return to Covid Spread Starter File and update the definitions for `quadratic1`, `quadratic2`, and `quadratic3`.
Then click "Run" to load your updated definition.

7) Use `fit-model` to determine the S -value of each model using the `MA-table`.
Hint: If you forgot the contract for `fit-model`*, look it up in the Contracts Pages!*

S for `quadratic1`: ______________ S for `quadratic2`: ______________ S for `quadratic3`: ______________

What does this model actually mean?

After experimenting, the best quadratic model I came up with for this dataset shows that __________ (x-variable) are correlated to __________ (y-variable)

The vertex of the parabola drawn by this model is a ______________ (minima or maxima?) at about ______________ (x, y), which predicts that ______________

____

The error in the model is described by an ***S-value*** of about ______________ (S) ______________ (units). I ______________ (strongly agree, agree, disagree, strongly disagree)

that this model is a good fit considering that ______________ (y-variable) in this dataset range from ______________ (lowest y-value) to ______________ (highest y-value).

Are Quadratic Models a Good Fit for This Data?

8) Would you feel good about making predictions based on these models? Why or why not? ______________

____

____

What Kind of Model? (Tables)

Decide whether each table is best described by a linear, quadratic, or exponential model.

If you chose exponential: What is the growth factor? Doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?

HINT: Can you draw the arrows to calculate the first difference? The second? *What does it mean if neither one is constant?*

x	y
1	5
2	10
3	15
4	20
5	25
6	30
7	35

1) Linear Quadratic Exponential ________ factor

x	y
0	10
1	100
2	1000
3	10000
4	100000
5	1000000
6	10000000

2) Linear Quadratic Exponential ________ factor

x	y
70	-169
71	-126
72	-81
73	-34
74	15
75	66
76	119

3) Linear Quadratic Exponential ________ factor

x	y
-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

4) Linear Quadratic Exponential ________ factor

x	y
0	3
1	6
2	12
3	24
4	48
5	96
6	192

5) Linear Quadratic Exponential ________ factor

x	y
-5	466656
-4	7776
-3	1296
-2	216
-1	36
0	6
1	1

★ Linear Quadratic Exponential ________ factor

Graphing Exponential Models: $f(x) = a(b)^x + k$

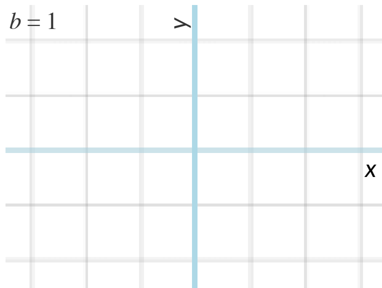
For this page, you'll need to have ***Modeling Covid Spread (Desmos)*** *open to* ***Slide 3: Exploring Exponential Models****.*
The curve you'll see is the graph of $g(x)$, an exponential function. Another, **identical** curve $f(x)$ is hiding behind it.

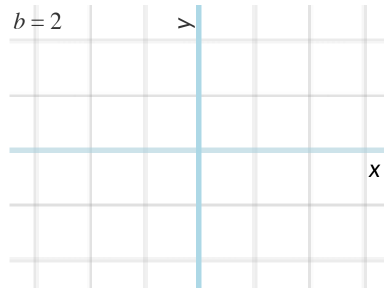
1) Use the starting values of a, b, and k in Desmos to complete the following equation: $g(x) = f(x) = \underset{a}{\underline{\quad}}(\underset{b}{\underline{\quad}})^x + \underset{k}{\underline{\quad}}$

Base b

2) Make sure $k = 0$ and $a = 1$. Experiment with b. **For what values of b is the function *undefined***, with the line disappearing? ____________

3) Keeping $a = 1$ and $k = 0$, change b to 0.5, 1, and 2, graphing each curve below. For each curve, **label the coordinates at x=1, 2, and 3.**





4) What does b tell us about an exponential function, when $b > 1$? ______________________________

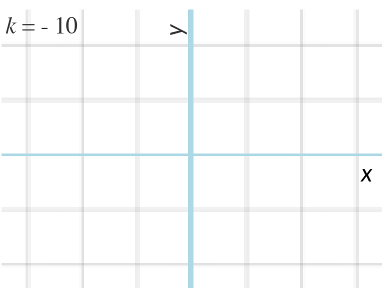
____

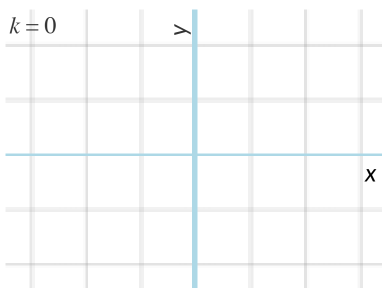
5) What does b tell us about an exponential function, when $0 < b < 1$? ______________________________

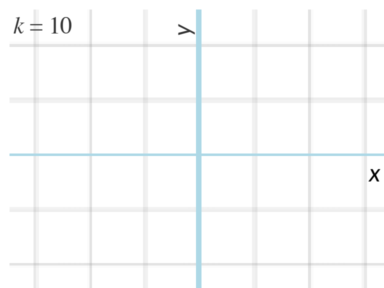
____

Vertical Shift...and Horizontal Asymptote k

6) Keeping $a = 1$ and $b = 2$, try changing the value of k to -10, 0, and 10, graphing each curve in the squares below. For each curve, find and label the y-value where the curve is "most horizontal", then **draw a horizontal line at that y-value.**





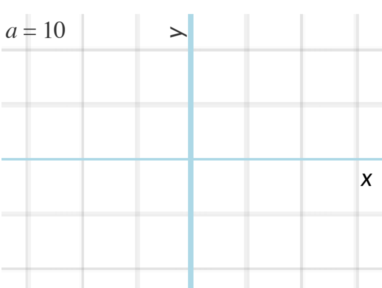


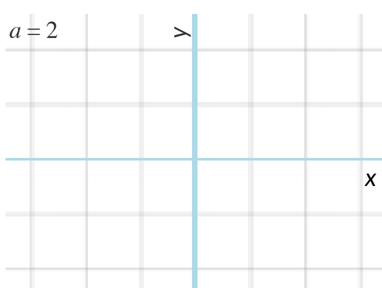
7) What does k tell us about an exponential function? ______________________________

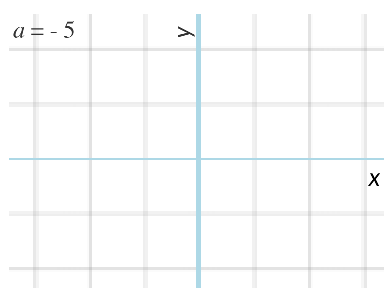
____

Initial Value a

8) Set $k = 0$ and $b = 2$. Change the value of a to 10, 2, and -5, graphing each curve in the squares below.
For each curve, label the y-intercept (x=0).



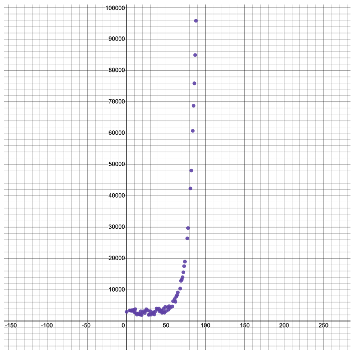




9) What does a tell us about an exponential function? ______________________________

What Kind of Model? (Graphs & Scatter Plots)

Are these scatter plots best be described by linear, quadratic, or exponential models? **If it's exponential, draw the asymptote!**

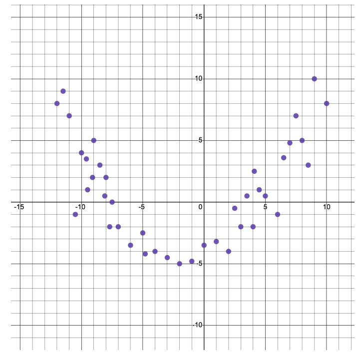


1) Linear Quadratic Exponential

How did you know? ______________________________

____

y-intercept ≈ ________ asymptote (if exponential): ________

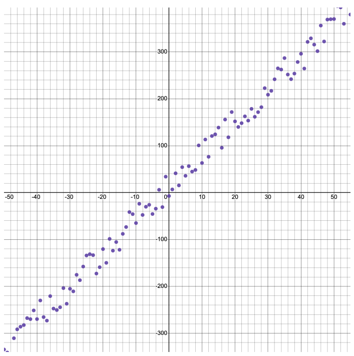


2) Linear Quadratic Exponential

How did you know? ______________________________

____

y-intercept ≈ ________ asymptote (if exponential): ________

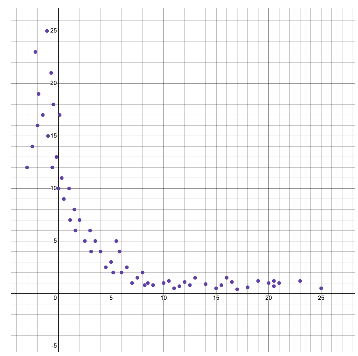


3) Linear Quadratic Exponential

How did you know? ______________________________

____

y-intercept ≈ ________ asymptote (if exponential): ________

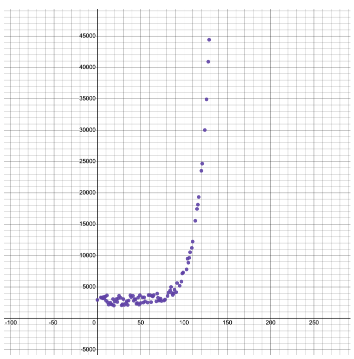


4) Linear Quadratic Exponential

How did you know? ______________________________

____

y-intercept ≈ ________ asymptote (if exponential): ________

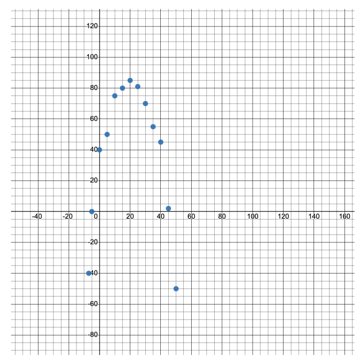


5) Linear Quadratic Exponential

How did you know? ______________________________

____

y-intercept ≈ ________ asymptote (if exponential): ________



6) Linear Quadratic Exponential

How did you know? ______________________________

____

y-intercept ≈ ________ asymptote (if exponential): ________

What Kind of Model? (Descriptions)

Decide whether each situation is best described by a linear, quadratic, or exponential function.

If the function is exponential: Identify the growth factor and the initial value.

Car Values

A particular kind of car sells for $32,000, and its resale value drops by 12.5% each year.

1) Is the function increasing or decreasing? ____________________

2) When the car is brand-new (x=0), how much is it worth? ________________

3) How much is it worth after…

(1 year) x=1	(2 years) x=2	x=3	x =4

4) What is the **form** of this function? linear ☐ quadratic ☐ exponential☐

5) **If it's exponential,**

Fill in the blanks to write a function that shows the value of the car after a given number of years:

$f(x) = \underset{\text{initial value } a}{\underline{\hspace{4cm}}} \times \underset{\text{growth factor } b}{\underline{\hspace{4cm}}}^{x} + \underset{\text{horizontal asymptote } k}{\underline{\hspace{4cm}}}$

Is it exponential *growth* ? ☐ or *decay* ? ☐

Lemonade Stand

Sally is selling lemonade, for $1.25 a glass in hopes of finally be able to get the power drill she's been wanting. She starts with $20 cash.

6) Is the function increasing or decreasing? ____________________

7) When Sally starts the day (x=0), how many dollars does she have? ________________

8) How many dollars will she have after…

(first sale) x=1	(second sale) x=2	x =3	x = 4

9) What is the **form** of this function? ☐ linear ☐quadratic ☐ exponential

10) **If it's exponential,**

Fill in the blanks to write a function that shows how much Sally has saved after a given number of sales:

$f(x) = \underset{\text{initial value } a}{\underline{\hspace{4cm}}} \times \underset{\text{growth factor } b}{\underline{\hspace{4cm}}}^{x} + \underset{\text{horizontal asymptote } k}{\underline{\hspace{4cm}}}$

Is it exponential *growth* ? ☐ or *decay* ? ☐

What Kind of Model? (Descriptions 2)

Decide whether each situation is best described by a linear, quadratic, or exponential function.
If the function is exponential: Identify the growth factor and the initial value.

High Fives

Two students started a club. At every meeting, students in attendance must high-five each of the other students. Club membership has been growing steadily by one student each meeting.

1) Is the function increasing or decreasing? ____________________

2) When the 2 students started the club (x=0), how many high-fives happened? ________________

3) How many high-fives happen at the subsequent meetings…

(3 students) x=1	(4 students) x=2	x=3	x=4

4) What is the **form** of this function? linear □ quadratic□ exponential□

5) **If it's exponential**,
Fill in the blanks to write a function that shows the how many high-fives happen for a given number of students:

$f(x) = \underset{\text{initial value } a}{\underline{\hspace{3cm}}} \times \underset{\text{growth factor } b}{\underline{\hspace{3cm}}}^{x} + \underset{\text{horizontal asymptote } k}{\underline{\hspace{3cm}}}$

Is it exponential *growth* ? □ or *decay* ? □

Going Viral

A student posted a video of a dog doing a back flip into a pile of laundry, and the meme went viral! Each person that sees the meme falls in love with it, and shares it with 10 new friends.

6) Is the function increasing or decreasing? ____________________

7) When the student posts it (x=0), how many total times has it been shared? ________________

8) How many times will it have been shared after…

(the next person shares) x=1	(their friends share) x=2	x=3	x=4

9) What is the **form** of this function? linear □ quadratic □ exponential□

10) **If it's exponential**,
Fill in the blanks to write a function that shows how many times the meme has been shared after a given number of "share cycles":

$f(x) = \underset{\text{initial value } a}{\underline{\hspace{3cm}}} \times \underset{\text{growth factor } b}{\underline{\hspace{3cm}}}^{x} + \underset{\text{horizontal asymptote } k}{\underline{\hspace{3cm}}}$

Is it exponential *growth* ?□ or *decay* ? □

What Kind of Model? (Definitions)

Decide whether each situation is best described by a linear, quadratic, or exponential function.

If the function is exponential: Identify the growth factor and the initial value.

$f(x) = 6x^2 - 5$

1) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$\text{miles}(\text{hours}) = \dfrac{22 \times \text{hours} + 14}{12 - 9}$

2) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$\text{cost}(w) = 5(1.2^w) + 16$

3) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$t(g) = 42 - 2g^2$

4) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$\text{price}(d) = d^2 + 6d$

5) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$j(x) = \dfrac{1}{2}^x + 22$

6) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$f(a) = 20000 - 4.1^a$

7) Linear Quadratic Exponential

How did you know? ______________________________

____

If it's exponential, what's the ________ (growth factor) ________ (initial value)?

$g(x) = 8(3^{-4x})$

8) Linear Quadratic Exponential

How did you know? ______________________________

____

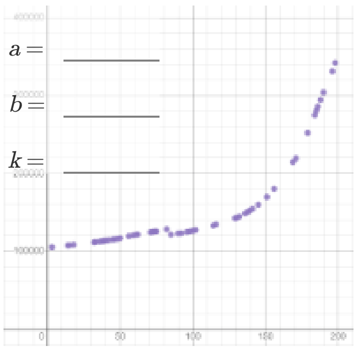
If it's exponential, what's the ________ (growth factor) ________ (initial value)?

Exponential Models of Covid in Massachusetts

*You'll need to have **Slide 4: Exponential Model for MA** of **Modeling Covid Spread (Desmos)** open on your computer.*

Change the values of k, then a, then b to find three promising exponential models.
Graph each one and label your values on the graphs below. ***growth rate*** *(r) can be calculated from the base:* $base = (1 + rate)$

Exponential Model 1



1) What is the initial value (# infections as of June 9th) of this model? ____________________

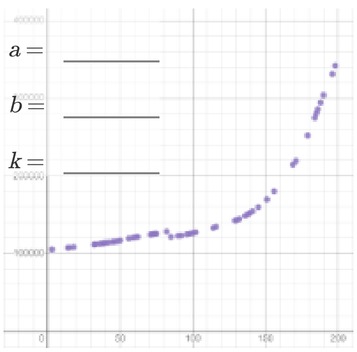
2) What is the ***growth rate*** of this model? ____________________

3) What is the y-intercept (and horizontal asymptote) of this model? ____________________

4) For what range of dates is this model *most* accurate? ____________________

5) For what range of dates is this model *least* accurate? ____________________

Exponential Model 2



6) What is the initial value (# infections as of June 9th) of this model? ____________________

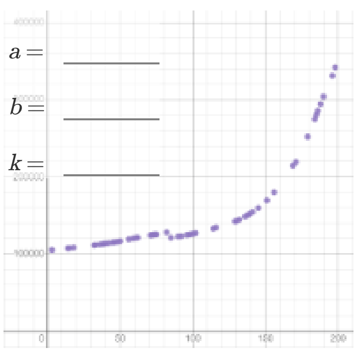
7) What is the ***growth rate*** of this model? ____________________

8) What is the y-intercept (and horizontal asymptote) of this model? ____________________

9) For what range of dates is this model *most* accurate? ____________________

10) For what range of dates is this model *least* accurate? ____________________

Exponential Model 3



11) What is the initial value (# infections as of June 9th) of this model? ____________________

12) What is the ***growth rate*** of this model? ____________________

13) What is the y-intercept (and horizontal asymptote) of this model? ____________________

14) For what range of dates is this model *most* accurate? ____________________

15) For what range of dates is this model *least* accurate? ____________________

Selecting a Model

16) Choose the model that you think best fits the data, and write it in function notation below:

$\text{exponential}(x) =$ ____
a(b)ˣ + k

Limits of Computational Modeling

Make sure you are working in your copy of the Covid Spread Starter File.

Benefits and Downsides of working with Approximations

1) What are some possible **benefits** to approximating large and small numbers when doing computations?

2) What are some possible **downsides** to approximating large and small numbers when doing computations?

Exponentiation and "RoughNums" in Pyret

3) **Write each of the expressions below in Pyret, then evaluate them and write down the answer.** *The first one has been started for you!*

	Pyret Code	Evaluates to...			Pyret Code	Evaluates to...
10^2	`expt(10, 2)`	`100`		1/3		
$2^{1/2}$				3^3		
$27^{1/3}$				$3^{1/3}$		

4) When do you think Pyret switches to RoughNums, instead of Numbers?

5) In Pyret, evaluate the following expressions: `1 == 1`, `~1 == ~1`
Why do you think Pyret gives an error when comparing identical RoughNums?

6) The pros of using `~1` involve speed. **What are the potential downsides of using `~1` to speed up a calculation?**

7) How likely are these downsides to apply to our Bootstrap work in code.pyret.org (CPO)?

Predicting Exponential Growth

Estimating with our Model

1) In the space below, define `exponential` for the best model from Exponential Models of Covid in Massachusetts.

`fun exponential(x): (` ______________ (a) `* expt(` ______________ (b) `, (~1 * x))) +` ______________ (k) `end`

2) Fill in the table below, to show how many positive cases you **estimate** based on your model after X days.
DO NOT use a calculator, keyboard or mouse! Instead, use **only your eyes** to look at the formula above or the Desmos graph on your screen.

After...	ESTIMATED model prediction	How confident are you? (1=Very confident, 5=Not at all)
50 days		
150 days		
250 days		
350 days		
450 days		

Fitting Exponential Models in Pyret

3) Define `exponential` in Covid Spread Starter File. Click "Run" and use `fit-model` to determine how closely `exponential` fits the `MA-table`. *Hint: If you forgot the contract for* `fit-model`*, look it up in the Contracts Pages!*

According to this exponential model, on June 9, 2020 (day zero) there were a total of about (________ (a) + ________ (k) =) ________ (a + k)

________ (y-units) in MA. This number increased by a factor of ________ (Growth Factor: b) or ________ (Growth Rate: (b - 1) × 100) % every day.

The error in the model is described by an ***S-value*** of ~ ________ (S) ________ (units). I agree (strongly agree, agree, disagree, strongly disagree)

that this model is a good fit considering that ________ (y-units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

4) **Use Pyret** to compute your model's predictions after each number of days and fill in the table below.

After...	ACTUAL model prediction	How close were you? (1=Very close, 5=Very far)
50 days		
150 days		
250 days		
350 days		
450 days		

5) If your predictions got *worse* as the number of days increased, why do you think that is? ______________________________

____

Campus Housing Data

A college is looking at enrollment and housing data for students who've decided what their major will be, vs. those who are undecided:

	# On Campus	# Off Campus	% On Campus
Undecided	120	80	120/200 = 60%
Decided	80	100	80/180 = 44%

1) According to the table, how many Undecided Majors live *off*-campus? ______________________

2) According to the table, how many Decided Majors live *on*-campus? ______________________

3) Who is more likely to live on campus: Decided or Undecided Majors? ______________________

4) Do you think there is a relationship between deciding on a major and living on or off campus? If so, why?

Models for Vermont

*For this page, you'll need to have the Covid Spread Starter File open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

Find the definition of `is-MA` in your starter file. The code is shown here:

```
# is-MA :: Row -> Boolean
# consumes a Row, and checks if state == "MA"
fun is-MA(r): r["state"] == "MA" end
```

1) Under the definition of `is-MA`, define a *new* function called `is-VT`, which tests to see if the `state` value is equal to "VT." Click run and try it out!

Find the definition of `MA-table` in your starter file. The code is shown here:

```
##########################################################
# Define some grouped and/or random samples
MA-table = filter(covid-table, is-MA)
```

2) Under the definition of `MA-table`, define a *new* grouped sample called `VT-table` containing all the rows for the state of Vermont.` Click run and try it out!

3) Use `lr-plot` to obtain the best-possible linear model for the relationship between `day` and `positive` in the `VT-table`, then fill in the blanks below:

The optimized linear model for this dataset predicts an ______________ (increase / decrease) of about ________ (slope) ______________ (y-variable) per ______________ (x-variable).

The error in the model is described by an ***S-value*** of about ______________ (S) ______________ (units). I ______________ (strongly agree, agree, disagree, strongly disagree)

that this model is a good fit considering that ______________ (y-variable) in this dataset range from ______________ (lowest y-value) to ______________ (highest y-value).

Exponential Model for Vermont (VT)

*For this section, in addition to Pyret, open **Slide 5: Exponential Model for VT** of **Modeling Covid Spread (Desmos)**.*

4) Turn to **Slide 5: Exponential Model for VT** of **Modeling Covid Spread (Desmos)** and adjust the sliders until you've come up with the best exponential model you can for the Vermont dataset. Record your model below:

____

5) Return to Covid Spread Starter File. At the bottom of the Definitions Area, define `exponential-VT` to be the model you just found.

Click "Run" to load your definition, then fit the model using `VT-table`.

According to this model, on June 9, 2020 (day zero) there were about ________ (a) + ________ (k) ______________ (y-units) in VT, for a total of about

______________ (a + k). This number grew exponentially, increasing by a factor of ______________ (Growth Factor: b), or ______________ (Growth Rate: (b - 1) × 100) % every day.

The error in the model is described by an ***S-value*** of about ______________ (S) ______________ (units).

I ______________ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ______________ (y-units) in this dataset range from

______________ (lowest y-value) to ______________ (highest y-value).

Do People in Rich Countries Live Longer?

- **Per-capita GDP** is a measure of what a "typical income" is for people in each country (US Dollars).
- **Median lifespan** is a measure of what a "typical lifespan" is for people in each country

A point at (75, 62) would represent a country where the average GDP is $75,000/year and the median lifespan is 62 years old.
The table below shows the per-capita GDP and median life expectancy for three different countries.

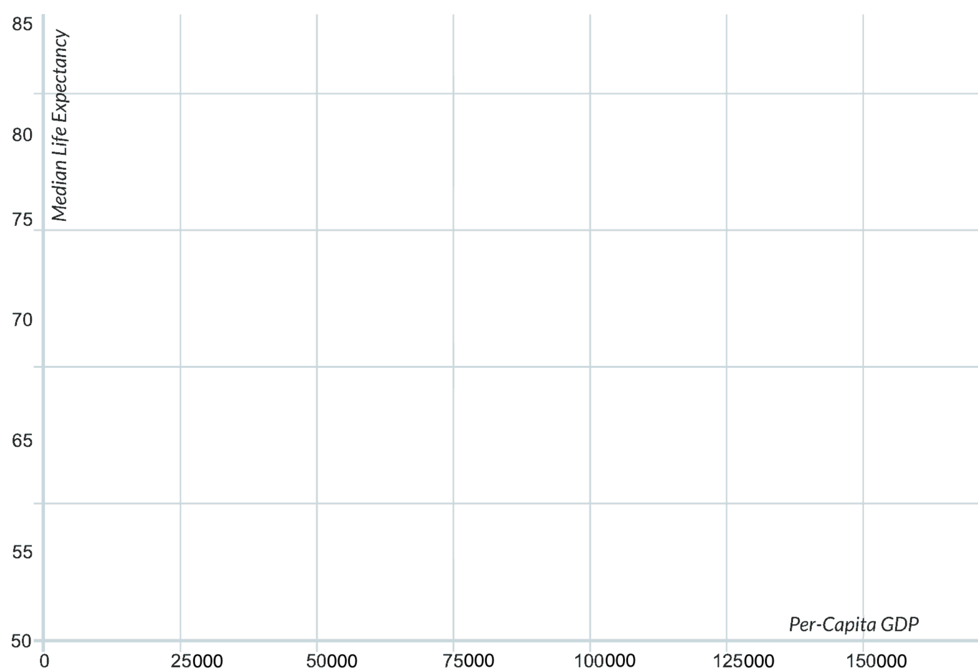
Country	Per-Capita GDP ($)	Median Lifespan (years)
Mozambique	$1,348	54.1
Pakistan	$5,085	68.4
Poland	$29,176	77.9
Luxembourg	$103,511	82.4

1) **Plot these four countries** as points on the graph below.

2) Use your *best guess* to **draw a curve** that fits all four points.

3) **Plot the points on your curve** corresponding to the four per-capita GDPs below. What does your curve predict the median lifespan to be, at each of these points?

Country	Per-Capita GDP ($)	Median Lifespan (years)
Country A	$5,000	
Country B	$25,000	
Country C	$125,000	
Country D	$150,000	



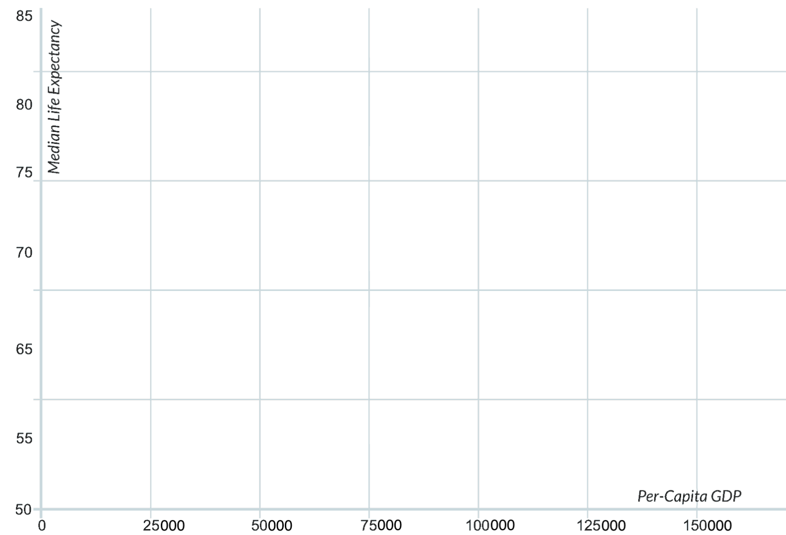
4) What do you **Notice** about the shape of the curve? Does it look like one of the functions we've seen before? ____________________

Exploring the Countries Dataset

*For this section, you'll need the [Countries of the World Starter File] open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. The columns in this dataset are described in the left column below:*

- **country** - name of the country
- **gdp** - total Gross Domestic Product of the country. GDP is often used to measure the economic health of a country.
- **population** - number of people in the country
- **pc-gdp** - the average GDP *per-person* , in US Dollars
- **has-univ-healthcare** - indicates if the country has universal healthcare
- **median-lifespan** - the median life expectancy of people in the country

In Pyret, make a scatter plot showing the relationship between `pc-gdp` and `median-lifespan`, and sketch it below.



1) What do you **Notice** about the point cloud? ______________________________

2) What do you **Wonder** about the point cloud? ______________________________

3) Do you see a relationship? If so, describe it (e.g. - linear or nonlinear? Strong or weak?)

4) Find the point for Equatorial Guinea, an outlier with `pc-gdp` = $38,058 and `median-life-expectancy` = 65. Why might this point be so far from the rest?

5) Find the point for Japan, another outlier with `pc-gdp` = $43,030 and `median-life-expectancy` = 85.5. Why might this point be so far from the rest?

6) Are there any countries that stand out? Why or why not? ______________________________

7) Suppose a wealthy country is suffering heavy casualties in a war. Draw a star on the scatter plot, showing where you might expect it to be.

Fitting Models for the Countries Dataset

For this page you will be working with both the [Countries of the World Starter File] and the ***Desmos*** *file* ***Fitting Wealth-v-Health and Exploring Logarithmic Models***.

1) Find the optimized **linear model** for this data using `lr-plot`. Then update the definition for `linear` on line 29 of your starter file.

$linear(x) =$ ________ (slope (m)) x + ________ (y-intercept / vertical shift) ________ (S-value)

The optimized linear model for this dataset predicts that a ________ (x-units) ________ (increase / decrease) in per-capita gdp (x-variable) will increase ________ (y-variable) by ________ (y-units). The error in the model is described by an S-value of about ________ (S) ________ (y-units), which is ________ (insignificant / reasonable / significant / extreme) considering ________ (y-units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

2) Find the best **quadratic model** you can, using the first slide *(Wealth-v-Health Quadratic)* in the Desmos activity. Then return to pyret. Update the `quadratic` function defined on line 35. Click "Run" and use `fit-model` to calculate the ***S-value***.

$quadratic(x) =$ ________ (quadratic coefficient (a)) $(x -$ ________ (horizontal shift (h)) $)^2 +$ ________ (vertical shift (k)) ________ (S-value)

The vertex of the parabola drawn by my model is a ________ (minima or maxima?) at about (________ (x, y)).

- *Before* this point, as ________ (x-variable) increases, ________ (y-variable) ________ (increases or decreases?).
- *After* this point, as ________ (x-variable) increases, ________ (y-variable) ________ (increases or decreases?).

The error in the model is described by an S-value of about ________ (S) ________ (y-units), which is ________ (insignificant / reasonable / significant / extreme) considering ________ (y-units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

3) Find the best **exponential model** you can, using the second slide *(Wealth-v-Health Exponential)* in the Desmos activity. Then return to pyret. Update the `exponential` function defined on line 39. Click "Run" and use `fit-model` to calculate the ***S-value***.

$exponential(x) =$ ________ (initial value (a)) (________ (growth factor (b)) ^{x}) + ________ (vertical shift (k)) ________ (S-value)

According to this exponential model, a country with a ________ (x-variable) of zero ________ (x-unit) would have a ________ (y-variable) of ________ (a) + ________ (k) ________ (y-units), for a total of about ________ (a + k). This number grows exponentially, increasing by a factor of ________ (Growth Factor: b) or ________ (Growth Rate: (b - 1) × 100) % with every ________ (x-unit) increase in ________ (x-variable).

The error in the model is described by an S-value of about ________ (S) ________ (y-units), which is ________ (insignificant / reasonable / significant / extreme) considering ________ (y-units) in this dataset range from ________ (lowest y-value) to ________ (highest y-value).

4) Are any of these models a good fit for this data? Why or why not? ________________

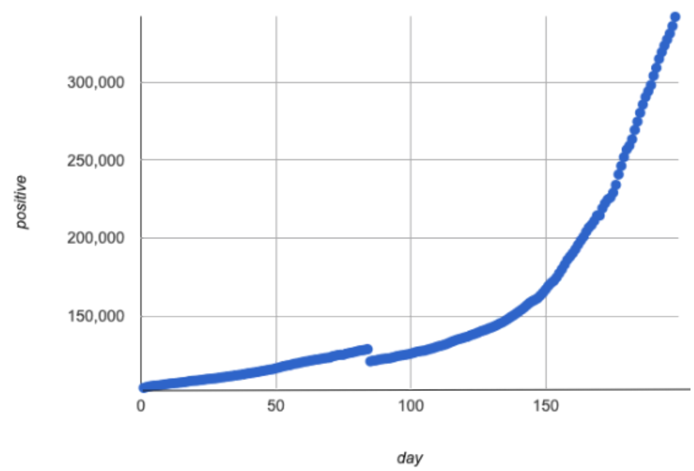
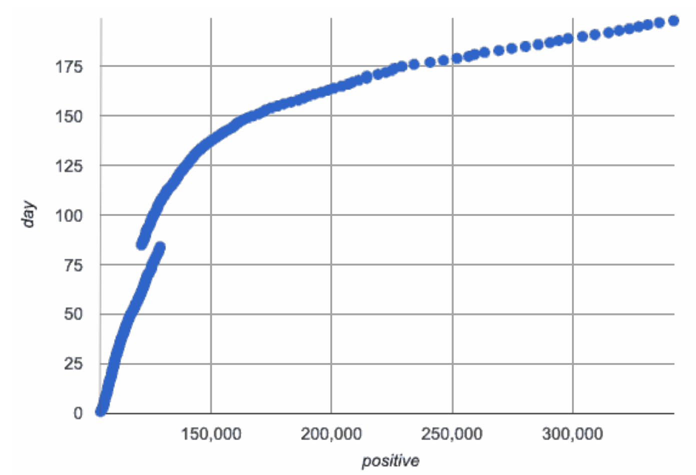
____

____

Swapping the Axes: Notice and Wonder

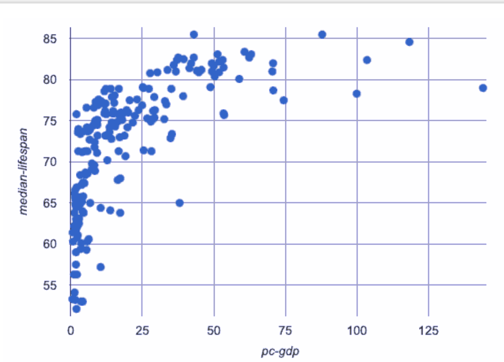
The scatter plots below were made with the same data... we just *swapped* the x- and y-axes.

- The first scatter plot shows an **exponential relationship**: the number of Covid Infections as a function of Days
- The second scatter plot shows a **logarithmic relationship**: Days as a function of Infections

Days → Infections	Infections → Days
	
1) What question might you answer from this graph?	2) What question might you answer from this graph?

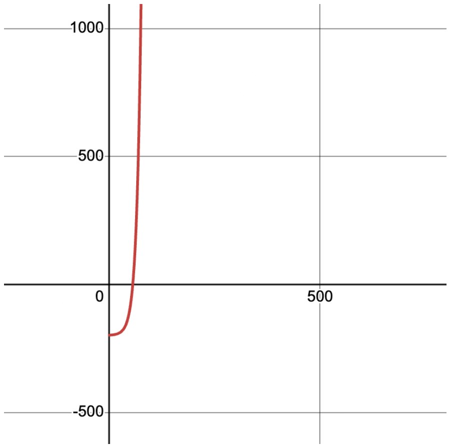
What do you Notice?	What do you Wonder?

3) This third scatter plot is our current dataset. Which of the scatter plots above does it resemble? *Days → Infections* *Infections → Days*

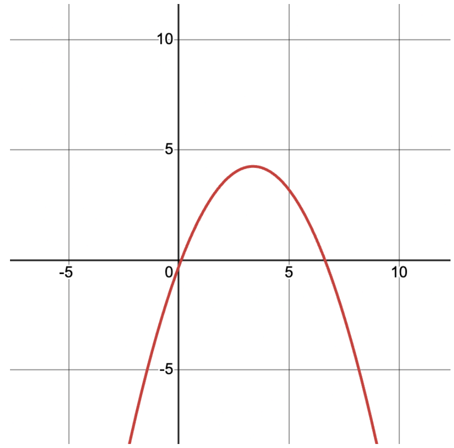
Per-Capita GDP → Median Life Expectancy


What Kind of Model? (Graphs & Scatter Plots)

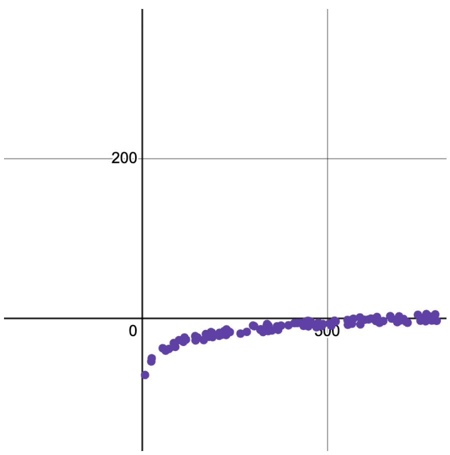
- Decide whether each representation is best described by a quadratic, exponential, or logarithmic function.
- *If you think it's exponential OR logarithmic*: Draw a diagonal line for $y = x$, and then sketch the reflection of the curve.



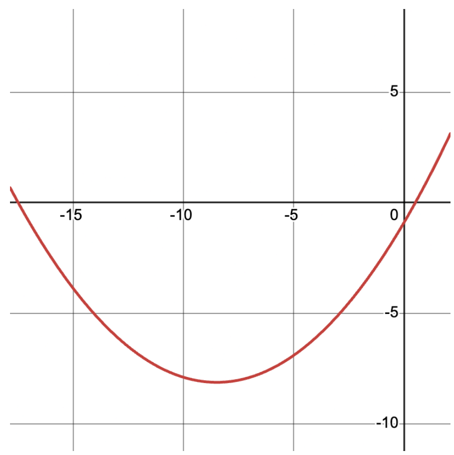
1) Quadratic Exponential Logarithmic



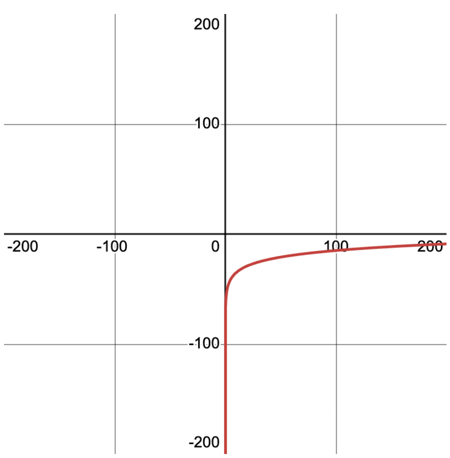
2) Quadratic Exponential Logarithmic



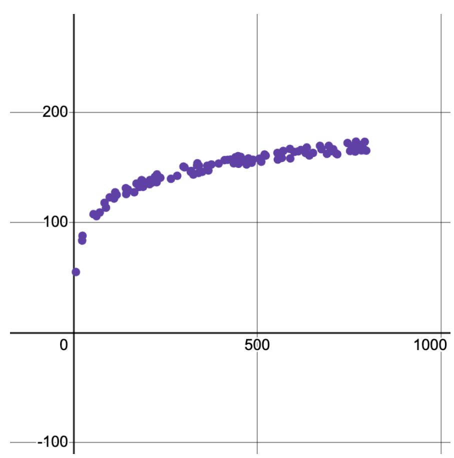
3) Quadratic Exponential Logarithmic



4) Quadratic Exponential Logarithmic



5) Quadratic Exponential Logarithmic



6) Quadratic Exponential Logarithmic

What Kind of Model? (Tables)

Decide whether each representation is best described by a quadratic, exponential, or logarithmic function.

- **If the function is exponential, find the base (also called the *growth factor*):** How much does y increase (2x? 10x?) for a single increase in x?
- **If the function is logarithmic, find the base:** How much does x need to increase (2x? 10x?) just to get a single increase in y?

HINT: Can you draw the arrows to calculate the first difference? The second? *What does it mean if neither one is constant?*

x	y
1	0
10	1
100	2
1000	3
10000	4
100000	5
1000000	6

1) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
0	1
1	10
2	100
3	1000
4	10000
5	100000
6	1000000

2) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
70	-169
71	-126
72	-81
73	-34
74	15
75	66
76	119

3) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
5	1
10	2
20	3
40	4
80	5
160	6
320	7

4) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

5) Quadratic Exponential ____ (base) Logarithmic ____ (base)

x	y
1	0
6	1
36	2
216	3
1296	4
7776	5
466656	6

6) Quadratic Exponential ____ (base) Logarithmic ____ (base)

Graphing Logarithmic Models: $f(x) = a \log_b x + k$

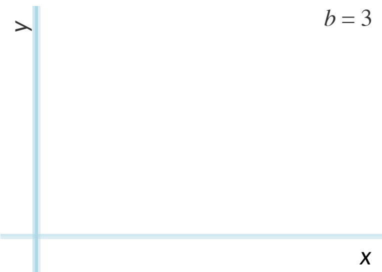
*You'll need to have **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)** open to **Slide 4: Exploring Logarithmic Functions**.*
The curve you'll see is the graph of $r(x)$, an logarithmic function. Another, **identical** curve $s(x)$ is hiding behind it.

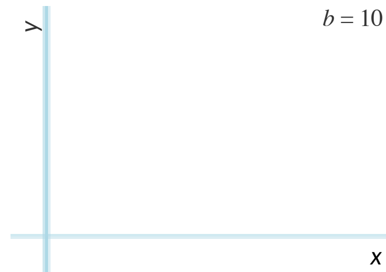
1) Use the starting values of a , b , and k to complete the following equation: $r(x) = s(x) = f(x) = \underset{\text{log coefficient}}{\_\_\_\_\_\_\_\_} \; log \underset{\text{base}}{\_\_\_\_} (x) \; + \underset{\text{vertical shift}}{\_\_\_\_\_\_\_\_}$

Base b

*Keep **k** at 0 and **a** at 1 . Change the value of **b** as indicated on each grid below.*

2) Sketch each graph and **label the coordinates where** $x = 1$**,** $y = 1$**,** $y = 2$ **and** $y = 3$.





3) How does the value of b impact the shape of a logarithmic function? ______________________________

4) What connections can you draw between the value of b and exponents? ______________________________

____

Vertical Shift k

*Set **a** to 1 and **b** to 2 . Change the value of **k** as indicated on each grid below.*

5) Sketch each graph and **label the coordinate where** $x = 1$.

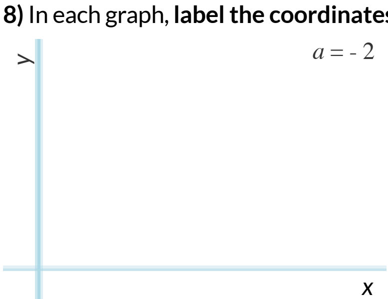
6) How does the value of k impact the shape of a logarithmic function? ______________________________

7) Why does $y = k$ when $x = 1$? ____

Logarithmic Coefficient a

*Set **k** to 0 and **b** to 10 , then zoom out out so you can see as far as $x = 1{,}000$.*
Change $s(x)$ to $s(x) = 1 \log_{10}(x) + 0$ so that the blue curve lands on top of the red curve.

8) In each graph, **label the coordinates where** $x = 10$ **and** $x = 100$ **and** $x = 1000$**.**



9) What is the value of x when $1\log_2(x) = 4$? __________ What about when $2\log_4(x) = 4$? __________ When $3\log_8(x) = 4$? __________

★ How are a and b related? ____

What Kind of Model? (Descriptions)

Decide whether each situation describes a quadratic, exponential, or logarithmic function. **HINT:** draw a table and plug in some points!

1) Earthquakes release enormous amounts of energy, which we can compare to the energy released by blowing up pounds of dynamite.
e.g. The force of blowing up 12,000 pounds of dynamite produces a 4.0 on the Richter scale! → $\text{richter}(12{,}000) = 4.0$
$\text{richter}(400{,}000) = 5.0$ $\text{richter}(12{,}540{,}000) = 6.0$ $\text{richter}(398{,}000{,}000) = 7.0$

Quadratic Exponential Logarithmic

2) A car accelerates at a constant rate of 5mph/s. After 1 second, $\text{distance}(1) = 2.5\text{miles}$.
$\text{distance}(2) = 10$ $\text{distance}(3) = 22.5$ $\text{distance}(4) = 40$

Quadratic Exponential Logarithmic

3) Moore's law says that the number of transistors in a microprocessor will double roughly every 1.5 years.
Starting with 16 transistors, how many years will it take to reach 4,294,967,296 transistors?

Quadratic Exponential Logarithmic

4) The population of a colony of bacteria can double every 20 minutes, as long as there is enough space and food. Starting with 1 bacteria…
$f(20) = 2$ $f(40) = 4$ $f(60) = 8$ $f(80) = 16$

Quadratic Exponential Logarithmic

5) Sequan puts $100 in a savings account, earning 4% interest. After a year…
$\text{savings}(1) = \$104$ $\text{savings}(2) = \$108.16$ $\text{savings}(2) = \$112.49$

Quadratic Exponential Logarithmic

6) If the *width and length* of a rectangle doubles, how much does the *area* change?

Quadratic Exponential Logarithmic

From Exponents to Logs

Fill in the blanks and empty cells for each table below. The first row of each table has been completed for you.

Find the Related Logarithmic or Exponential Statement

	Exponential Equation	Logarithmic Equation
Example:	$5^3 = 125$	$\log_5 125 = 3$
1)	$9^2 = 81$	
2)		$\log_2 32 = 5$
3)	$x^y = z$	
4)		$\log_a b = c$

Phrasing 1

	Expressions	Translation	Evaluates to:
Example:	$\log_2(1)$	"To get 1, I raise 2 to what power?"	0
5)	$\log_2(8)$	"To get 8, I raise 2 to what power?"	
6)	$\log_5(25)$	"To get ________, I raise ________ to what power?"	
7)	$\log_5(1)$	"To get ________, I raise ________ to what power?"	
8)		"To get 81, I raise 3 to what power?"	
9)	$\log_4(1)$		

Phrasing 2

	Expressions	Translation	Evaluates to:
Example:	$\log_2(8)$	"The power to which you raise 2 to get 8"	3
10)	$\log_3(1)$	"The power to which you raise ________ to get ________"	
11)		"The power to which you raise 0.1 to get 0.01"	
12)	$\log_2(16)$		

Create Your Own

Come up with your own logarithmic expressions! *Use the phrasing you like best* to translate them, then solve them.

	Expressions	Translation	Evaluates to:
13)			
14)			

(optional)

Balancing Function Growth and Axis Growth (Linear)

*Make sure you're on **Slide 5: Changing the Scale (Linear)** of **Fitting Wealth-v-Health and Exploring Logarithmic Models**.*

- Both the x- and y-axis are labeled with a sequence of numbers *(e.g. 0, 2, 4, 6 ...)*.

1) What kind of growth do the sequences on these axes show? (circle one) Linear Quadratic Exponential Logarithmic

2) The function f is ______________ (Linear, Quadratic, Exponential, or Logarithmic), and its slope is ____. Which two points are plotted on f? ______________

Faking a Change in Slope

- Click on the wrench button (🔧) in the top-right corner of the Desmos graph to **open the "Graph Settings" window.**
- **Change the scale of the y-axis** so that $-2 \leq y \leq 2$, making sure you can see both labeled points

3) What kind of growth does the sequence of numbers on the **y-axis** show *now?* Linear Quadratic Exponential Logarithmic

4) Did our two points on the line change? ____ Did the slope of f change? ____. How did the graph change? ______________

____

5) What is going on here? ______________

____

6) **Without changing the y-axis**, change the *x-axis* to put the line roughly back where it was. What scale did you use? ______ $\leq x \leq$ ______

Faking a Change to Logarithmic

- Open the "Graph Settings" window, and **expand the "More Options" section** by clicking the triangle (▶).
- **Change the y-axis** scale from `Linear` to `Logarithmic`. *(The y-axis labels should change to something like 0.1, 1, 10, 100...)*
- **Drag the graph** to make sure you can see both labeled points

7) What kind of growth does the sequence of numbers on the **y-axis** show *now?* Linear Quadratic Exponential Logarithmic

8) Did our two points on the line change? ____ Did the slope of f change? ____. How did the graph change? ______________

____

9) What is going on here? ______________

____

Faking a Change to Exponential

- Change the y-axis scale back to `Linear`, and the `x-axis` scale to `Logarithmic`.
- **Drag the graph** to make sure you can see both labeled points

10) What kind of growth does the sequence of numbers on the **x-axis** show *now?* Linear Quadratic Exponential Logarithmic

11) Did our two points on the line change? ____. Did the slope of f change? ____. How did the graph change? ______________

____

12) What is going on here? ______________

____

★ **Make a prediction:** What do you think will happen if *both axes* are switched to `Logarithmic`? ______________

____

13) Change both scales to `Logarithmic`. Was your prediction accurate? ____. What happened, and why? ______________

____

____

Balancing Function Growth and Axis Growth (Logs and Exponents)

Make sure you're on ***Slide 6: Changing Scale (Exponential and Logarithmic)*** *of* ***Fitting Wealth-v-Health and Exploring Logarithmic Models****.*

Balancing Logarithmic Growth

- The folder for a logarithmic function g is shown in red, and the folder for h is "turned off". **Do not** turn it on yet!
- Both the x- and y-axis are be labeled with a sequence of numbers *(e.g. 0, 2, 4, 6 ...)* .

1) What kind of growth do the sequences on these axes show? (circle one)   Linear   Quadratic   Exponential   Logarithmic

2) The function g is ______________________________ (Linear, Quadratic, Exponential, or Logarithmic), and its base is ______. Which two points are plotted on g? ________________

- Click on the wrench button (🔧) in the top-right corner of the Desmos graph to **open the "Graph Settings" window.**

3) The x-axis goes from _____ $\leq x \leq$ _____. Can you change these numbers to make g look linear? ______ Why? ________________________

____

- Click on the wrench button (🔧) and change the **x-axis** from `Linear` to `Logarithmic`

4) Did our two points on the line change? ____ How did the graph change? ________________________________

5) What is going on here? ____

6) **Make a prediction:** What would the graph of g look like with a `Linear` **x-axis** a `Logarithmic` **y-axis**? ________________________

____

7) Try it out! Was your prediction accurate? ____. What happened, and why? ________________________________

____

Balancing Exponential Growth

- Set the scale for both axes back to `Linear`.
- "Turn off" the folder for g, and "turn on" the folder for our exponential function h.

8) The function h is ______________________________ (Linear, Quadratic, Exponential, or Logarithmic), and its base is ______. Which two points are plotted on h? ________________

9) Without switching either axis to `Logarithmic`, can we change the minimum and maximum x or y to make h look `Linear`? __________

10) **Make a prediction:** Which axis should we switch to `Logarithmic`, in order to make h look `Linear`? ____________________

11) Try it out! Was your prediction accurate? ____. What happened, and why? ________________________________

____

- Change the **y-axis** back to `Linear`, and change the x-axis to `Logarithmic`

12) What happened to the graph of h, and why? ____

____

★ Desmos has two choices for scale:

- `Linear` *(each interval is the same size as the one before it)*
- `Logarithmic` *(each interval is 10x larger than the one before it)*

If we wanted to make h appear linear **using the x-axis**, what kind of scale would we need? ________________________

____

Fitting Logarithmic Models

To complete this page you will need Desmos and the Countries of the World Starter File open on your computer.

Fitting a Logarithmic Model *f(x) = a log$_b$ x + k*

*You should be on **Slide 7: Wealth-v-Health (Logarithmic)** of **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)**.*

- The x-axis should labeled with a sequence of numbers that looks something like this: *20000, 40000, 60000, 80000, 10000, 120000...*

1) What kind of growth does the sequence on the x-axis show? (circle one) Linear Quadratic Exponential Logarithmic

2) Use the sliders for a and k to make the best-fitting logarithmic model you can find. Write it below. *(Note: Pyret's `log` always uses $b = 10$)*

$\text{logarithmic}(x) = \underset{\text{log coefficient (a)}}{\underline{\hspace{3cm}}} \log_{10}(x) + \underset{\text{vertical shift (k)}}{\underline{\hspace{3cm}}}$

```
fun logarithmic(x): (_______ * log(x)) + _______ end
```

3) Define `logarithmic(x)` in the Countries of the World Starter File to be this model, and fit it using `fit-model`.

The error in the model is described by an S-value of about ________ (S) ________ (units). I ____agree or disagree - not strong____ (strongly agree, agree, disagree, strongly disagree) that this model is a good fit considering that ________________ (y-variable) in this dataset ranges from ________ (lowest y-value) to ________ (highest y-value).

Scaling the x-Axis

- Click on the wrench button (🔧) in the top-right corner of the Desmos graph to **Open the "Graph Settings" window.**
- **Expand the "More Options" section** by clicking the triangle (▶).
- **Change the x-axis scale** from `Linear` to `Logarithmic`. *(The x-axis labels should change to something like 100, 1000, 10^4, 10^5 ...)*

4) What kind of growth does the sequence on the x-axis show? (circle one) Linear Quadratic Exponential Logarithmic

5) What is the shape of the point cloud, after changing the x scale to `Logarithmic`? Linear Quadratic Exponential Logarithmic

6) Adjust the sliders for a and k to improve the model. *Toggle back and forth between `logarithmic` and `linear` x-axis scales as you work.*

When you are satisfied with your model, record both forms of the definition below.

$\text{logarithmic2}(x) = \underset{\text{log coefficient (a)}}{\underline{\hspace{3cm}}} \log_{10}(x) + \underset{\text{vertical shift (k)}}{\underline{\hspace{3cm}}}$

```
fun logarithmic2(x): (_______ * log(x)) + _______ end
```

7) In Pyret, define `logarithmic2(x)` to match this model. Then use the `fit-model` function to find its ***S-value***: ________________

8) How much ________ (more / less) error do we expect in predictions made using `logarithmic2` than with the `logarithmic` model?

Percent Change $= \dfrac{\text{Difference between the S-values}}{\text{S-value for logarithmic model}} \times 100 = \underline{\hspace{1cm}}$

We expect ________ percent ________ (more / less) error from predictions made with `logarithmic2` than with the `logarithmic` model!

9) Do we know for sure that either of these models is optimal? Explain. ____

____

10) Why does transforming the **x-axis** makes our data look linear? ____

____

____

Transforming the Data

*For this page, you'll need **Slide 8: Wealth-v-Health (Transformed)** of **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)**.*

1) Compare the two tables below from the Desmos file we are about to work with.

Wealth vs. Health | **Log(Wealth) vs. Health** | **What do you Notice or Wonder?**

Wealth vs. Health

x_1	y_1
1.99051	52.1
11.76559	78.6
15.19295	77.2
6.26897	60.6
24.95776	76.9

Log(Wealth) vs. Health

$g(x_1)$	y_2
3.2989644	52.1
4.0706137	78.6
4.1816421	77.2
3.7971962	60.6
4.3972056	76.9
4.313631	77.5
3.9688606	75.1

What do you Notice or Wonder?

____

____

____

____

____

____

Fitting a Model

This slide contains two tables, a list and a function:

- `Wealth vs. Health` - the same table we've seen before.
- y_2 - a copy of the table's y_1 column
- $g(x)$ - a function that takes in a number x, and produces $log_{10}(x)$
- `Log(Wealth) vs. Health` - a new table!
  - *x_2 uses $g(x)$ on the x_1 column in the first table.*
  - *y_2 is our copy of the y_1 column in the first table.*

2) Notice how the red dots (representing the data points from the original table) are dispersed on the graph.

- Hide these data points from the graph by clicking on the "points" circle (⁖) at the top of the y_1 column.
- Then click on the triangle (▶) in front of the folder name to hide the table.
- Turn ON the points for the `Log(Wealth) vs. Health` table by clicking on the "points" circle (⁖) at the top of the y_2 column

How does the new graph look different from the original graph? ______________________________

____

3) Scroll to the bottom of the `Log(Wealth) vs. Health` table and click the 🔍 to rescale the graph.

4) Looking at the point cloud, the best model for this dataset would be (circle one): linear quadratic exponential

5) Why do you think transforming the **x-values** makes our data look linear? ______________________________

____

6) Move the sliders for m and b at the bottom left to find the best-fitting linear model you can:

$f(x) =$ ______________ $x +$ ______________
slope (m) | y-intercept / vertical shift

Let's compare the model settings from your linear and logarithmic models.

Linear (From above)	______________ slope (m)	______________ y-intercept / vertical shift
Logarithmic (From [Fitting Logarithmic Models])	______________ log coefficient (a)	______________ vertical shift (k)

7) How are they similar? ______________________________

____

Logarithmic Models

Open your copy of the Countries of the World Starter File and click "Run".

Transforming: From Logarithmic ***Plots*** to Linear Ones

1) Find the definition of `g(r)`. What does this function do? ______________________________

____

2) Find the Contract for `build-column` on the Contracts Page.

What is its **Range**? ______________ What is its **Domain**? ______________________________

3) At the end of the program, you'll find this code:

```
countries-transformed = build-column(countries-table, "log(pc-gdp)", g)
```

What do you think it does? ______________________________

____

4) Click "Run", and evaluate `countries-transformed` in the Interactions Area on the right to test it out!

a. What is different about this Table? *Hint: Find the last column!* ______________________________

b. Where did the column on the right come from? ______________________________

5) Use this new table to make an `lr-plot` comparing `log(pc-gdp)` and `median-lifespan`, with `country` as the label. Record the regression line and S value below:

$y =$ ______________ (slope) $x +$ ______________ (vertical shift) S: ______________

Inverting: From Linear ***Models*** to Logarithmic Ones

6) Use the model settings of the *linear* model you just made to complete the *logarithmic* model below:

$logarithmic3(x) =$ ______________ (log coefficient (a)) $\log_{10}(x) +$ ______________ (vertical shift (k))

```
fun logarithmic3(x): ( ______ * log(x)) + ______ end
```

7) Let's interpret this model:

A country where the ______________________________ (explanatory variable (x)) *is* ______ (base (b)) *times higher than another is also*

predicted to have a ______________________________ (response variable (y)) *that is* ______________ (log coefficient (a)) ______________ (y-axis units) *longer.*

8) Add the definition of `logarithmic3` to your starter file. Use `fit-model` to calculate the ***S-value*** and complete the table below:

	Linear	Quadratic	Exponential	Logarithmic
S-value	~5.926626			

9) Compare the two smallest S values using percent change. $\dfrac{\text{Difference between the S-values}}{\text{S-value for second best model}} \times 100 =$ ____

Predictions made with the logarithmic model are expected to have ______ percent ______________ (more / less) error than predictions made with the

______________ model!

Does Wealth impact lifespan equally if there's Universal Healthcare?

*For this page, you'll need the [Countries of the World Starter File] open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

1) Add `fun no-universal(r): not(r["has-univ-healthcare"]) end` at the bottom of the Definitions Area.

Read the function definition carefully. What do you think it does? ______________________________

____

2) Click "Run" and evaluate `no-universal(albania)` in the Interactions Area. What does Pyret return? ______________

3) What does that mean? ____

4) Add `countries-wo-univ = filter(countries-transformed, no-universal)` to the Definitions Area and click "Run".

What does evaluating `countries-wo-univ` in the Interactions Area produce? ______________________

____

5) Using the two definitions you just added as models:

- **Define a new function** called `yes-universal`, which returns the value in the `has-univ-healthcare` column.
- **Define a new table** called `countries-w-univ`, which shows all the countries with universal healthcare.
- Click "Run" to load these new definitions once you have them both typed into the Definitions Area.

6) Fill in the table below by:

- Building an `lr-plot` for each of these tables with the transformed-column `log(pc-gdp)`.
- Using what you learn from `lr-plot` to write logarithmic models for each table.
- Using `fit-model` to find S for each of your logarithmic models and their corresponding untransformed `countries-w-univ` and `countries-wo-univ` tables.

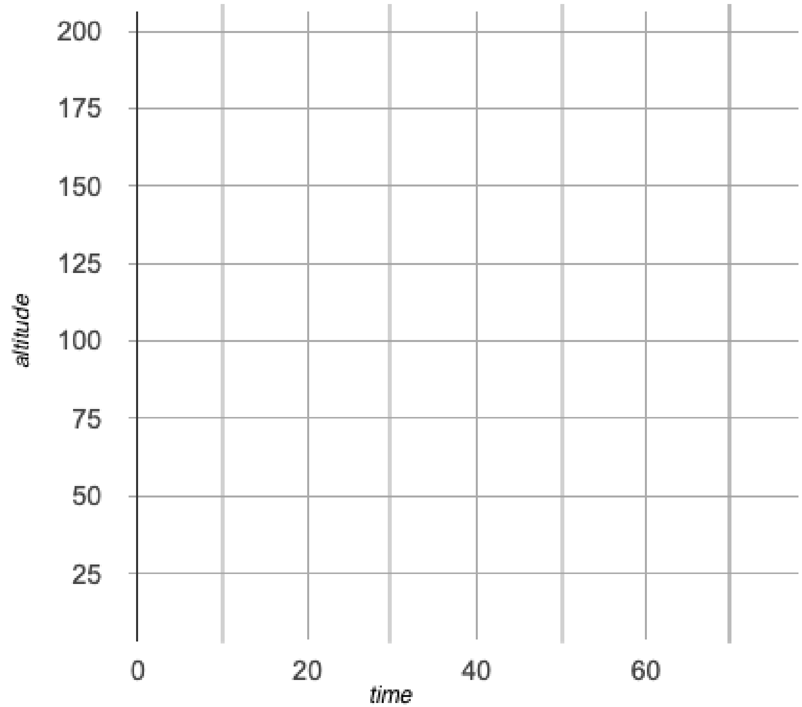
	With Universal Healthcare	Without Universal Healthcare
Linear Model	$f(x) =$ ______ (slope) x + ______ (y-intercept)	$f(x) =$ ______ (slope) x + ______ (y-intercept)
Logarithmic Model	$f(x) =$ ______ (log coefficient) $\log_{10}(x) +$ ______ (y-intercept)	$f(x) =$ ______ (log coefficient) $\log_{10}(x) +$ ______ (y-intercept)
S	______ years	______ years
What does each model predict the increase in `median-lifespan` to be for each 10x increase in `pc-gdp`?		
predicted increase	______ years	______ years

7) Was the relationship stronger for `countries-w-univ` or `countries-wo-univ`? ______________________

8) For which table is `pc-gdp` expected to have a bigger impact on `median-lifespan`? ______________________

Exploring Periodic Data

`time` (minutes)	`altitude` (feet)
0	5.0
5	55.0
10	154.9
15	205.0
20	155.2
25	55.2
30	5.0
35	54.7
40	154.6
45	205.0
50	155.5
55	55.5
60	5.0



1) What do you **Notice** about the data in the table? ______________________________

____

____

2) What do you **Wonder**? ______________________________

____

____

3) The ride goes from ____________ feet to ____________ feet in ________ minutes.
(lowest point) (highest point)

4) The ride does a **full cycle** in ____________ minutes. *(A full cycle can either be measured from high-point to high-point or low-point to low-point.)*

5) Plot each of the points in the table (left) on the coordinate plane (right) to create a ***scatter plot***.

6) Working from left to right, *connect the dots* one pair at a time using straight lines. This will create a data visualization known as a **line-graph**.

7) Draw a dotted horizontal line on the graph, precisely halfway between the highest and lowest point. What is its altitude? ____________ feet.

8) Describe the relationship you see between `time` and `altitude`. (Is it linear, quadratic, exponential, etc.?) ______________________________

____

____

9) What kind of ride do you think the teacher was on, and why? ______________________________

____

____

Reasoning about Unit Clocks

A *unit clock* (shown below) has a radius of 1, and is centered at the origin $(0, 0)$. As time passes, the point (A, B) rotates around the circle.

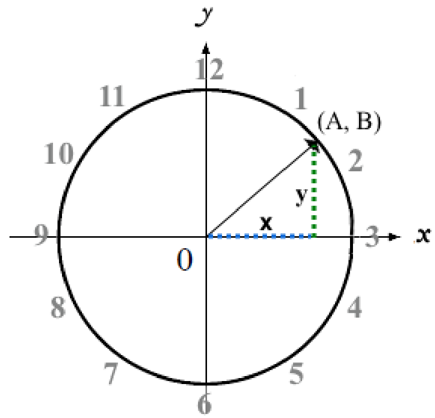
A and B, around the clock

1) The line connecting (A,B) to the origin is the hypotenuse of a right triangle. How long is this line, no matter what time it is? _______________

The tables below show the values of A (left table) and B (right table) at different times.

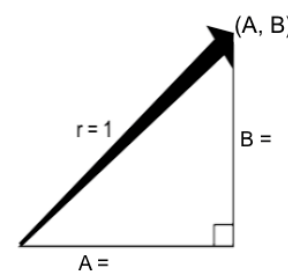
2) The values for 12, 3, and 6 o'clock are already shown in the tables below. Fill in the values of A and B at 9 o'clock.

Time	A
12:00	0
1:30	
3:00	1
4:30	
6:00	0
7:30	
9:00	
10:30	
12:00	0



Time	B
12:00	1
1:30	
3:00	0
4:30	
6:00	-1
7:30	
9:00	
10:30	
12:00	1

3) On the unit clock above (and the right triangle to the right) the hand is pointing to (A, B) at **1:30**, when $A = B$. Calculate the lengths of A and B in the space below. Then label them on the right triangle diagram.



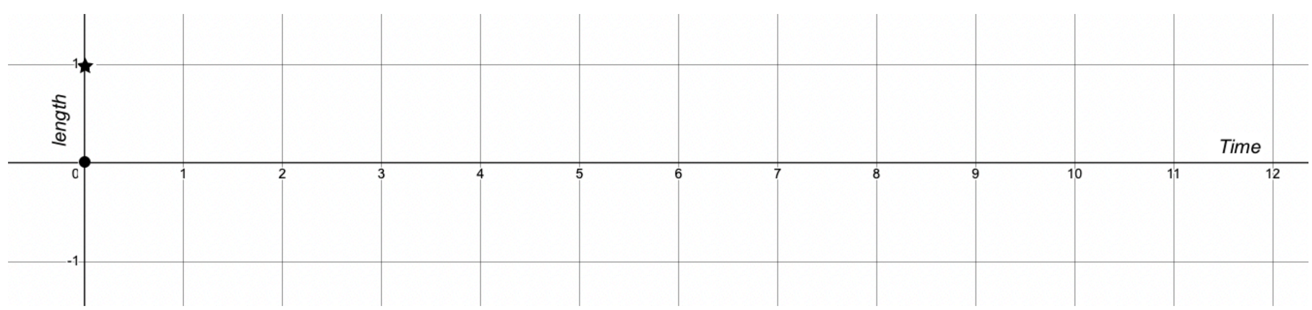
$A^2 + B^2 = 1$ and $A = B$, so…

4) Fill in the rest of the table with values of A and B at 4:30, 7:30, and 10:30.

A and B, over time

5) In the graph below, draw a **dot** for the coordinates (*time* , A) in each row of the table. Connect them from left-to-right, to form a curve.

6) In the graph below, draw a **star** for the coordinates (*time* , B) in each row of the table. Connect them from left-to-right, to form a curve.



Unit Clock Starter File

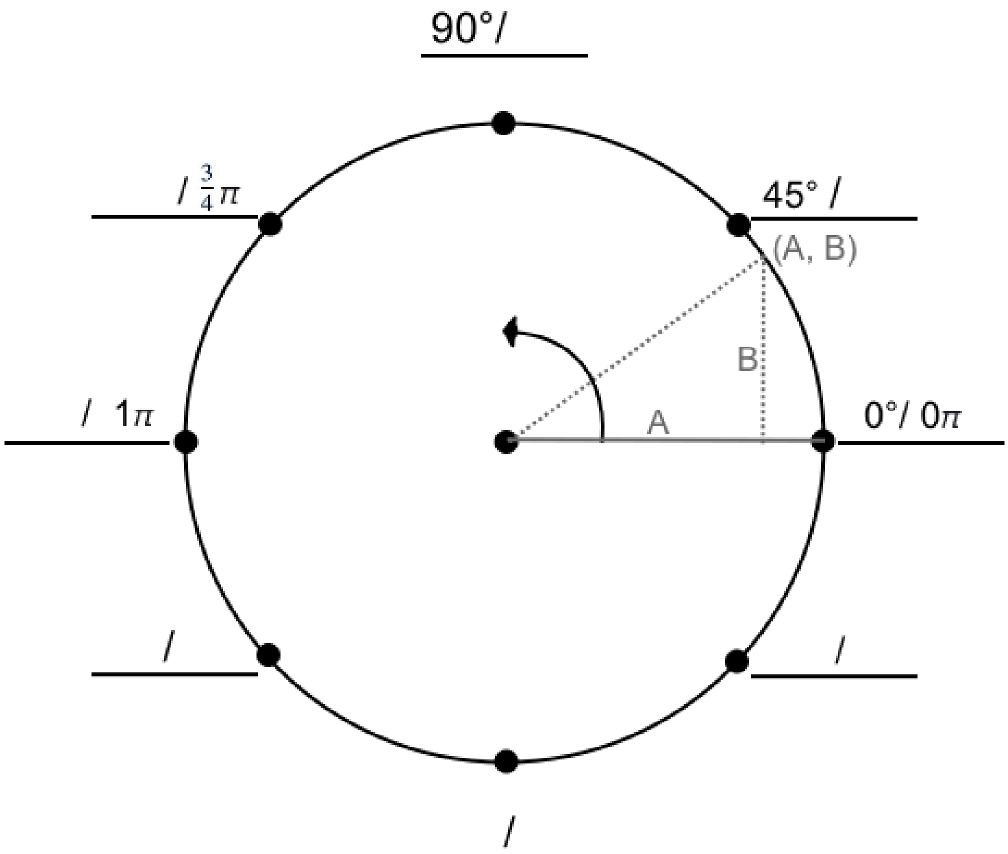
Open the Unit Clock Starter File. The questions below refer to the animation you'll see when you click "Run"

7) The **green** curve measures… ____

8) The **blue** curve measures… ____

Converting Between Angles

1) In the circle below, fill in the blanks to label the number of degrees and radians at each point.



2) **Use Pyret's `sin` and `cos` functions** to complete the table below.
Note: These function use radians not degrees.

Angle (θ)	`sin(θ)`	`cos(θ)`
0° / $0\,\pi$		
45° / 0.25π	`sin(0.25 * PI) =`	
90° / 0.50π		
135° / 0.75π		
180° / 1.00π		
225° / 1.25π		
270° / 1.50π		
315° / 1.75π		

3) Which function computes the **horizontal** leg of the right triangle (A)? ______

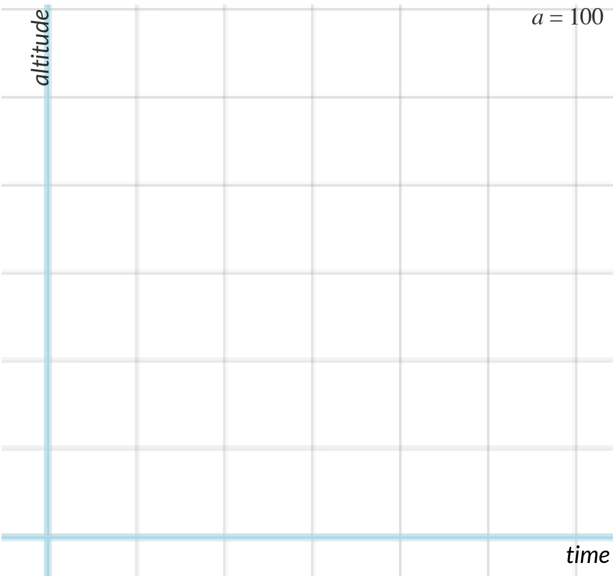
4) Which function computes the **vertical** leg of the right triangle (B)? ______

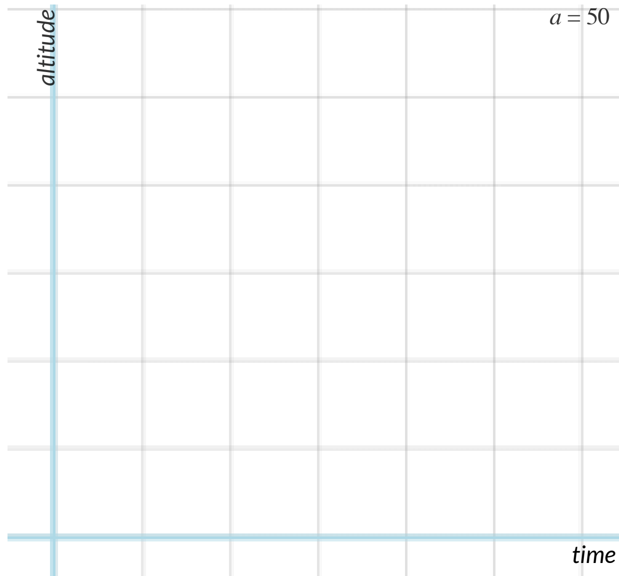
Graphing Periodic Models: Amplitude (a)

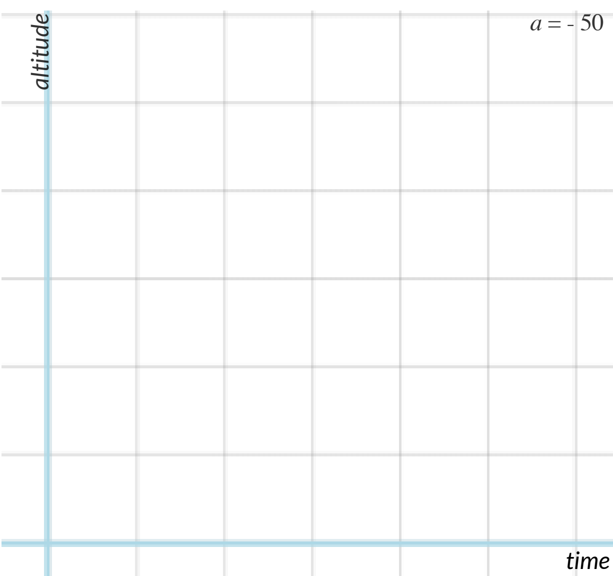
The standard form of periodic models is $f(x) = a\ sin(b \cdot (x - h)) + k$. Let's explore the role of ***amplitude*** a in periodic functions! *Open the Desmos File **Exploring Periodic Functions** to Slide 1: **Modeling the Ferris Wheel Dataset (sine)**. You should see four sliders for* a , b , h , *and* k .

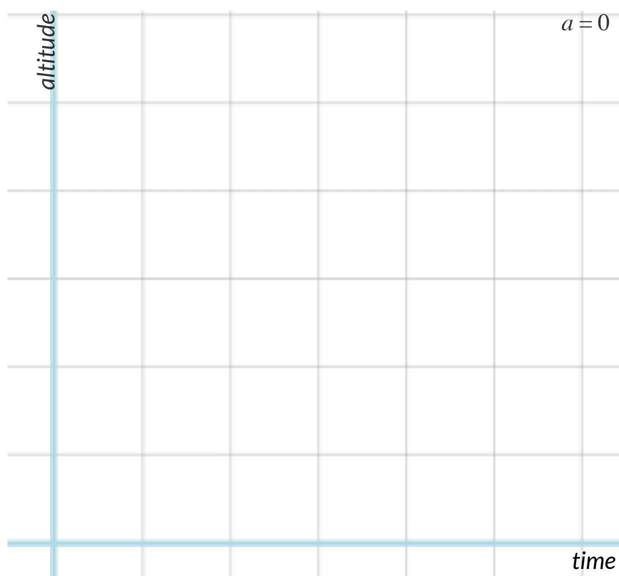
1) Adjust the sliders to fit the data as best you can, then record your model settings: __________, __________, __________ and __________.
a b h k

2) Changing **ONLY** the slider for a , experiment with values at 100, 50, -50, and 0, graphing each curve below. **For each curve, label the coordinates at time=** 15**,** 30**, and** 45**.**









3) What does a tell us about a periodic function? ______________________________

____

The distance between two adjacent *peaks* or *valleys* is called the *period*: the interval over which the pattern repeats itself.

4) What effect does changing a have on the ***period*** of a periodic function? ______________________________

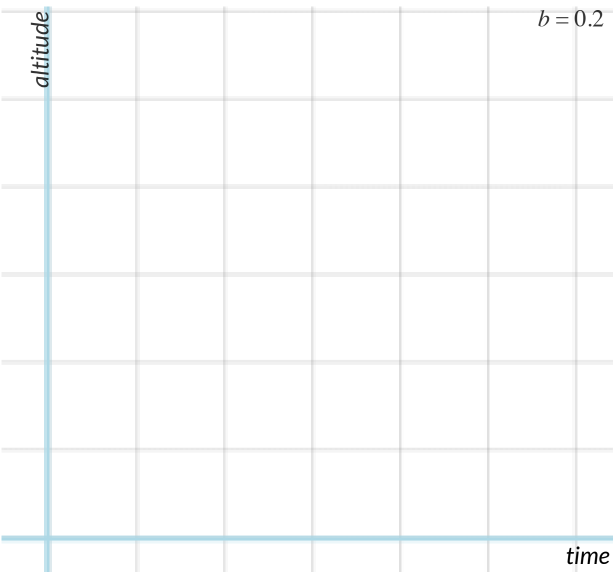
Graphing Periodic Models: Frequency (*b*)

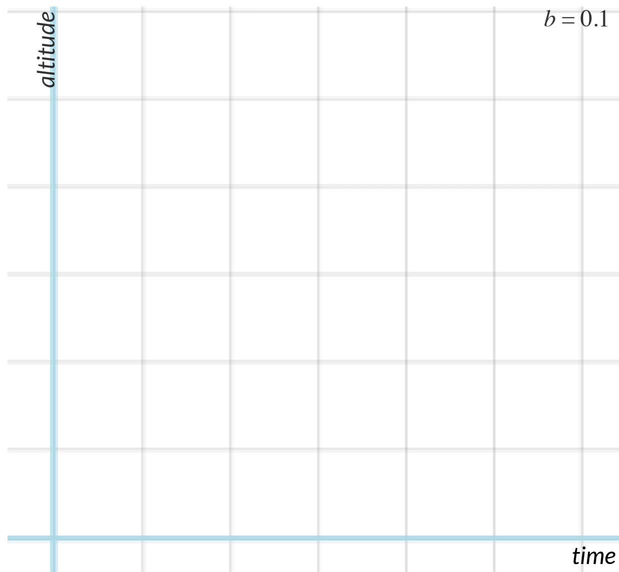
The standard form of a periodic model is $f(x) = a\ sin(b \cdot (x - h)) + k$. On this page, we'll explore the role of ***amplitude*** a in periodic functions. *Open the Desmos File* ***Exploring Periodic Functions****. You should be on* ***Slide 1: Modeling the Ferris Wheel Dataset (sine)*** *and see four sliders for* a, b, h, *and* k.

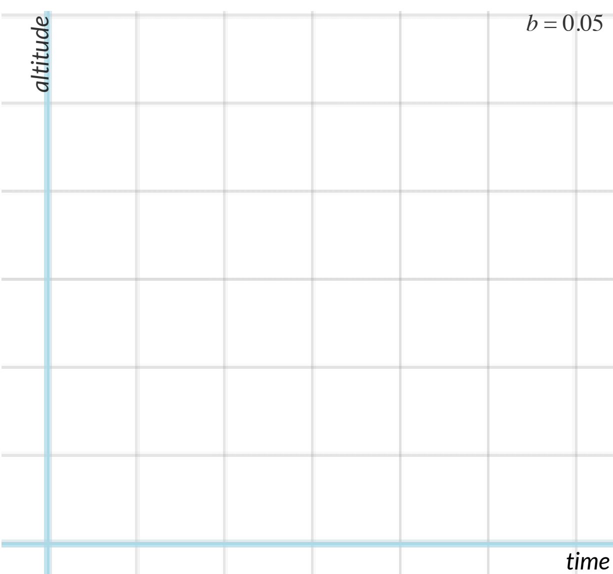
1) Adjust the sliders to fit the data as best you can, then record your model settings: __________, __________, __________ and __________.
a *b* *h* *k*

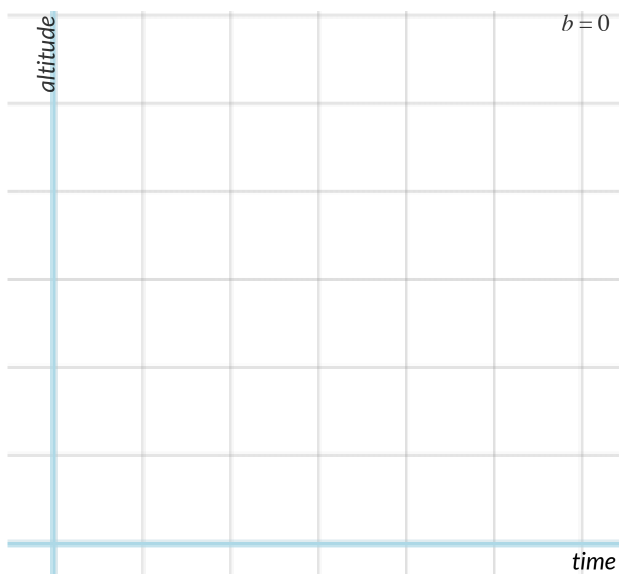
2) Click on one of the ***peaks*** (highest-points) on the graph of your periodic function. Desmos will add a gray dot to *all* of the peaks.

3) Leaving the other sliders where they best fit the data, change **ONLY** the slider for b, experimenting with values at 0.2, 0.1, 0.05, and 0, graphing each curve below.
For each curve, label two adjacent peaks.









The distance between two adjacent *peaks* or *valleys* is called the *period*: the interval over which the pattern repeats itself.

4) What is the ***period*** when $b = 0.2$? __________ when $b = 0.1$? __________ When $b = 0.5$? __________ ★ When $b = 0$? __________

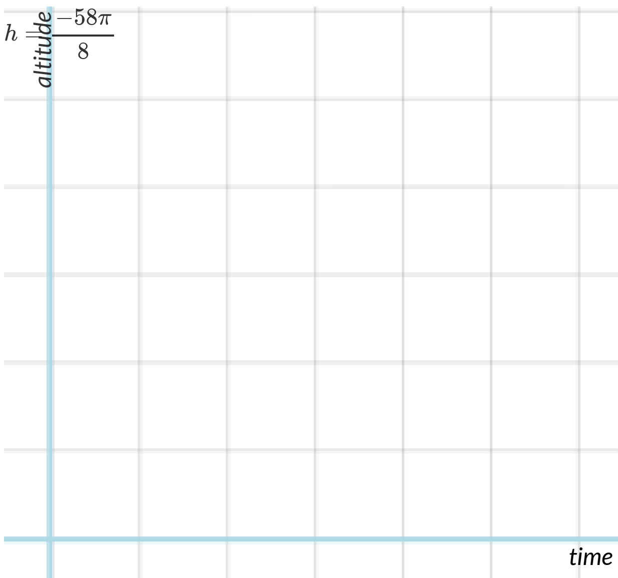
5) As the ***frequency*** (b) doubles, the ***period*** ____________________. As the ***frequency*** (b) gets cut in half, the ***period*** ____________________

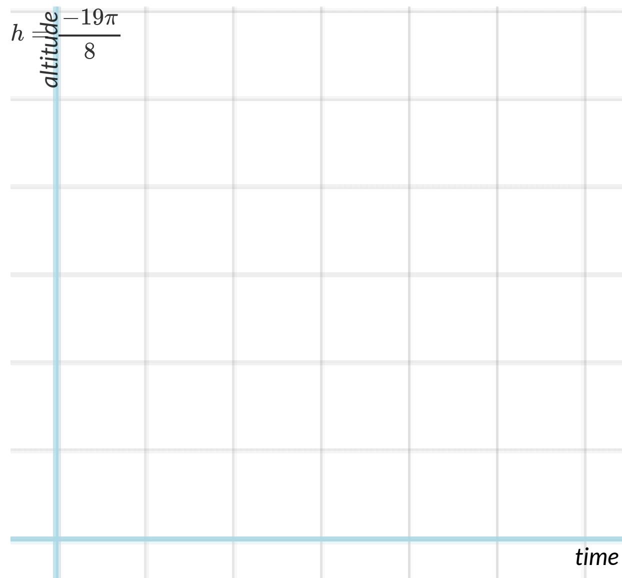
Graphing Periodic Models: Horizontal/Phase Shift (*h*)

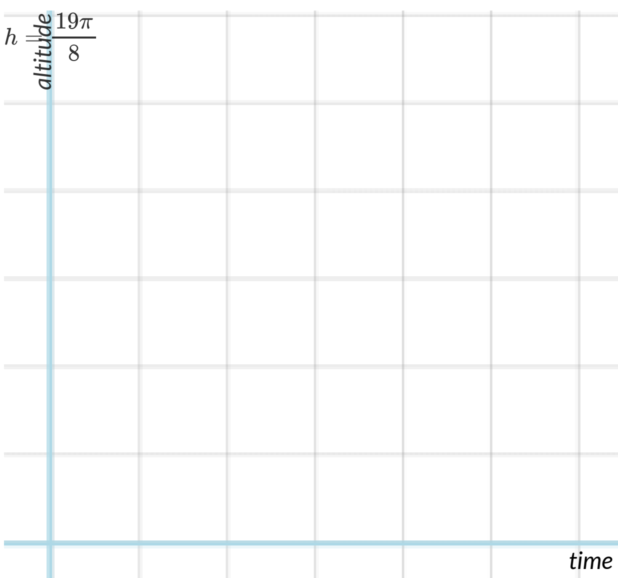
The standard form of a periodic model is $f(x) = a\,sin(b\cdot(x - h)) + k$. On this page, we'll explore the role of ***amplitude*** a in periodic functions. *Open the Desmos File **Exploring Periodic Functions**. You should be on **Slide 1: Modeling the Ferris Wheel Dataset (sine)** and see four sliders for* a, b, h, *and* k.

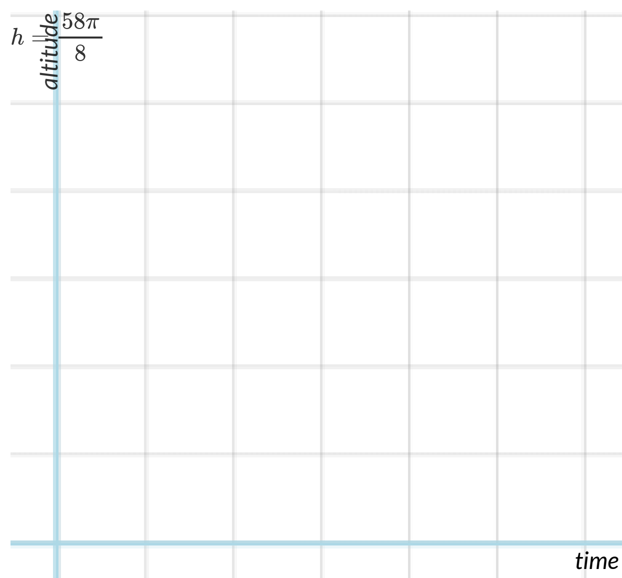
1) Adjust the sliders to fit the data as best you can, then record your model settings: __________ (a), __________ (b), __________ (h) and __________ (k).

2) Leaving the other sliders where they best fit the data, change **ONLY** the slider for h, experimenting with values at $\frac{-58\pi}{8}$, $\frac{-19\pi}{8}$, $\frac{19\pi}{8}$, and $\frac{58\pi}{8}$, graphing each curve below. **For each curve, label the coordinates at time= 15, 30, and 45.**









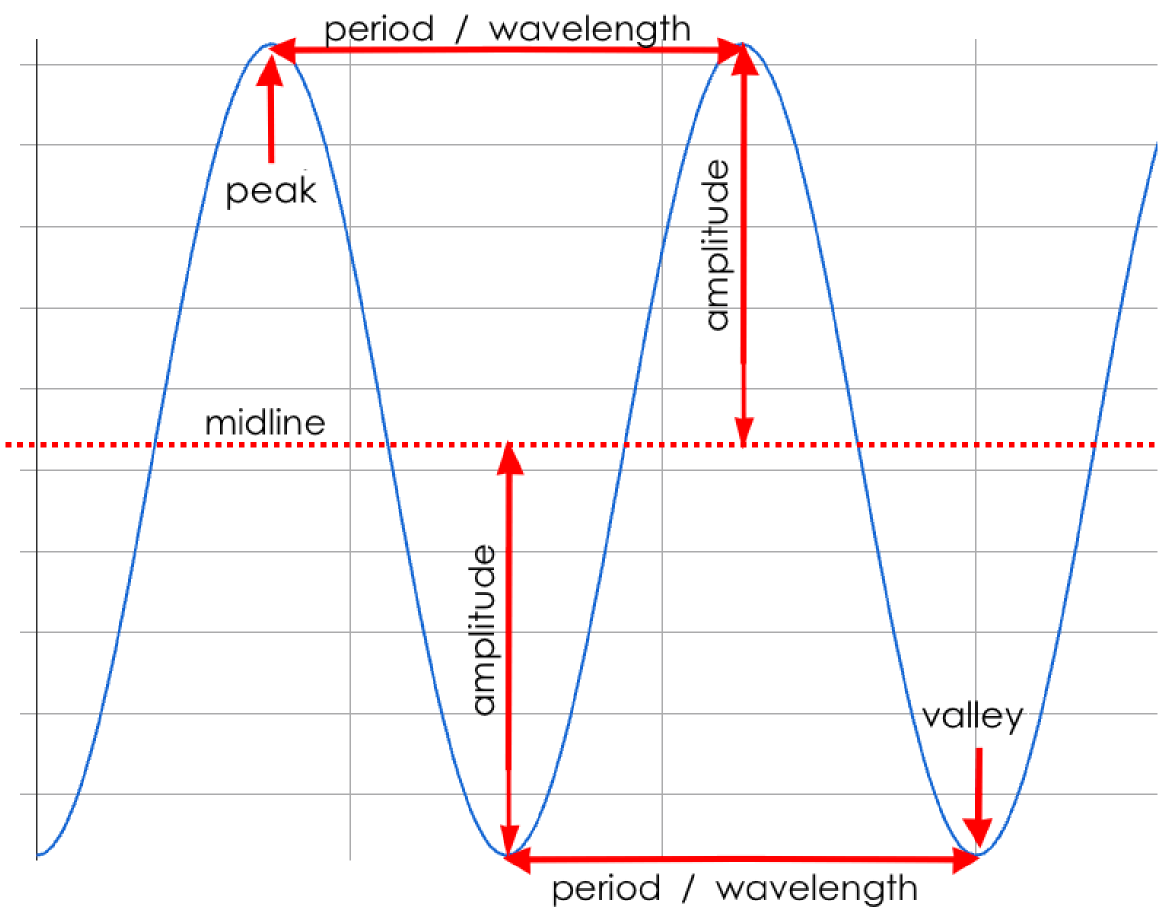
3) Describe the change in the graph when h *increases*: ____

4) Describe the change in the graph when h *decreases*: ____

5) The model fits as long as h changes by increments of $\frac{77\pi}{8}$, because ________________________________

____

Describing Periodic Functions



Based on what you can learn from the diagram, describe each of the terms below in your own words.

Peaks - ____

____

____

Valleys - ____

____

____

Period - ____

____

____

Midline - ____

____

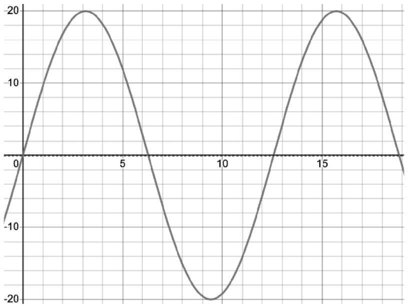
____

Amplitude - ____

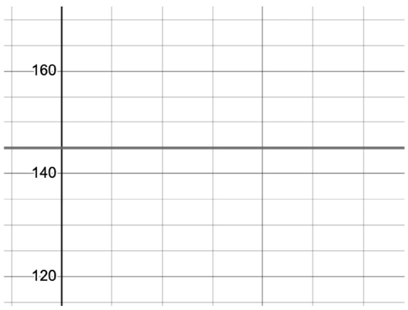
____

____

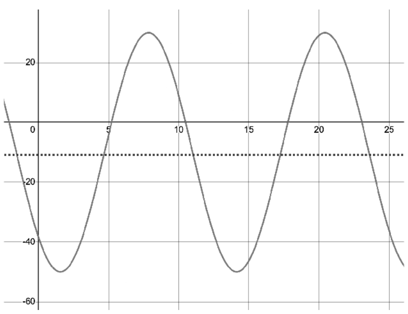
Matching Periodic Descriptions



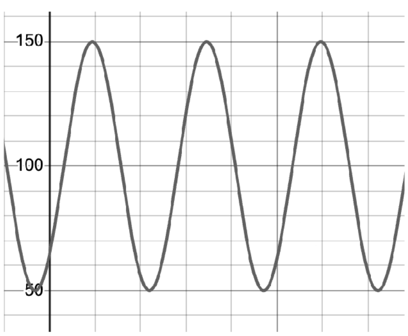
1 A This function has an amplitude of 50



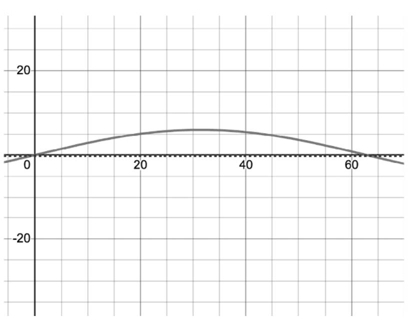
2 B This function has a midline of -10



3 C This function has peaks at 20 and valleys at -20



4 D This function has a wavelength of more than 60



5 E This function has peaks, valleys, and a midline at 145

Modeling the Ferris Wheel Data

Modeling with sin

For this section, use ***Slide 1: "Modeling the Ferris Wheel Dataset (sine)"*** *of the* ***Exploring Periodic Functions*** *Desmos File.* You'll find **the data from the Ferris wheel plotted in blue**, along with a basic periodic model of the form $f(x) = a\sin(b(x - h)) + k$.

1) Use the sliders to estimate the best periodic fit.

2) The ***peaks*** are at ______________ feet, ***valleys*** are at ______________ feet, ***midline*** is at ______________ feet and the ***amplitude*** is ______________ feet

3) The ***period*** of the data is ________________ minutes. If $\text{period} = \dfrac{2\pi}{\text{frequency}}$, what is the ***frequency***? ________________ cycles per minute

4) Adjust the slider for horizontal shift to find the best fit, then write the model below in Function and Pyret notation. **Write h in terms of pi.**

Function Notation	$f(x) =$ ______________ (amplitude) $\times$ sin (______________ (frequency) (x - ______________ (horizontal shift))) + ______________ (vertical shift)
Pyret Notation	`fun f(x): (` ______________ `* sin(` ______________ `* (x -` ______________ `))) +` ______________ `end`

Translating from sin to cos

For this section, advance to ***Slide 2: "Translating from sine to cosine"*** *of the* ***Exploring Periodic Functions*** *Desmos File.* You'll see a function f(x) defined here graphed in blue, which uses cos instead of sin.

5) Adjust the sliders so that the function q perfectly overlaps the function p. What is the value of a? __________ b? __________ k? ______

6) What was the value of h, expressed as a decimal? ________________ What was the value of h, expressed a fraction of pi? ________

7) In row 1 below, write the definition of q using the values of a, b, and k that you found earlier. **Change the definition of p** and write it in the second row, then show how q would need to change to overlap it.

Function using sin	**Function using cos**	**Vertical Shift k**
$p(x) = 10\ sin(1 \times (x - 0)) + 2$	$q(x) =$	
$p(x) =$	$q(x) =$	

8) Do you think that all basic cosine functions can be expressed as sine functions? Why or why not? ______________________________

____

Modeling with cos

For this section, advance to ***Slide 3: "Modeling the Ferris Wheel Dataset (cosine)"*** *of the* ***Exploring Periodic Functions*** *Desmos File.*

9) Translate your sin-based model to a cos-based one. Express the horizontal shift in terms of pi.

Function Notation	$g(x) =$ ______________ (amplitude) $\times$ cos (______________ (frequency) (x - ______________ (horizontal shift))) + ______________ (vertical shift)
Pyret Notation	`fun g(x): (` ______________ `* cos(` ______________ `* (x -` ______________ `))) +` ______________ `end`

Make Your Own Ferris Wheel!

The Ferris wheel is being upgraded!

Match each upgrade on the left to the property or properties that it will change on the right.

	A	*midline*
1) The wheel is being raised *higher*		
	B	*vertical shift*
2) The wheel is being made to spin *faster*	C	*frequency*
	D	*amplitude*
3) The wheel is being made *larger*		
	E	*period*

Design a New Wheel

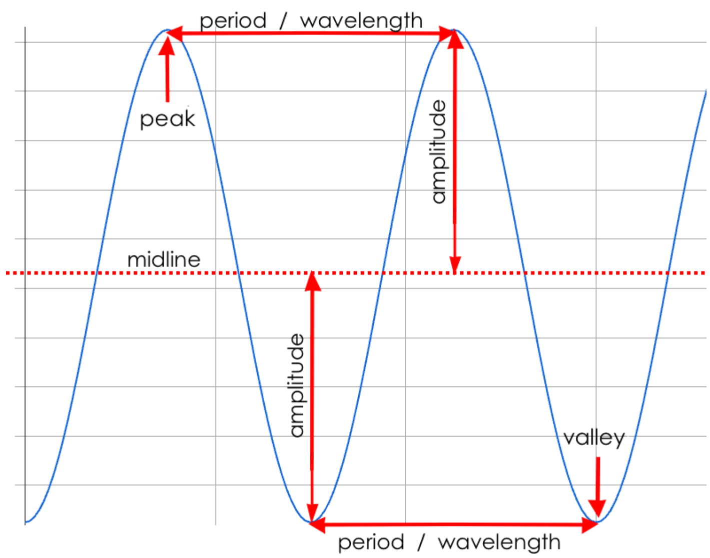
4) *Design your own Ferris wheel!* Fill in the table below, then **trade papers with someone else**.

Radius	Altitude of Center	Speed

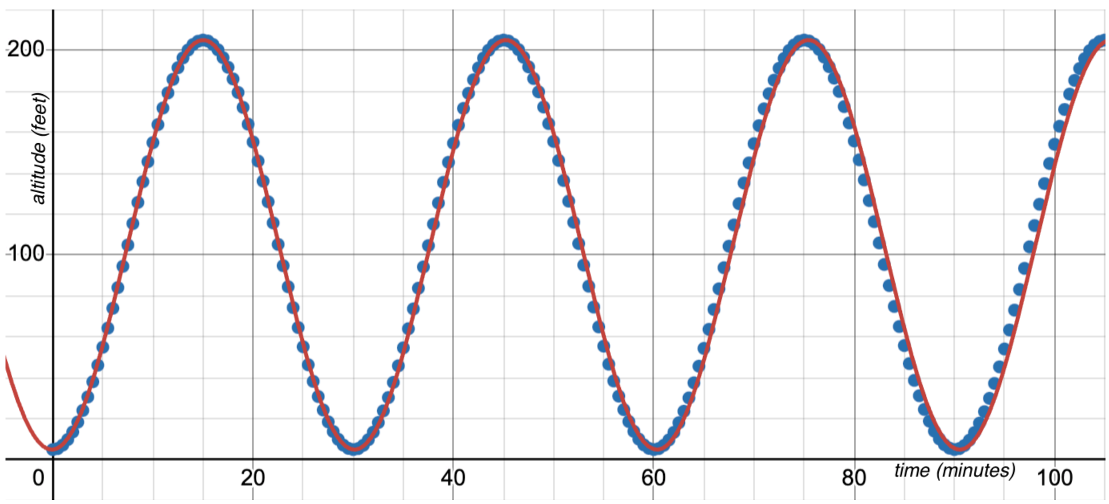
5) Based on the table above, what function will model the height of the wheel over time?

Function Notation	$f(x) = \underset{\text{amplitude}}{\underline{\qquad\qquad}} \times \sin(\underset{\text{frequency}}{\underline{\qquad\qquad}}(x - \underset{\text{horizontal shift}}{\underline{\qquad\qquad}})) + \underset{\text{vertical shift}}{\underline{\qquad\qquad}}$
Pyret Notation	`fun f(x): (________ * sin(________ * (x - __________))) + ________ end`

Describing Periodic Models



Ferris Wheel Data



1) The dataset appears to have ***peaks*** at (write the (x,y) coordinates) ______________________________

2) The dataset appears to have ***valleys*** at (write the (x,y) coordinates) ______________________________

3) The ***period*** of this dataset appears to be ______________________________

4) The ***midline*** of this dataset appears to be ______________________________

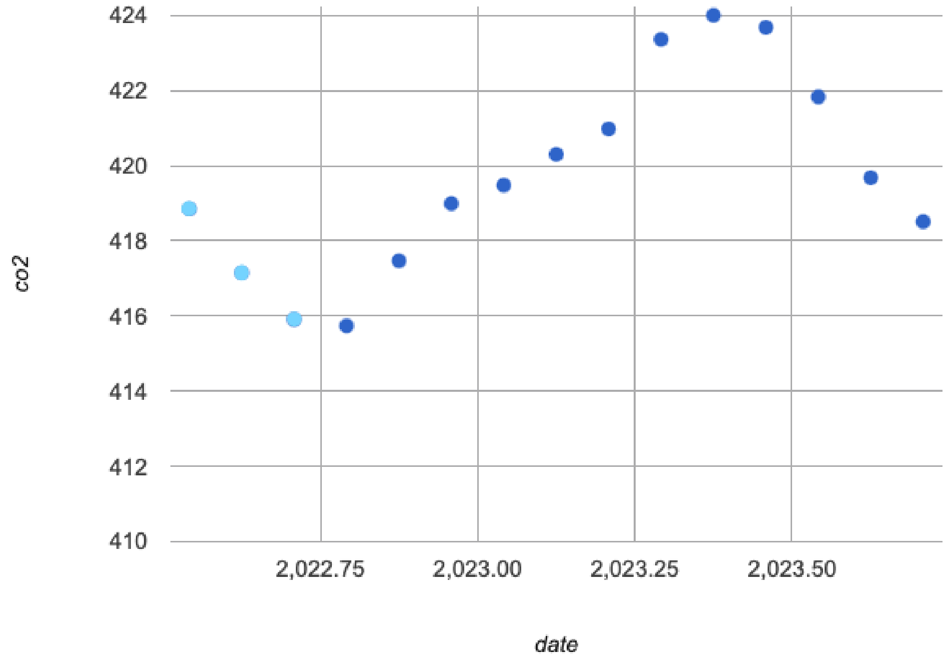
5) The ***amplitude*** of this dataset appears to be ______________________________

(optional)

Modeling Recent Carbon Dioxide Levels

*The data below was generated from the [Carbon Dioxide Starter File](). The **dark dots** show the amount of CO_2 in the atmosphere (in parts per million) recorded between December 2022 to November 2023. NOTE: the `date` column is the **decimal year** (so "June 15th, 2023" would be 2023.5).*

`date`	`co2` (ppm)
2022.708	415.91
2022.792	415.74
2022.875	417.47
2022.958	418.99
2023.042	419.48
2023.125	420.30
2023.208	420.98
2023.292	423.36
2023.375	424.00
2023.458	423.68
2023.542	421.83
2023.625	419.68
2023.708	418.51



1) Connect the **dark dots** on the scatter plot from left to right to form a line-graph of the data in `recent-table`.

2) The amount of $\mathrm{CO_2}$ varies from ______________ to ______________.
the lowest ***valley*** | the highest ***peak***

3) This represents a change in atmospheric $\mathrm{CO_2}$ of ______________________________ parts per million.
difference between highest and lowest

4) Find the ***amplitude*** (a) by cutting the vertical distance you calculated in half: ______________________________
$\mathrm{CO_2}$ in the atmosphere in parts per million

5) Draw the ***midline*** on the graph. *(It should be a horizontal line passing in the middle between the lowest **valley** and the highest **peak**.)*

6) The midline makes the ***vertical shift*** (k) visible. What is the ***vertical shift*** (k) of the model? ______________________________
$\mathrm{CO_2}$ in the atmosphere in parts per million

HINT: The vertical shift can also be calculated by adding the amplitude(a) to the valley.

7) The ***phase shift*** (h) is the decimal year when the data **first** crosses the midline. Estimate The ***phase shift*** (h) _______________
years

8) Calculate the ***period*** between the valleys: 2023.708 - 2022.792 = ____________________
date for the lowest values in 2023 | date for the lowest values in 2022 | years (round to the nearest full year)

9) *If* $\textbf{period} = \dfrac{2\pi}{\text{frequency}}$ *then* $\textbf{frequency} = \dfrac{2\pi}{\text{period}}$. How do you know that this statement is true?

____

____

10) What is the ***frequency*** (b) of this model? ______________________________________

Modeling Recent Carbon Dioxide Levels (continued)

This page relies on the [Carbon Dioxide Starter File](). Make sure you have it open on your computer!

Define Your Periodic Model

1) Define a periodic model using the values you computed for a, k, h and b on [Modeling Recent Carbon Dioxide Levels]().

Function Notation	$periodic\text{-}sin(x) = ($ ____________ (amplitude (a)) $\times$ $sin\ ($ ____________ (frequency (b)) $(x -$ ____________ (horizontal shift (h)) $))) +$ ____________ (vertical shift (k))
Pyret Notation	`fun periodic-sin(x): (_____________ * sin(_____________ * (x - _____________))) + _____________ end`

2) Then update the `periodic-sin` function definition in your starter file to match what you've just written.

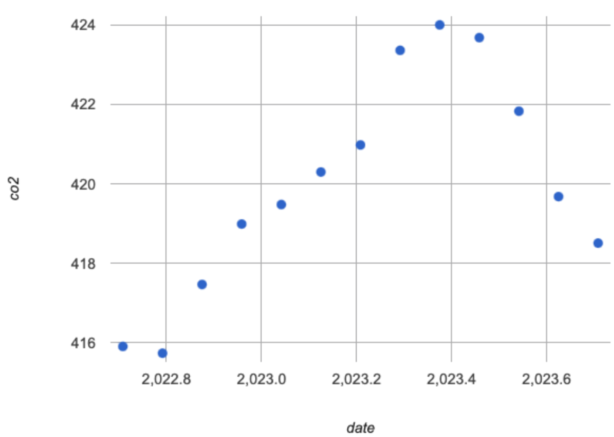
★ Try translating the definition you wrote for `periodic` so that it uses cosine instead of sine :

Function Notation	$periodic\text{-}cos(x) = ($ ____________ (amplitude (a)) $\times$ $cos\ ($ ____________ (frequency (b)) $(x -$ ____________ (horizontal shift (h)) $))) +$ ____________ (vertical shift (k))
Pyret Notation	`fun periodic-cos(x): (_____________ * cos(_____________ * (x - _____________))) + _____________ end`

Fit Your Periodic Model to the Data

3) Use `fit-model` to fit your `periodic` model to the data in the `recent-table`. What S-value do you get? ____________________

4) On the scatter plot below, make a sketch of what you see in pyret showing how the `periodic` model fits the data.



5) Then fill in the blanks below to tell us what the model means.

Between the end of 2022 and 2023, the amount of CO_2 in the air fluctuated between __________ (lowest) *and* __________ (highest) *parts-per-million.*

This pattern appears to be ***periodic****, with an amplitude of* __________ (amplitude) *rising and falling around a* ***midline*** *of* __________ (midline). *We expect this pattern to repeat every* _______ (period) ________ (x-units). *The error in the model is described by an* ***S-value*** *of about* _______ (S) ________ (y-units). *I* ____________________ (strongly agree, agree, disagree, strongly disagree) *that this model is a good fit considering that* __________ (y-variable) *in this dataset ranges from about* __________ (lowest y-value) *to* __________ (highest y-value).

Choosing the Best Model for the Data

Deconstructing a Model

1) The four functions below are defined in the Hybrid CO2 Models Starter File. Use `fit-model` to fit `periodic-cos` and `periodic-cos2` to the `recent-table` data.

```
fun periodic-cos(x): (4.13 * cos(6.28 * (x - 2023.35))) + 419.87 end
fun wave-cos(x):     (4.13 * cos(6.28 * (x - 2023.35)))          end
fun midline-cos(x):                                       419.87 end
fun periodic-cos2(x):     wave-cos(x)      +      midline-cos(x) end
```

2) Read these 4 functions carefully, and explain why `periodic-cos2` will produce the *same graph* as `periodic-cos`. ______________________________

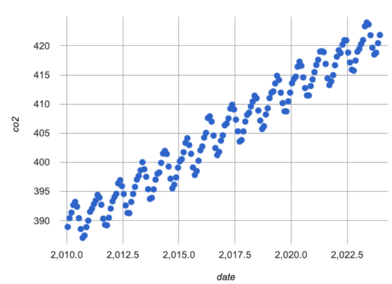
____

____

Review: Other Models We've Seen

	Linear	Quadratic	Exponential	Logarithmic
Sketch				
Key Characteristics				

What kind of model?



3) What do you Notice about this scatter plot? What do you Wonder? ______________________________

____

4) Which of the other models you described above would make the most sense to fit to this scatter plot? Why? ______________________________

____

____

Modeling Modern Carbon Dioxide Levels

This page focuses on the `modern-table` *of the [Hybrid CO2 Models Starter File](), which tracks atmospheric* CO_2 *(parts per million) from 2010-2023.*

Decomposing Your Periodic Model

Towards the bottom of the Definitions Area, find the section of the starter file where you're asked to "Define your periodic-sin functions…"

1) Define `periodic-sin` to be **the periodic model you found earlier**, for CO_2 levels from 2022-2023.

- You should already have defined it in Carbon Dioxide Starter File.
- You can also look at Modeling Recent Carbon Dioxide Levels (continued), the workbook page from the previous lesson.

2) Using the deconstruction of `periodic-cos` as your model, change the other three functions in this section to show how to separate the wave and midline of your `periodic-sin` model and define `periodic-sin2` using function composition.

Fitting the Optimal Linear Model

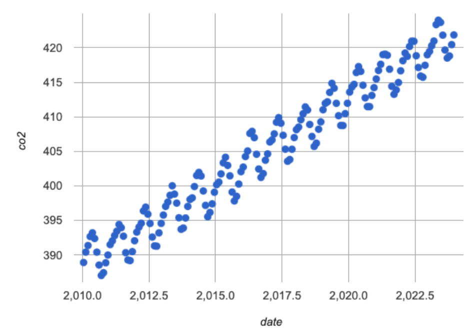
3) Use `lr-plot` to find the best linear model for the `modern-table`, and record the function below:

```
fun linear-modern(x): (_______________ * x) + _______________ end
```

4) Change the `linear-modern` function in the starter file to match the function above. Then use `fit-model` to fit it to the `modern-table`.

The ***S-value*** is: ____________

5) Sketch the model on the graph to the right.



Fitting your Periodic Model

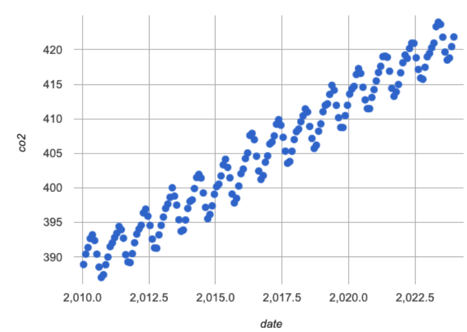
6) Use `fit-model` to fit `periodic-sin` to the data in the `modern-table`.

The ***S-value*** is: ____________

7) Sketch the model on the graph to the right. What would need to change about your model, to fit this data? ____

____

____



Imagining the *Best Possible* Model

8) Sketch the *best possible model* you can imagine for this data on the graph to the right, and describe it. Do parts of it look linear? Quadratic? Exponential? Logarithmic? Periodic?

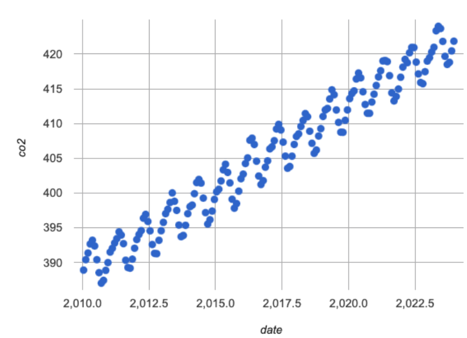
____

____

____

____

____



Building a Hybrid Model

Open your copy of [Hybrid CO2 Models Starter File] and click "Run".

Building a hybrid model for the `modern-table`

Both `periodic-sin` and `periodic-cos` models are built to follow a *horizontal* midline with the equation $f(x) = 419.87$.

1) What line do we wish the model would follow instead? ______________________________

2) Find `hybrid-modern` in the starter file and define it using function composition.
Hint: Like `periodic-sin2` this function will use `wave-sin`. What will it use instead of `midline-sin`?

3) Fit `hybrid-modern` to the `modern-table`, and describe what you see: ______________________________

____

____

Comparing Models

4) What ***S-value*** describes the expected error in our `hybrid-modern` model for this data? ____________ (*S*-value) ____________ (y-units)

5) What ***S-value*** describes the expected error in our `linear-modern` model for this data? ____________ (*S*-value) ____________ (y-units)

6) How much ________ (more / less) error do we expect in predictions made using `hybrid-modern` than with the `linear-modern` model?

Percent Change = $\frac{\text{Difference between the S-values}}{\text{S-value for linear model}} \times 100 = \underline{\quad}$

We expect ________ percent ________ (more / less) error from predictions made with `hybrid-modern` than with `linear-modern` model!

Interpreting the Hybrid Model

This model predicts that:

- *The overall trend will be a(n)* ____________ (increase/decrease) *of* ____________ (rate of change) ______________________ (x-variable) *every* ________ (x-units).
- *Within this trend, the data fluctuates following a periodic pattern that rises and falls around this* ***midline***.

  The wave has an amplitude of ____________ (amplitude) *and repeats every* ________ (period) ________ (x-units).

The error in the model is described by an ***S-value*** *of about* ________ (S) ________ (y-units). *I* ______________________ (strongly agree, agree, disagree, strongly disagree) *that this model is a good fit considering that* ____________ (y-variable) *in this dataset ranges from about* ____________ (lowest y-value) *to* ____________ (highest y-value).

More Models

Open your copy of [Hybrid CO2 Models Starter File] and click "Run".

Building a hybrid model for the entire `co2-table`

1) How well would you expect our `hybrid-modern` model to fit the data in the *full* `co2-table`, with data covering a span of 50 years?

____

2) Let's test it out. What ***S-value*** do you get? ____

At the bottom of the Definitions Area, find the section titled "HYBRID MODEL for the full co2-table".

3) Define `hybrid-all` as a model that fits the full dataset, referring back to `hybrid-modern` to help you think through which functions you will need to define in order to be able to compose your function definition.

4) Use `fit-model` to fit your new `hybrid-all` model to the `co2-table`. What is the ***S-value*** of `hybrid-all` with this data? __________

5) Compute the change in ***S-values*** between `hybrid-all` and `hybrid-modern`, when used with the `co2-table`:

$$\textit{Percent Change} \quad = \quad \frac{\text{Difference between the S-values}}{\text{S-value for linear model}} \times 100 \quad = \quad \underline{\qquad\qquad}$$

"For `co2-table`, we expect ________ percent __________ (more / less) error from predictions made with `hybrid-all` than with `hybrid-modern`."

Comparing *S*-values doesn't always make sense

You've had a lot of practice comparing the ***S-values*** of two different models on the same dataset (including what you just did on this page!), to quantify the error between them. **But can we compare the *S-values* for one model fit to *two different datasets?***

6) In the first row of the table below, we've fit the `periodic-cos` model to all three datasets.

	`recent-table`	**`modern-table`**	**`co2-table`**
S-value of `periodic-cos`	~1.2ppm	~17.76ppm	~55.89ppm
lowest y-value (CO_2 in ppm)	415.74ppm	387.03ppm	327.28ppm
highest y-value (CO_2 in ppm)	424ppm	424ppm	424ppm

The `periodic-cos` model was built from `recent-table`, *so there's no surprise* it was a good fit for `recent-table`! But for datasets that reach farther and farther back in time, that model fits worse and worse.

7) It's tempting to compare the S-values in this table, to quantify how much more error we'd expect from the `periodic-sin` model for the `co2-table`. **Why shouldn't we?**

____

____

____

★ Just as there's nothing to say that midlines have to be horizontal, there's also nothing to say that midlines need to be straight! If you look closely, the `co2-table`'s midline almost looks quadratic… can you come up with a `hybrid-curve` model for the `co2-table` that can beat the ***S-value*** you just found?

____

Linear Models

Slope-Intercept Form: $f(x) = \underbrace{m}_{\text{slope}}(x) + \underbrace{k}_{\text{vertical shift}}$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Standard Form: $A(x) + B(y) = C$

Quadratic Models

Vertex Form: $f(x) = \underbrace{a}_{\text{quadratic coefficient}}(x - \underbrace{h}_{\text{horizontal shift}})^2 + \underbrace{k}_{\text{vertical shift}}$

Factored Form: $f(x) = a(x - r_1)(x - r_2)$

Standard Form: $f(x) = a(x)^2 + b(x) + c$

Exponential & Logarithmic Models

$$f(x) = \underbrace{a}_{\text{initial value}} (\underbrace{b}_{\text{base}})^x + \underbrace{k}_{\text{vertical shift}}$$

Logarithmic Models

$$f(x) = \underbrace{\text{a}}_{\text{log coefficient}}\ log_{\underbrace{b}_{\text{base}}}(x) + \underbrace{\text{k}}_{\text{vertical shift}}$$

Periodic Models

Sine

$$f(x) = \underset{\text{amplitude}}{\underline{\quad a \quad}} \sin(\underset{\text{frequency}}{\underline{\quad b \quad}} ((\quad x \quad - \underset{\text{horizontal shift}}{\underline{\quad h \quad}})) \quad + \underset{\text{vertical shift}}{\underline{\quad k \quad}}$$

Cosine

$$f(x) = \underset{\text{amplitude}}{\underline{\quad a \quad}} \cos(\underset{\text{frequency}}{\underline{\quad b \quad}} (\quad x \quad - \underset{\text{horizontal shift}}{\underline{\quad h \quad}})) \quad + \underset{\text{vertical shift}}{\underline{\quad k \quad}}$$

Contracts for Algebra 2

Contracts tell us how to use a function, by telling us three important things:

1. The **Name**
2. The **Domain** of the function - what kinds of inputs do we need to give the function, and how many?
3. The **Range** of the function - what kind of output will the function give us back?

For example: The contract `triangle :: (Number, String, String) -> Image` tells us that the name of the function is `triangle`, it needs three inputs (a Number and two Strings), and it produces an Image.
With these three pieces of information, we know that typing `triangle(20, "solid", "green")` will evaluate to an Image.

Name		Domain		Range
`# build-column`	::	(Table *table-name*, String *column*, (Row -> Value) *builder-function*)	->	Table
build-column(animals-table, "kilos", kilograms)				
`# circle`	::	(Number *radius*, String *fill-style*, String *color*)	->	Image
circle(50, "solid", "purple")				
`# ellipse`	::	(Number *width*, Number *height*, String *fill-style*, String *color*)	->	Image
ellipse(100, 50, "outline", "orange")				
`# filter`	::	(Table *table-name*, (Row -> Boolean) *tester-function*)	->	Table
filter(animals-table, is-dog)				
`# fit-model`	::	(Table *table-name*, String *labels*, String *xs*, String *ys*, (Num -> Num) *model-function*)	->	Image
fit-model(animals-table, "name", "pounds","weeks", f)				
`# isosceles-triangle`	::	(Number *size*, Number *vertex-angle*, String *fill-style*, String *color*)	->	Image
isosceles-triangle(50, 20, "solid", "grey")				
`# log`	::	(Number *n*)	->	Number
log(4)				
`# log-base`	::	(Number *base*, Number *n*)	->	Number
log-base(2, 4)				
`# overlay`	::	(Image *top*, Image *bottom*)	->	Image
overlay(circle(10, "solid", "black"), square(50, "solid", "red"))				
`# radial-star`	::	(Num *points*, Num *outer*, Num *inner*, Str *fill-style*, Str *color*)	->	Image
radial-star(6, 20, 50, "solid", "red")				
`# rectangle`	::	(Number *width*, Number *height*, String *fill-style*, String *color*)	->	Image
rectangle(100, 50, "outline", "green")				
`# regular-polygon`	::	(Number *size*, Number *vertices*, String *fill-style*, String *color*)	->	Image
regular-polygon(25,5, "solid", "purple")				

Name		Domain		Range
# rhombus	::	(Number [size], Number [top-angle], String [fill-style], String [color])	->	Image
rhombus(100, 45, "outline", "pink")				
# right-triangle	::	(Number [leg1], Number [leg2], String [fill-style], String [color])	->	Image
right-triangle(50, 60, "outline", "blue")				
# rotate	::	(Number [degrees], Image [img])	->	Image
rotate(45, star(50, "solid", "dark-blue"))				
# S	::	(Table [table-name], String [xs], String [ys], (Num -> Num) [model-function])	->	Number
S(animals-table, "name", "pounds","weeks", f)				
# scatter-plot	::	(Table [table-name], String [labels], String [xs], String [ys])	->	Image
scatter-plot(animals-table, "name", "pounds","weeks")				
# sqr	::	(Number)	->	Number
sqr(4)				
# sqrt	::	(Number)	->	Number
sqrt(4)				
# square	::	(Number [size], String [fill-style], String [color])	->	Image
square(50, "solid", "red")				
# star	::	(Number [radius], String [fill-style], String [color])	->	Image
star(50, "solid", "red")				
# star-polygon	::	(Number [size], Number [point-count], Number [step-count], String [fill-style], String [color])	->	Image
star-polygon(100, 10, 3 ,"outline", "red")				
# string-contains	::	(String [haystack], String [needle])	->	Boolean
string-contains("hotdog", "dog")				
# string-length	::	(String)	->	Number
string-length("rainbow")				
# text	::	(String [message], Number [size], String [color])	->	Image
text("Zari", 85, "orange")				
# triangle	::	(Number [size], String [fill-style], String [color])	->	Image
triangle(50, "solid", "fuchsia")				
# triangle-asa	::	(Number [top-left-angle], Number [left-side], Number [bottom-angle], String [fill-style], String [color])	->	Image
triangle-asa(90, 200, 10, "solid", "purple")				
# triangle-sas	::	(Number [bottom-R-side], Number [top-R-side], Number [top-side], String [fill-style], String [color])	->	Image
triangle-sas(50, 20, 70, "outline", "dark-green")				

	::		->

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