Evaluating Exponent Expressions with Negative Values

(Also available in Pyret)

Students use Circles of Evaluation to visualize the structure of expressions with exponents and negative values.

Lesson Goals	Students will be able to… • Determine the order in which we negate and apply exponents in expressions both with and without grouping symbols. • Evaluate expressions involving exponents and negative values. • Evaluate exponent expressions with negatives and variables.
Student-facing Lesson Goals	• Let's use Circles of Evaluation to visualize the structure of expressions with exponents and negative values.
Prerequisites	• Simple Data Types • Translating Between Words and Math • Contracts • Equivalence • The Commutative Property • The Associative Property • Variables • Evaluating Exponents
Materials	• Lesson Glossary • Lesson Slides • Printable Lesson Plan (a PDF of this web page)

Exponent Expressions with Negatives

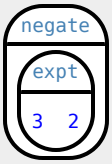
Overview

Students use Circles of Evaluation to visualize expressions with ***exponents*** and negatives.

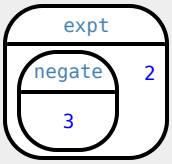
Launch

- What do you Notice about the two Circles of Evaluation below? What do you Wonder?

$-(3^2)$



$(-3)^2$



- Possible Noticings: One expression is negative, the other is positive. Both expressions have one Circle inside another Circle.
- Possible Wonderings: Will I always get a negative outcome if there are no parentheses? Will I always get a positive outcome if there are parentheses?
  - *-3^2 has the **same** Circle of Evaluation as one of the expressions, above. Which expression do you think is equivalent to -3^2?*
- Student responses will vary. The Circle of Evaluation on the left is equivalent. In an expression like the one above, we apply the exponent before we negate. Negating can be viewed as multiplying by -1. Exponentiation precedes multiplication.

Investigate

Today, we are going to apply exponents in two different contexts:

- Sometimes, we apply an exponent to a *negative value*.
- Sometimes, we apply an exponent… and *then* we negate the outcome.

Circles of Evaluation will help us visualize the difference between these two scenarios.

- On True or False? Exponents and Negatives, draw two Circles of Evaluation to represent each equation. Then, use your Circles of Evaluation to determine if the equation is true or false. The first one is done for you.
- When you're finished, look over your work to see what patterns you notice and then respond to the questions at the bottom of the page.

Synthesize

- An exponent expression has a negative base *inside* parentheses. Which do we apply first: `expt` or `negate`?
  - *First, we apply* `negate`, *then* `expt`.
- An exponent expression has a negative base and no grouping symbols. Which do we apply first: `expt` or `negate`?
  - *First, we apply* `expt`, *then* `negate`.

- Why were *some* of the equations on the page true, but not all of them?
    - *Possible response: Exponent expressions with negatives but no grouping symbols were always negative, because the negation came last. Exponent expressions with grouping symbols were sometimes negative and sometimes positive, depending on how many times we multiplied the base by itself.*

Variable Exponent Expressions with Negatives

Overview

Students extend their knowledge of Circles of Evalution to evaluate expressions with ***variables***, exponents, and negatives.

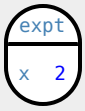
Launch

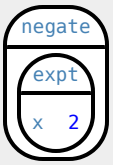
Invite students to consider the role of variables in expressions with negatives and exponents by facilitating a discussion using the prompts below.

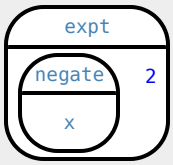
- Is - m sometimes, never, or always negative? Explain your response.
  - *If the value of m is negative, then - m is positive. If the value of m is positive, then - m is negative. So, - m can be positive, negative, or zero.*
- Is - m^2 sometimes, never, or always negative? Explain your response.
  - *First we apply the exponent, then we negate. The outcome is always negative (unless m is zero).*

Investigate

- What do you Notice and what do you Wonder about the three Circles of Evaluation below?







- Possible responses: They all include x and 2. Two of the three Circles of Evaluation include nested Circles. The functions used are `expt` and `negate`. I wonder if they are equivalent or not.
  - *Write an algebraic expression to correspond with each Circle of Evaluation.*
- Circle 1: x^2
- Circle 2: - x^2
- Circle 3: $(-x)^2$
- Debrief with students to confirm that they can represent exponent expressions with Circles of Evaluation before moving on to the next activity, where students will apply this understanding.

- On Evaluate and Compare, create a Circle of Evaluation for each expression provided. Once you have drawn a Circle of Evaluation, use it to help you evaluate the expression *twice* - once for $x = 5$ and once for $x = -5$.
- When did the expressions produce the same outcome for both $x = 5$ and $x = -5$?
  - The expressions produced the same outcome for 1 and 2, the expressions that involved squaring rather than cubing._
- When you're done, turn to Variable Expressions with Exponents and Negatives. Draw a Circle of Evaluation to represent the expression, then evaluate using the assigned value.

Synthesize

Was it more challenging to work with *algebraic* exponent expressions (with variables) than it was to work with numeric exponent expressions? Why or why not?