Name: _____

Algebra 2

Fall 2024 Student Workbook - Pyret Edition

Workbook v0.9-beta

Brought to you by the Bootstrap team:

- Emmanuel Schanzer
- Kathi Fisler
- Shriram Krishnamurthi
- Dorai Sitaram
- Joe Politz
- Ben Lerner
- Nancy Pfenning
- Flannery Denny
- Rachel Tabak
- Anders Hulleberg

Introduction to Computational Data Science

Many important questions ("What's the best restaurant in town?", "Is this law good for citizens?", etc.) are answered with *data*.
Data Scientists try to answer these questions by writing *programs that ask questions about data*.

Data of all types can be organized into **Tables**.

- Every Table has a **header row** and some number of **data rows**.
- **Quantitative data** is numeric and measures *an amount*, such as a person's height, a score on a test, distance, etc. A list of quantitative data can be ordered from smallest to largest.
- **Categorical data** is data that specifies *qualities*, such as sex, eye color, country of origin, etc. Categorical data is not subject to the laws of arithmetic — for example, we cannot take the "average" of a list of colors.

Categorical or Quantitative?

- **Quantitative data** measures an *amount* and can be ordered from smallest to largest.
- **Categorical data** specifies *qualities* and is not subject to the laws of arithmetic — for example, we cannot take the "average" of a list of colors.

  Note: Numbers can sometimes be categorical rather than quantitative!

For each piece of data below, circle whether it is **Categorical** or **Quantitative**.

1)	Hair color	categorical	quantitative
2)	Age	categorical	quantitative
3)	ZIP Code	categorical	quantitative
4)	Date	categorical	quantitative
5)	Height	categorical	quantitative
6)	Sex	categorical	quantitative
7)	Street Name	categorical	quantitative

For each question, circle whether it will be answered by **Categorical** or **Quantitative** data.

8)	We'd like to find out the average price of cars in a lot.	categorical	quantitative
9)	We'd like to find out the most popular color for cars.	categorical	quantitative
10)	We'd like to find out which puppy is the youngest.	categorical	quantitative
11)	We'd like to find out which cats have been fixed.	categorical	quantitative
12)	We want to know which people have a ZIP code of 02907.	categorical	quantitative

★ We decide to sort the animals in *ascending order* (smallest-to-largest) by `age`. Then we sort the table in *alphabetical order* (A-to-Z) by `name`.

Does that mean `name` is a quantitative column? Why or why not? _____

Questions and Column Descriptions

1) Take some time to look through the Animals Dataset. What stands out to you? Which animals are interesting? What patterns do you notice? Put your observations in the **Notice** column below.

2) Do any of these observations make you wonder? If so, write your question next to the observation in the **Wonder** column. If not, think of another question to write down.

Notice	Wonder	Answered by this dataset?
I notice that *Kujo took a long time to be adopted*	*Is it because he was so big?*	*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*
I notice that		*Yes No*

Describe the table, and two of the columns, by filling in the blanks below.

1. This dataset is about _____; it contains _____ data rows.
2. Some of the columns are:

   a. _____ (column name), which contains _____ (*categorical or quantitative*) data. Some example values are:

   _____.

   b. _____ (column name), which contains _____ (*categorical or quantitative*) data. Some example values are:

   _____.

What Questions Can You Answer with the Given Data?

The following is a dataset of a bicycle rider's training rides.

date	miles	time (w/stops)	weather	average speed	max speed
04/10/2018	10	44	"cloudy"	13	30
05/30/2018	15	66	"sunny"	13.5	22
06/12/2018	12	61	"rainy"	11.2	25
07/04/2018	24	103	"sunny"	14	26
07/12/2018	24	120	"windy"	12.5	26

1) Decide whether each questions below *can* or *cannot* be answered with the given data and circle your selection.

Question	Answered by this dataset?	
How many miles did the cyclist ride June 12th?	*Yes*	*No*
What tire pressure produces the highest average speed?	*Yes*	*No*
What is the average time it takes this cyclist to ride 1 mi?	*Yes*	*No*
Does this cyclist ride slower when it is rainy?	*Yes*	*No*
Does this cyclist ride faster when they are late to an appointment?	*Yes*	*No*
How many miles has the cyclist ridden in total as part of their training?	*Yes*	*No*

2) In the space provided below each question, explain *how* you could answer the question using the data or *why you cannot* answer the question.

★ Are there any questions that you could find the answers to more than one way?

Opening Questions

Sports

- Who is the best quarterback of all time?
- Are baseball pitchers throwing harder than ever?
- How much more do male soccer players earn than females?
- How common is it for former Olympic athletes to become coaches?
- How much does an extra inch of height help a basketball player?

Pop Culture

- What percentage of people have seen the movie that won last year's Best Picture Award?
- Who tends to be more popular: bands or solo singers?
- Are younger actors paid more than older actors?
- Are movies with female leads as profitable as movies with male leads?
- Does winning a Grammy increase sales?

Politics

- Is "Stop and Frisk" a racist policy?
- Do Republican politicians tend to come from different states than Democratic ones?
- Do people in countries that have universal healthcare live longer than people in countries that don't?
- Was press coverage slanted for or against a particular candidate?

Education

- Do small schools perform better than large ones?
- Which has a stronger correlation with student achievement: race or wealth?
- Do bilingual classes result in better outcomes for ESL/ELL students?
- How does quality of education differ in various regions of the United States?

Introduction to Programming

The **Editor** is a software program we use to write Code. Our Editor allows us to experiment with Code on the right-hand side, in the **Interactions Area**. For Code that we want to *keep*, we can put it on the left-hand side in the **Definitions Area**. Clicking the "Run" button causes the computer to re-read everything in the Definitions Area and erase anything that was typed into the Interactions Area.

Data Types

Programming languages involve different ***data types***, such as Numbers, Strings, Booleans, and even Images.

- Numbers are values like `1`, `0.4`, `1/3`, and `-8261.003`.
  - Numbers are *usually* used for quantitative data and other values are *usually* used as categorical data.
  - In Pyret, any decimal *must* start with a 0. For example, `0.22` is valid, but `.22` is not.
- Strings are values like `"Emma"`, `"Rosanna"`, `"Jen and Ed"`, or even `"08/28/1980"`.
  - All strings *must* be surrounded by quotation marks.
- Booleans are either `true` or `false`.

All values evaluate to themselves. The program `42` will evaluate to `42`, the String `"Hello"` will evaluate to `"Hello"`, and the Boolean `false` will evaluate to `false`.

Operators

Operators (like `+`, `-`, `*`, `<`, etc.) work the same way in Pyret that they do in math.

- Operators are written between values, for example: `4 + 2`.
- In Pyret, operators must always have spaces around them. `4 + 2` is valid, but `4+2` is not.
- If an expression has different operators, parentheses must be used to show order of operations. `4 + 2 + 6` and `4 + (2 * 6)` are valid, but `4 + 2 * 6` is not.

Applying Functions

Applying functions works much the way it does in math. Every function has a name, takes some inputs, and produces some output. The function name is written first, followed by a list of ***arguments*** in parentheses.

- In math this could look like $f(5)$ or $g(10, 4)$.
- In Pyret, these examples would be written as `f(5)` and `g(10, 4)`.
- Applying a function to make images would look like `star(50, "solid", "red")`.
- There are many other functions, for example `num-sqr`, `num-sqrt`, `triangle`, `square`, `string-repeat`, etc.

Functions have ***contracts***, which help explain how a function should be used. Every Contract has three parts:

- The *Name* of the function - literally, what it's called.
- The *Domain* of the function - what *type(s) of value(s)* the function consumes, and in what order.
- The *Range* of the function - what *type of value* the function produces.

Strings and Numbers

*Make sure you've loaded code.pyret.org (CPO), clicked "Run", and are working in the **Interactions Area** on the right. Hit Enter/return to evaluate expressions you test out.*

Strings

String values are always in quotes.

- Try typing your name *(in quotes!)*.
- Try typing a sentence like "I'm excited to learn to code!" *(in quotes!)*.
- Try typing your name with the opening quote, but *without the closing quote.* Read the error message!
- Now try typing your name *without any quotes.* Read the error message!

1) Explain what you understand about how strings work in this programming language. _____

_____

Numbers

2) Try typing `42` into the Interactions Area and hitting "Enter". Is `42` the same as `"42"` ? Why or why not?

_____

3) What is the largest number the editor can handle?

_____

4) Try typing `0.5` . Then try typing `.5` . Then try clicking on the answer. Experiment with other decimals.

Explain what you understand about how decimals work in this programming language. _____

_____

5) What happens if you try a fraction like `1/3` ? _____

_____

6) Try writing **negative** integers, fractions and decimals. What do you learn? _____

_____

_____

Operators

7) Just like math, Pyret has ***operators*** like `+` , `-` , `*` and `/` .
Try typing in `4 + 2` and then `4+2` (without the spaces). What can you conclude from this?

_____

8) Type in the following expressions, **one at a time**: `4 + 2 * 6` `(4 + 2) * 6` `4 + (2 * 6)` What do you notice?

_____

9) Try typing in `4 + "cat"` , and then `"dog" + "cat"` . What can you conclude from this?

_____

_____

Booleans

Boolean-producing expressions are yes-or-no questions, and will always evaluate to either `true` ("yes") or `false` ("no").

What will the expressions below evaluate to? *Write down your prediction, then type the code into the Interactions Area to see what it returns.*

	Prediction	Result		Prediction	Result
1) `3 <= 4`			2) `"a" > "b"`		
3) `3 == 2`			4) `"a" < "b"`		
5) `2 < 4`			6) `"a" == "b"`		
7) `5 >= 5`			8) `"a" <> "a"`		
9) `4 >= 6`			10) `"a" >= "a"`		
11) `3 <> 3`			12) `"a" <> "b"`		
13) `4 <> 3`			14) `"a" >= "b"`		

15) In your own words, describe what `<` does. _____

_____

16) In your own words, describe what `>=` does. _____

_____

17) In your own words, describe what `<>` does. _____

_____

	Prediction:	Result:
18) `string-contains("catnap", "cat")`		
19) `string-contains("cat", "catnap")`		

20) In your own words, describe what `string-contains` does. Can you generate another expression using `string-contains` that returns true?

_____

★ There are infinite string values ("a", "aa", "aaa"…) and infinite number values out there (…-2,-1,0,-1,2…). But how many different *Boolean* values are there? _____

Functions for Tables

Open the Animals Starter File and click "Run".
In the Interactions Window on the right, type `animals-table` *and hit "Enter" to see the default view of the table.*

sort

Suppose we wanted to see the names of the animals in alphabetical order…
The `sort` function takes in three pieces of information:

1. A table
2. A column we want to sort the table by (declared using a String)
3. The order in which we want the column sorted (declared using a Boolean)

Test out these two expressions in the Interactions Area and record what you learn about ordering below:

- `sort(animals-table, "species", true)`
- `sort(animals-table, "species", false)`

1) `true` sorts the table… _____

2) `false` sorts the table… _____

Suppose we wanted to sort the `animals-table` by the `weeks` column to determine which animals were adopted quickest…

3) Would you use `true` or `false`? Explain. _____

4) Test it out, and write your thinking about ***quantitative*** columns at the end of your explanations of `true` and `false` above.

5) Which animal(s) were adopted the quickest? _____

6) Some functions produce Numbers, some produce Strings, some produce Booleans. What did the `sort` function produce? _____

There are many other functions available to us in Pyret. We can describe them using contracts. The Contract for `sort` is:

```
# sort :: Table, String, Boolean -> Table
```

- Each Contract begins with the function name: _____ *in this case* `sort`
- Lists the data types required to satisfy its Domain: _____ *in this case* Table, String, Boolean
- And then declares the data type of the Range it will return. _____ *in this case* Table
- Contracts can also be written with more detail, by adding *variable names* in the Domain:

```
# sort :: ( Table , String , Boolean) -> Table
          table-name  column-name  order
```

Suppose we wanted to sort the `animals-table` by the `legs` column to determine which animals had the most legs…

7) Fill in the blanks below with the code you'd use (We've put pieces of the Contract below each line to help you!):

_____ (_____ , _____ , _____)
function-name — table-name :: Table — column-name :: String — order :: Boolean

8) Which animal(s) had the most legs? _____

9) Think of another question you might answer quickly by sorting the table.

_____

10) What code would you write to answer your question?

_____ (_____ , _____ , _____)
function-name — table-name :: Table — column-name :: String — order :: Boolean

Functions for Tables (continued)

count

```
# count :: Table, String -> Table
```

1) What is the Domain of `count`? _____

2) What is the Range of `count`? _____

3) What do you suspect the String in the Domain will describe? _____

Suppose we wanted to know how many animals had 4 legs...
Type `count(animals-table, "legs")` into the Interactions Area and click "Enter"

4) What did the expression produce? _____

5) How many animals had 4 legs? _____

6) Think of another question you might be able to answer with the `count` function.

_____

7) Fill in the blanks with the code you'd write.

_____ (_____, _____)
function-name — table-name :: Table — column-name :: String

8) Tables that summarize data with a count are commonly used in the real world. Give two examples of where you've seen them before:

- Example 1: _____
- Example 2: _____

9) Newscasters and journalists often incorporate data into their reporting. How else might they display this information, besides using a table?

_____

first-n-rows

10) Type `first-n-rows(animals-table, 5)`. What happens? _____

11) If we wanted a table of the first 3 rows of the `animals-table`, what code would you write? _____

12) What is the Contract for `first-n-rows`? _____

★ What happens when you type `first-n-rows(sort(animals-table, "pounds", true), 5)`?

_____

Note: In this case, the output of `sort(animals-table, "pounds", true)` *is the Table* `first-n-rows` *is taking in!*

★★ See if you can figure out how to compose the code that would generate a table of the 10 oldest animals!

_____ (_____, _____)
function-name — Table — Number

Circles of Evaluation: Count, Sort, First-n-rows

For each scenario below, draw the Circle of Evaluation and then use it to write the code.
When you're done, test your code out in the Animals Starter File and make sure it does what you'd expect it to.

```
# count :: Table, String -> Table
# first-n-rows :: Table, Number -> Table
# sort :: Table, String, Boolean -> Table
```

1) **We want to see the 10 animals who were adopted the quickest.**
Circle of Evaluation:

code: _____

2) **We want to see the heaviest animal.**
Circle of Evaluation:

code: _____

3) **We want to take the first 8 animals from the table and put them in alphabetical order (by name).**
Circle of Evaluation:

code: _____

4) **You notice that the lightest 16 animals weigh under 10 pounds and you want to know the count (*by species*) of those animals.**
Circle of Evaluation:

code: _____

Catching Bugs when Sorting Tables

Learning about a Function through Error Messages

1) Type `sort` into the Interactions Area of the Animals Starter File and hit "Enter". What do you learn? _____

2) We know that all functions need an open parenthesis and at least one input! Type `sort(animals-table)` in the Interactions Area and hit Enter/return. Read the error message. What hint does it give us about how to use this function?

_____

What Kind of Error is it?

syntax errors - when the computer cannot make sense of the code because of unclosed strings, missing commas or parentheses, etc.
contract errors - when the function isn't given what it needs (the wrong type or number of arguments are used)

3) In your own words, the difference between ***syntax errors*** and ***contract errors*** is: _____

_____

Finding Mistakes with Error Messages

The code below is BUGGY! Read the code and the error messages, and see if you can catch the mistake WITHOUT typing the code into Pyret.

4) `sort(animals-table, name , true)`

```
The name name is unbound:
sort(animals-table, name , true)
It is used but not previously defined.
```

This is a _____ (contract / syntax) error. The problem is that _____

5) `sort(animals-table, "name" , "true")`

```
The Boolean annotation:
fun sort(t :: Table, col :: String, asc :: Boolean)
was not satisfied by the value
     "true"
```

This is a _____ (contract / syntax) error. The problem is that _____

6) `sort(animals-table "name" true)`

```
Pyret didn't understand your program around:
sort(animals-table "name" true)
You may need to add or remove some text to fix your program. Look carefully before the
highlighted text. Is there a missing colon (:), comma (,), string marker ("), or keyword? Is
there something there that shouldn't be?
```

This is a _____ (contract / syntax) error. The problem is that _____

7) `sort(animals-table, "name", true`

```
Pyret didn't expect your program to end as soon as it did:
sort(animals-table, "name", true
You may be missing an "end", or closing punctuation like ")" or "]" somewhere in your program.
```

This is a _____ (contract / syntax) error. The problem is that _____

8) `sort (animals-table, "name", true)`

```
Pyret thinks this code is probably a function call:
sort (animals-table, "name", true)
Function calls must not have space between the function expression and the arguments.
```

This is a _____ (contract / syntax) error. The problem is that _____

Contracts for Image-Producing Functions

Log into [code.pyret.org (CPO)] and click "Run". Experiment with each of the functions listed below, trying to find an expression that will build. Record the contract and example code for each function you are able to successfully build!

Name		Domain		Range
`# triangle`	`::`	`Number, String, String`	`->`	`Image`
`triangle(80, "solid", "darkgreen")`				
`# star`	`::`		`->`	
`# circle`	`::`		`->`	
`# rectangle`	`::`		`->`	
`# text`	`::`		`->`	
`# square`	`::`		`->`	
`# ellipse`	`::`		`->`	
`# regular-polygon`	`::`		`->`	

Challenge: Composing with Circles of Evaluation

What if we wanted to see your name written on a diagonal?

- We know that we can use the `text` function to make an Image of your name.
- Pyret also has a function called `rotate` that will rotate any Image a specified number of degrees.

```
# rotate :: Number, Image -> Image
```

But how could the `rotate` and `text` functions work together? Draw a Circle of Evaluation, translate it to code and test it out in the Editor!

Exploring Displays

Use the contracts provided below to make each type of display in the Animals Starter File. Then answer the questions about each display.

Bar Charts `# bar-chart :: Table, String -> Image`

_____ (_____, _____)
function-name: table-name :: Table; column-name :: String

Sketch a bar chart below.

Bar charts summarize 1 column of _____ data.
categorical/quantitative

This kind of display tells us…

_____

_____

_____

Pie Charts `# pie-chart :: Table, String -> Image`

_____ (_____, _____)
function-name: table-name :: Table; column-name :: String

Sketch a pie chart below.

Pie charts summarize 1 column of _____ data.
categorical/quantitative

This kind of display tells us…

_____

_____

_____

Box Plots `# box-plot :: Table, String -> Image`

_____ (_____, _____)
function-name: table-name :: Table; column-name :: String

Sketch a box plot below.

Box plots summarize 1 column of _____ data.
categorical/quantitative

This kind of display tells us…

_____

_____

_____

Histograms `# histogram :: Table, String, String, Number -> Image`

_____ (_____, _____, _____, _____)
function-name: table-name :: Table; labels :: String; values :: String; bin-width :: Number

Sketch a histogram below.

Histograms summarize 1 column of _____ data.
categorical/quantitative

This kind of display tells us…

_____

_____

_____

Circles of Evaluation: Composing Functions to Make Displays

Using the Contracts below as a reference, draw the Circle of Evaluation for each prompt.

```
# pie-chart :: Table, String -> Image
# bar-chart :: Table, String -> Image
# histogram :: Table, String, String, Number -> Image
# box-plot :: Table, String -> Image
# first-n-rows :: Table, Number -> Table
# sort :: Table, String, Boolean -> Table
```

1) Make a bar-chart of the lightest 16 animals by `sex`.

★ What other bar chart might you want to compare this to? _____

2) Take the heaviest 20 animals and make a histogram of `weeks` to adoption (use `"species"` for your labels).

★ What other histogram might you want to compare this to? _____

3) Make a box-plot of `age` for the 11 animals who spent the most `weeks` in the shelter.

★ What other box plot might you want to compare this to? _____

4) Make a pie-chart of `species` for the 18 animals who spent the fewest `weeks` in the shelter.

★ What other pie chart might you want to compare this to? _____

Exploring Displays (2)

Use the contracts provided below to make each type of display in the Animals Starter File. Then answer the questions about each display.

Line Graphs `# line-graph :: Table, String, String, String -> Image`

_____ (_____, _____, _____,
function-name — table-name :: Table — column-name :: String — column-name :: String

_____)
column-name :: String

Sketch a line graph below.

Line Graphs summarize 2 columns of _____ data.
categorical/quantitative

This kind of display tells us...

Scatter Plots `# scatter-plot :: Table, String, String, String -> Image`

_____ (_____, _____, _____, _____)
function-name — table-name :: Table — column-name :: String — column-name :: String — column-name :: String

Sketch a scatter plot below.

Scatter Plots summarize 2 columns of _____ data.
categorical/quantitative

This kind of display tells us...

LR Plots `# lr-plot :: Table, String, String, String -> Image`

_____ (_____, _____, _____, _____)
function-name — table-name :: Table — column-name :: String — column-name :: String — column-name :: String

Sketch an Linear Regression Plot below.

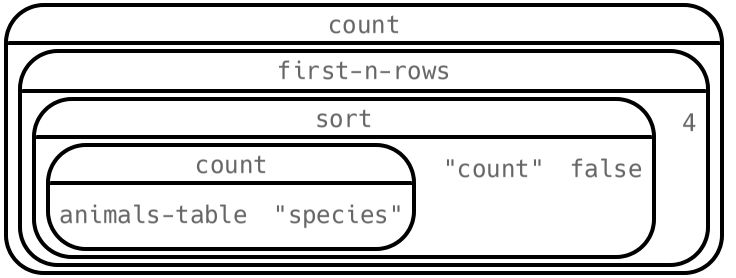
LR Plots summarize 2 columns of _____ data.
categorical/quantitative

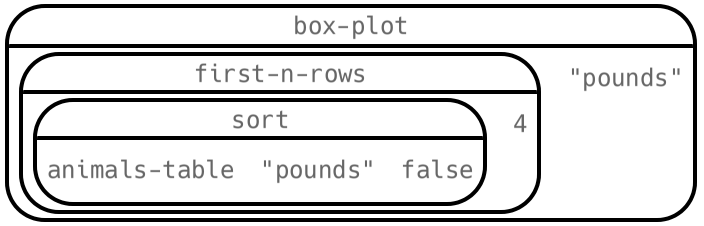
This kind of display tells us...

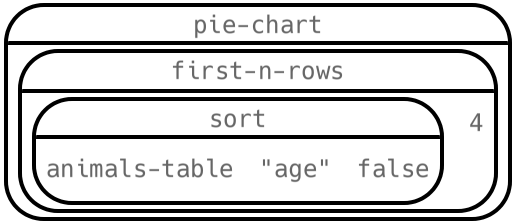
Composing Functions: Match Display Descriptions to Circles of Evaluation

Match each prompt on the left to the Circle of Evaluation used to answer it.

Make a pie-chart, showing the species of the 4 oldest animals.	1	A
Take the 4 heaviest animals and make a box plot of their weight.	2	B
Make a table showing the count of the species in this dataset, sorted from most to least.	3	C
Make a table showing the count of the 4 species with the most animals	4	D







Circles of Evaluation: Composing Functions to Make Displays (2)

Using the Contracts below as a reference, draw the Circle of Evaluation for each prompt.

```
# pie-chart :: Table, String -> Image
# bar-chart :: Table, String -> Image
# histogram :: Table, String, String, Number -> Image
# box-plot :: Table, String -> Image
# first-n-rows :: Table, Number -> Table
# sort :: Table, String, Boolean -> Table
```

1) Take the youngest 12 animals and make a box-plot of `pounds`.

What other box plot might you want to compare this to? _____

2) Make a pie-chart of `legs` for the 10 oldest animals.

What other pie chart might you want to compare this to? _____

★ Take the 20 lightest animals, then take the 10 youngest of *those* animals and make a bar-chart of `species`

What other pie chart might you want to compare this to? _____

Exploring the States Dataset

Open the Preview: State Demographics Starter File.
Then, click "Run" and type `states-table` into the Interactions Area on the right to see the dataset.

What do you Notice about this dataset?	What do you Wonder about this dataset?

1) What code will produce a table showing the number of states in each region? _____

2) Which states do you **think** have the most people? _____

3) What code will produce a table containing the five states with the largest population in 2020?

_____

4) Which states do you **think** have the most poverty? _____

5) What code will produce a table containing the ten states with the highest % of people in poverty?

_____

6) What code will produce a table containing the states with the lowest **median** income?

_____

7) What code will produce a table containing the states with the lowest **per-capita** ("average" or "mean") income?

_____

★ What does it mean if a state has a higher `per-capita income` than `median-income`? _____

_____

The two lines of code under # `Define some rows` *extract rows 0 and 1 from the table, and define them as* `alabama` *and* `alaska`.

8) Type `alabama` into the Interactions Area. What do you get back? _____

9) Underneath the definition of those rows, **add a new definition** for `california` and click "Run", so that Pyret reads your new definition.

10) Add a definition for your own state, then **click "Run"** and test it out in the Interactions Area!

11) Add any additional Notices or Wonderings you have about this dataset to the table at the top.

Looking for Patterns

Open the Preview: State Demographics Starter File.

Part 1

1) What columns do you think might be related to one another? (e.g. - is the number of veterans related to the amount of land-area? Is the population in 2010 related to the population in 2020?) List three possible relationships below.

a. I think that _____ may be related to _____

b. I think that _____ may be related to _____

c. I think that _____ may be related to _____

```
# scatter-plot :: (Table, String, String, String) -> Image
                  labels  explanatory response
```

2) Use the Contract above to make a scatter-plot for the **first relationship** you wrote above. (Use `"state"` as the label, so that clicking on a point will show you which state you're looking at.)

a. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? _____

_____

b. In your own words, describe the pattern you see in the scatter plot so someone else could sketch it. _____

_____

_____

3) Make a scatter-plot for the **second relationship** you wrote.

a. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? _____

_____

b. In your own words, describe the pattern you see in the scatter plot so someone else could sketch it. _____

_____

_____

4) Make a scatter-plot for the **third relationship** you wrote.

a. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? _____

_____

b. In your own words, describe the pattern you see in the scatter plot so someone else could sketch it. _____

_____

_____

Part 2

Wait to complete this until after diving deeper into statistical relationships!

Revisit the three scatter plots you made and add the following labels to the descriptions you wrote in Question 1:

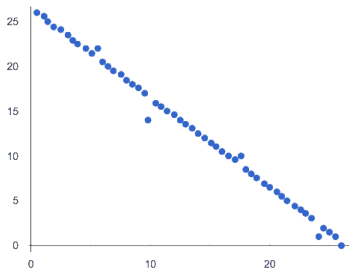
- Place an "L" by any relationships that you think might be linear.
- Place a "P" by any relationships that appear to be positive.
- Place an "N" by any relationships that appear to be negative.
- Place an "S" by the strongest-looking relationship.
- Place a "W" by the weakest-looking relationship.

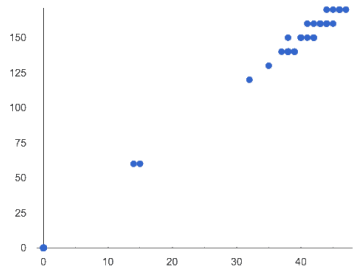
Identifying Form, Direction and Strength (Matching)

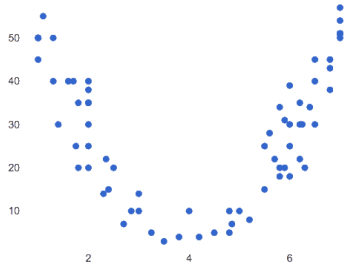
Match the description (left) with the scatter plot (right).

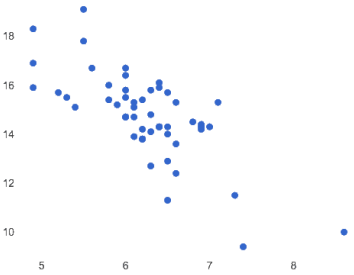
Note: The computer won't tell us if the relationship we see is linear, so we must train our eyes to decide this ourselves. For linear relationships, we should train our eyes to assess their direction and get a feel for their strength, rather than relying completely on what numbers the computer reports.

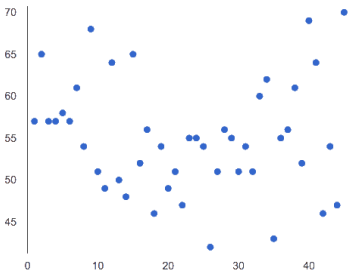
The relationship appears to be linear, negative, and of moderate strength.	1	A
This relationship is nonlinear.	2	B
The x and y variables in this dataset do not appear to be related.	3	C
The relationship appears to be linear, positive, and strong.	4	D
The relationship appears to be linear, negative, and strong.	5	E







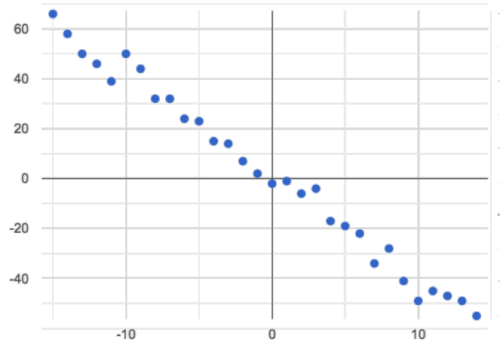




Identifying Form, Direction and Strength

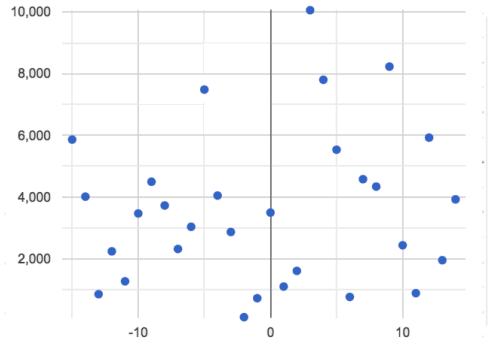
What do your eyes tell you about the Form, Direction, & Strength of these displays?
Note: If the form is nonlinear, we shouldn't report direction - a curve may rise and then fall.

A



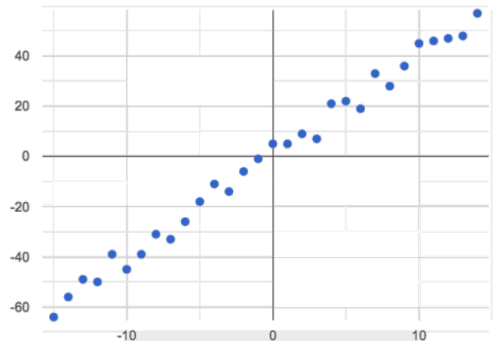
Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

B



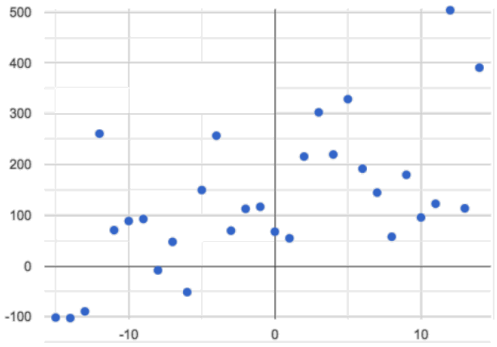
Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

C



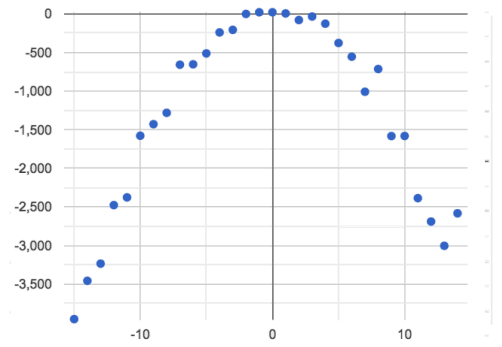
Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

D



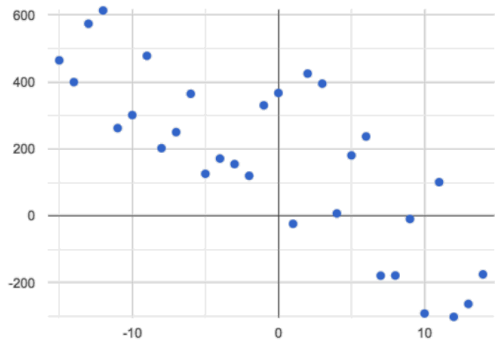
Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

E



Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

F



Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

Build a Model from Samples: College Degrees v. Income

Open the Preview: State Demographics Starter File.

1) Record the `pct-college-or-higher` and `median-income` values for the `alabama` and `alaska` rows, as (x,y) pairs below:

(_____ (AL pct-college-or-higher), _____ (AL median-income)) (_____ (AK pct-college-or-higher), _____ (AK median-income))

2) Using the space below, compute the equation of the line passing between these two points. **This line will be your linear model** (also known as the "predictor function", or "line of best fit"), which predicts `median-income` as a function of `pct-college-or-higher`.

3) Write the complete model below (in both Function and Pyret notation):

$\text{al-ak}(x) =$ _____ (slope(m)) x + _____ (y-intercept / vertical shift)

```
fun al-ak(x): (_____ * x) + _____ end
```

Return to your copy of the starter file and add the code you just wrote to the Definitions Area. Then Click "Run".
(If there are any errors or warnings, fix them and click "Run" again.)

4) In the Interactions Area, try plugging in the `pct-college-or-higher` value for Alabama by typing `al-ak(22.6)`.

- How well does it predict the correct median income for Alabama? _____
- What expression would predict median income for Alaska? _____.
- How well does it predict the correct median income for Alaska? _____
  Consider: If it doesn't predict it perfectly, why might that be?

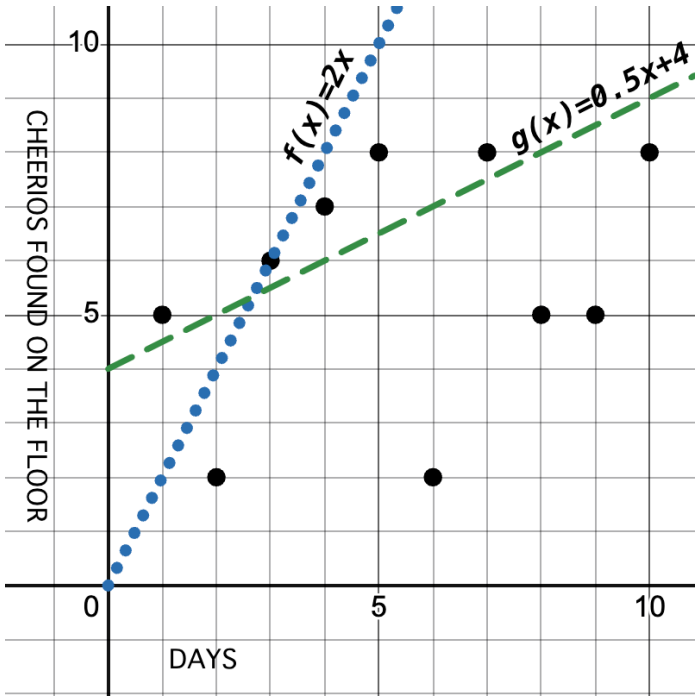
Try different `pct-college-or-higher` values from *other* states, to see how well our Alabama-Alaska model fits the rest of the country.

5) Identify a state for which this model works well: _____

6) Identify a state for which this model works poorly: _____

7) What median income does this model expect a state without ANY college graduates (0%) to earn? _____

How could we Measure Whether a Model is a Good Fit?



id	Days	Cheerios found on the floor
a	1	5
b	2	2
c	3	6
d	4	7
e	5	8
f	6	2
g	7	8
h	8	5
i	9	5
j	10	8

1) Do you think `f(x)` or `g(x)` is a better model for this data? _____

2) What makes you think that? _____

_____

_____

_____

3) What could we measure, to calculate *how much better of a model* it is? _____

_____

_____

_____

4) *Neither of these models is the best possible model!* What would have to be true of a third model, for us to know that it was a better fit than these two? _____

_____

_____

_____

Fit a Model: College Degrees v. Income

*Open the Fitting a Model: State Demographics Starter File and **Save a Copy** of the file that's just for you.*

The `al-ak` Model

Type `fit-model(states-table, "state", "pct-college-or-higher", "median-income", al-ak)` in the Interactions Area, then find the points for AL and AK along the predictor line. *Hint: You know their coordinates and they will help you know where to look!*

1) What do you Notice?

_____

_____

2) What do you Wonder?

_____

_____

3) Find S in the upper left corner. What is the S value (the number after S)? _____

Other Models

In the definitions area, find the section titled *Define some other models by modifying al-ak*.

- For now, all three definitions in this section are exactly the same as `al-ak`.
- You will be changing them according to the directions below.

4) If you wanted the model to be *less steep*, what slope could you use? _____

- Change the definition for `less-steep` to use the slope you wrote above.
  - Click "Run" to load your new definition. In the Interactions Area type:
    `fit-model(states-table, "state", "pct-college-or-higher", "median-income", less-steep)`
  - What is the S value of `less-steep`? _____

- Identify a y-intercept that would make the model fit the data better: _____
  - Adjust the definition to use the new y-intercept and click "Run".
  - Hit the up arrow in the Interactions Area and click return/Enter to fit the model again.
  - What is the S value of `less-steep` now? _____

5) Change the definition of `negative` so that it models the data with a slope that is *negative*.

- Click "Run" and type the code to fit this model to the data.
- What slope did you use? _____ What is the S value now? _____

6) Change the definition of `horizontal` so that it draws a horizontal model. Click "Run" and fit this model. What is the S value? _____

7) Change the y-intercept so that the horizontal line passes through more of the points. Click "Run" and fit this model.

- What y-intercept did you use? _____ What is the S value now? _____

8) What do you think S tells us? _____

_____

_____

_____

What does *S* tell us about the fit of these models?

For each model below, decide whether the fit is "poor", "ok", or "good". *Then* rank the models from 1 (best fit) to 8 (worst fit).

How good is the model?	Ranking
1 A data scientist is working with data from animals at a shelter. • The range of days to adoption in this dataset are from 0 to 400. • An S value of 300 means predicted adoption times could be off by 300 days. This is a(n) _____ (poor, ok, good) model for the dataset.	
2 A student is exploring a dataset on climate change. • The range of Arctic Sea Ice is from 3,920,000 to 7,670,000 square kilometers • An S value of 300 means predicted Arctic Sea Ice coverage could be off by 300 square kilometers. This is a(n) _____ (poor, ok, good) model for the dataset.	
3 A data scientist is working with data from US public schools. • The range of graduates per school per year is 2 to 2003. • An S value of 300 means predicted graduate values could be off by 300 students. This is a(n) _____ (poor, ok, good) model for the dataset.	
4 A student is exploring a dataset on earthquakes. • The range of earthquake depths in this dataset are from 4200m to 664000m. • An S value of 300 means predicted earthquake depths could be off by 300 meters. This is a(n) _____ (poor, ok, good) model for the dataset.	
5 A student is exploring a dataset on arrests in Los Angeles. • The age range in this dataset is from 0 to 92. • An S value of 1 means predicted ages could be off by 1 year. This is a(n) _____ (poor, ok, good) model for the dataset.	
6 A data scientist is working with data about snowflakes. • The range of snowflake weights is from 0.001 grams to 0.02 grams. • An S value of 1 means predicted values could be off by 1 gram. This is a(n) _____ (poor, ok, good) model for the dataset.	
7 A data scientist is working with data from animals at a shelter. • The range of ages is from 0.5 years to 16 years. • An S value of 1 means predicted ages could be off by 1 year. This is a(n) _____ (poor, ok, good) model for the dataset.	
8 A student is working with a dataset of adult blue whales. • The range of weights is 200,000 to 330,000 pounds. • An S value of 1 means predicted weights could be off by 1 pound. This is a(n) _____ (poor, ok, good) model for the dataset.	

Better Modeling: College Degrees v. Income

Open your copy of the Fitting a Model: State Demographics Starter File.

Build a Model through Trial & Error

Find `# Define some rows` in the Definitions Area.
Add two new definitions for `MA` (row 21) and `NV` (row 28), using the definitions for `alaska` and `alabama` as a model.

1) Record the `college-or-higher` and `median-income` values for `MA` and `NV`, as (x,y) pairs below:

(_____ , _____)
MA college-or-higher, MA median-income

(_____ , _____)
NV college-or-higher, NV median-income

2) Derive the `MA-NV model` (using the same steps you followed to derive the `AL-AK model` on Fit a Model: College Degrees v. Income) and write it below (in both Function and Pyret notation), then fit the model and record the S-value:

$ma\text{-}nv(x) =$ _____ x + _____ `fun ma-nv(x): ( _____ * x) + _____ end` S: _____
slope (m), y-intercept / vertical shift

3) Identify two other states that you think would make a better model: _____ and _____.

Add two new definitions for these states to your Fitting a Model: State Demographics Starter File.

4) Record the `college-or-higher` and `median-income` values for these states, as (x,y) pairs below:

(_____ , _____)
college-or-higher, median-income

(_____ , _____)
college-or-higher, median-income

5) Derive your model and write it below (in both Function and Pyret notation), then fit the model and record the S-value:

$other(x) =$ _____ x + _____ `fun other(x): ( _____ * x) + _____ end` S: _____
slope (m), y-intercept / vertical shift

6) Adjust the slope and y-intercept of your model to get the **smallest S possible.** Write the best model you find (and corresponding S) below:

$best(x) =$ _____ x + _____ `fun best(x): ( _____ * x) + _____ end` S: _____
slope (m), y-intercept / vertical shift

Optimizing and Interpreting Linear Models

Open your copy of the [Fitting a Model: State Demographics Starter File](#).

Build a Model Computationally

`lr-plot` computes the *optimal linear model* using all of the data points.

1) Evaluate `lr-plot(states-table, "state", "pct-college-or-higher", "median-income")`. What is S? _____

2) On the line below, write the optimal linear model that was computed through linear regression:

$optimal(x) =$ _____ (slope (m)) x + _____ (y-intercept / vertical shift)

```
fun optimal(x): (_____ * x) + _____ end
```

Interpret the Model

We started with a model based on Alabama and Alaska `fun al-ak(x): (5613.67 * x) + -83616.02 end` `S: ~36164.68` which we can interpret as follows:

The Alabama-Alaska (sensible name) *model predicts that a 1* percent (x-axis units) *increase in* percent college degrees (x-axis) *is associated with a* 5613 dollar (slope, y-units) increase (increase / decrease) *in* median household income (y-axis). *With an S-value of* ~36,164.68 (S-value) dollars (y-units) *and* median household income (y-axis) *ranging from* $39,031 (lowest y-value) *to* $73,538 (highest y-value), *this model fits* really, really poorly (really well, decently, poorly, etc.).

3) Describe the optimal model YOU created via linear regression:

The linear-regression (sensible name) model predicts that a 1 _____ (x-axis units) increase in _____ (x-axis) is associated with a _____ (slope, y-units) _____ (increase / decrease) in _____ (y-axis). With an S-value of _____ (S-value) dollars (y-units) and _____ (y-axis) ranging from _____ (lowest y-value) to _____ (highest y-value), this model fits _____ (really well, decently, poorly, etc.).

4) What does the **slope (m)** of this linear model tell us? _____

5) What does the **y-intercept / vertical shift** of this linear model tell us? _____

6) Suppose a state goes from 10% to 11% college graduation. According to this model,

- What kind of change would we expect to see in the median household income? _____
- What if it goes from 50% to 51%? _____
- What if it goes from 90% to 91%? _____

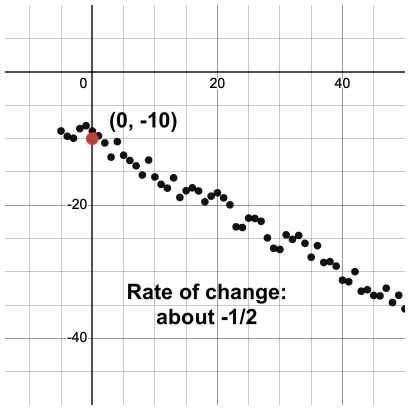
7) Does this model predict the same increase in income for *every* additional 1% `college-or-higher`? Why or why not? _____

Which Form is Best?

For each set of data provided below,

- Decide which form of the line would be the easiest to build from the available information.
- Write a definition of the linear model in that form.
- Translate the definition into Pyret notation.

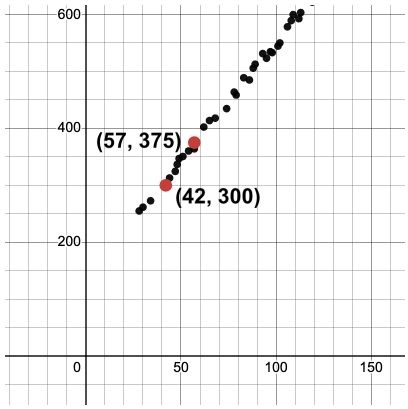
1



Linear Model: _____
Your model slope-intercept, point-slope, or standard form - which ever is easiest!

```
fun f(x) : _____ end
```

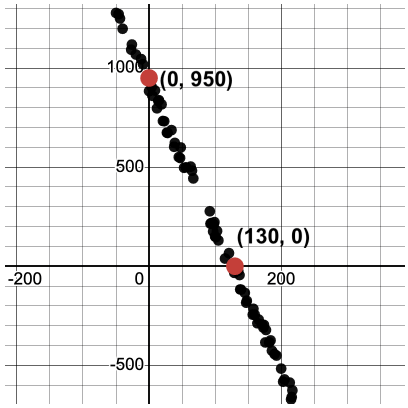
2



Linear Model: _____
Your model slope-intercept, point-slope, or standard form - which ever is easiest!

```
fun f(x) : _____ end
```

3

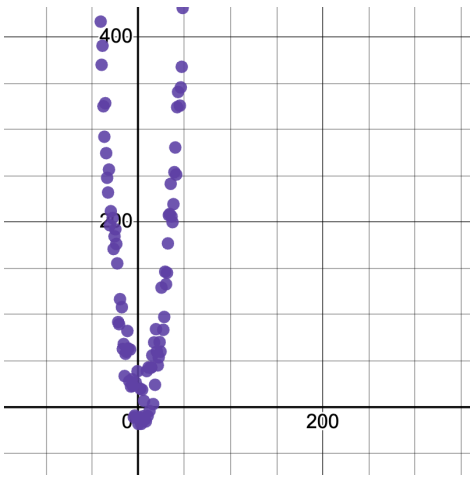


Linear Model: _____
Your model slope-intercept, point-slope, or standard form - which ever is easiest!

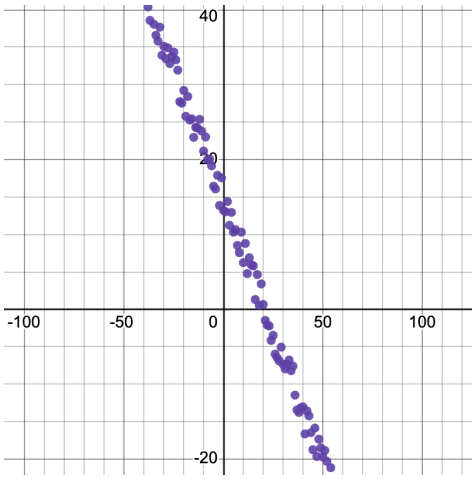
```
fun f(x) : _____ end
```

Linear, Non-linear, or Bust?

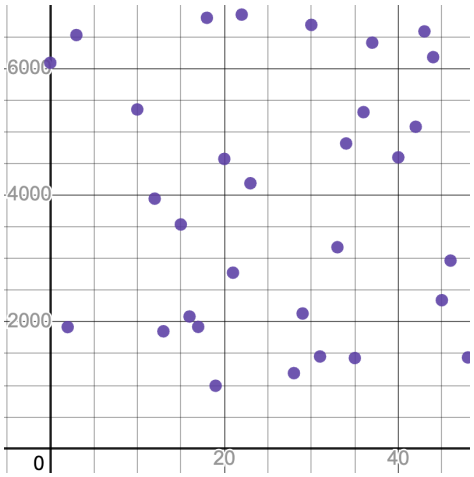
Decide whether each scatter plot appears to be best modeled by a linear function, a nonlinear function or there is no apparent relationship.



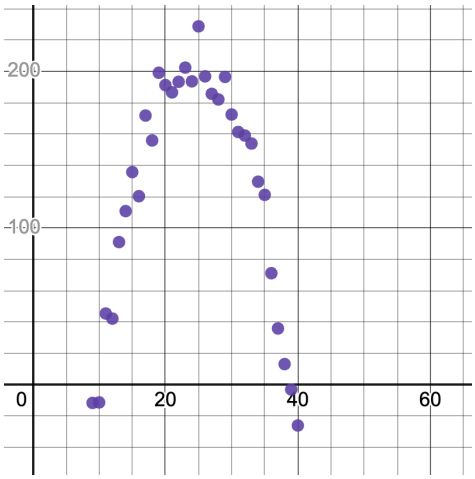
1) Linear Nonlinear No apparent relationship



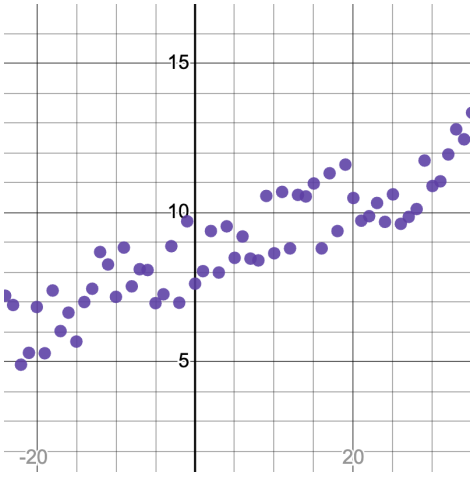
2) Linear Nonlinear No apparent relationship



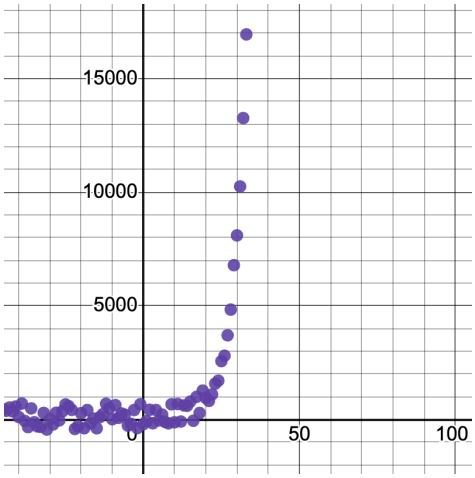
3) Linear Nonlinear No apparent relationship



4) Linear Nonlinear No apparent relationship



5) Linear Nonlinear No apparent relationship



6) Linear Nonlinear No apparent relationship

Defining a Linear Function from Two Points

The guided three-step process is designed to help you calculate slope and y-intercept from a pair of points.

Define the linear function through (-2,5) and (3,-10).

Step 1: Calculate the slope of the line by replacing the variables in the equation below with their corresponding coordinates.

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\quad - \quad}{\quad - \quad} = \_\_\_\_$$

Hint: $y_2 =$ - 10

Step 2: Use the slope intercept form of the line to calculate the y-intercept.

- replace m with the slope we just calculated
- replace x and y with the values from the first point: $(\text{ - } 2,5)$
- solve for b

Slope-intercept form of the line: $y = mx + b$

$\_\_\_\_\_ = \_\_\_\_\_ + b$

$\_\_\_\_\_ = b$

Note*: We could also have done **Step 2 using the second point: (3, - 10). Let's do that now to make sure we get the same result!*

$\_\_\_\_\_ = \_\_\_\_\_ + b$

$\_\_\_\_\_ = b$

Step 3: Use the ***slope*** and ***y-intercept*** we calculated to write our function definition!

$f(x) = \_\_\_\_\_ x +$ _____

Define the linear function through (-5,2) and (3,6).

Step 1: Calculate slope.

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\quad - \quad}{\quad - \quad} = \_\_\_\_$$

Step 2: Calculate the y-intercept. *Hint: You can use either point. Which would be simpler?*

$\_\_\_\_\_ = b$

Step 3: Write the function definition!

$f(x) = \_\_\_\_\_ x +$ _____

Build a Model from Samples: College Degrees v. Income (Scaffolded)

Open the Preview: State Demographics Starter File.

1) Record the `pct-college-or-higher` and `median-income` values for the `alabama` and `alaska` rows, as (x,y) pairs below:

(_____ AL pct-college-or-higher, _____ AL median-income) (_____ AK pct-college-or-higher, _____ AK median-income)

2) We want to understand how change (Δ) in the percent of the population that attended college relates to change (Δ) in the median income.

Compute Δ median-income = _____ AK median-income - _____ AL median-income = _____

Compute Δ pct-college-or-higher = _____ AK pct-college-or-higher - _____ AL pct-college-or-higher = _____

Compute the slope/rate of change between `AL` and `AK`: $\frac{\Delta\text{median-income}}{\Delta\text{pct-college-or-higher}} =$ _____ $=$ _____

Based solely on data from Alabama and Alaska, we are seeing that a _____ Δ pct-college-or-higher *percent increase in college graduates among the population translates to a* _____ Δ median-income *dollar increase in median income.*

3) Now, let's use the slope-intercept form of the line to calculate the y-intercept / vertical shift of the line passing through `AK` and `AL`.

_____ y = _____ slope (m) × _____ x + b y-intercept / vertical shift

- Find the x and y values from the `AK` row: (_____ **x** :: AK pct-college-or-higher, _____ **y** :: AK median-income)
- Now, replace y, m, and x in the equation above with values from the `AK` row and the slope we just calculated in question 2.
- Then solve the equation above for $b =$ _____

4) Use the slope and y-intercept you calculated to write the complete model below (in both Function and Pyret notation):

$al\text{-}ak(x) =$ _____ slope (m) x + _____ y-intercept / vertical shift

```
fun al-ak(x): (_____ * x) + _____ end
```

Return to your copy of the starter file and add the code you just wrote to the Definitions Area. Then Click "Run".

5) In the Interactions Area, try plugging in the `pct-college-or-higher` value for Alabama by typing `al-ak(22.6)`.

- How well does it predict the correct median income for Alabama? _____
- What expression would predict median income for Alaska? _____.
- How well does it predict the correct median income for Alaska? _____

Try different `pct-college-or-higher` values from *other* states, to see how well our Alabama-Alaska model fits the rest of the country.

6) Identify a state for which this model works well: _____

7) Identify a state for which this model works poorly: _____

8) What median income does this model expect a state with zero college graduates to earn? _____

Graphing Linear Models

Sketch three of your linear models from Build a Model from Samples: College Degrees v. Income, Fit a Model: College Degrees v. Income, and Better Modeling: College Degrees v. Income on the scatter plots below.
Then label the slope, y-intercept, and S value of each model!

1)

Slope

y-Intercept

S

2)

Slope

y-Intercept

S

3)

Slope

y-Intercept

S

Building More Linear Models

Open your copy of Fitting a Model: State Demographics Starter File. If you haven't already, ***Save a Copy*** *now.*

1) Which two columns will you explore? _____ (x-axis) and _____ (y-axis).

2) Fill in the code to make a scatter plot exploring the relationship between those columns:

```
scatter-plot(states-table, "state", _____, _____)
```

3) Pick two states to use for your first model: _____ and _____.

4) Based on these two points, define your model in Function and Pyret Notation:

Function Notation	Pyret Notation
$f(x) =$ _____ x + _____	`fun f(x): (`_____`* x) +`_____`end`

5) Type this model into Pyret, and fit it to your data use `fit-model`. What S value did you get? _____

6) What's the best model you can build? In the table below, record your models and the S values you got for them, then draw a star next to the one with the best S-value.

Model (Function or Pyret Notation - whatever you prefer!)	S

★ What does this model actually mean? Try completing the sentences below:

This model predicts that a 1 _____ (x-axis units) *increase in* _____ (x-axis) *is associated with a*

_____ (slope, y-units) _____ (increase / decrease) *in* _____ (y-axis). *Based on the S of*

_____ (S) *and* _____ (y-axis) *ranging from* _____ (lowest y-value) *to* _____ (highest y-value), *I would say this model fits*

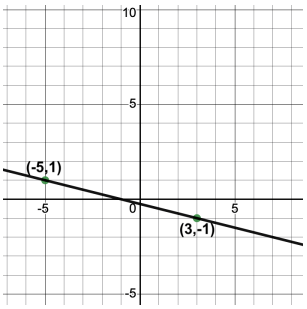
_____ (real well / not great / poorly / terribly).

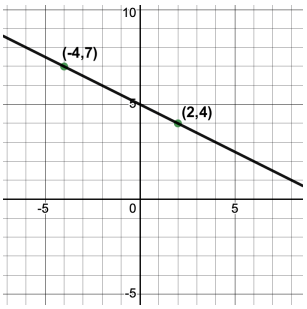
Matching Point-Slope Form to Graphs

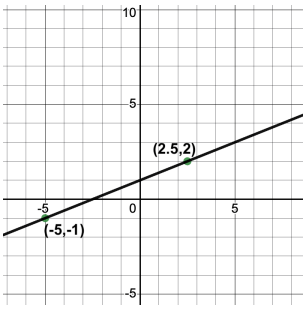
Point-Slope Form: $y - y_1 = m(x - x_1)$ m: *slope* y_1: *y-coordinate of a point* x_1: *x-coordinate of the same point*

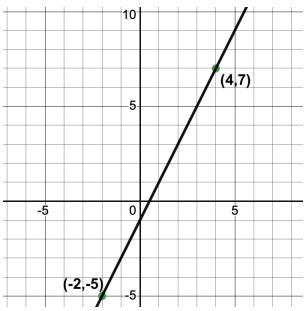
Each of the graphs below represents a line of best fit derived from some data. **Match each definition below to the linear model it describes.**

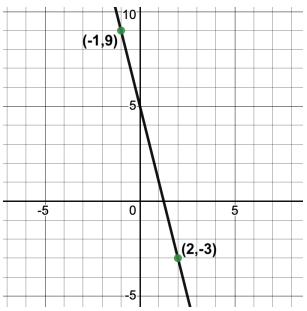
$y - 7 = -.5(x + 4)$	**1**	**A**
$y + 3 = -4(x - 2)$	**2**	**B**
$y + 5 = -0.25(x - 1)$	**3**	**C**
$y - 7 = 2(x - 4)$	**4**	**D**
$y + 5 = 0.4(x + 1)$	**5**	**E**







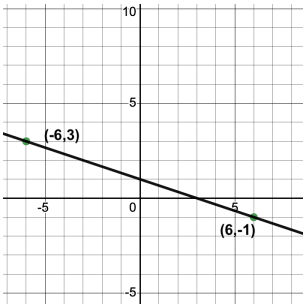


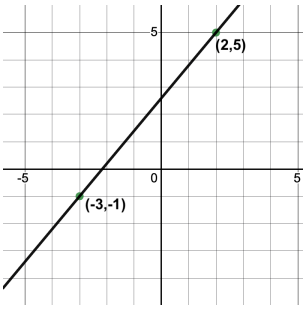


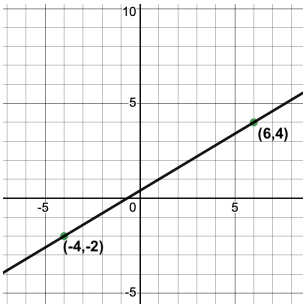
Matching Standard Form to Graphs

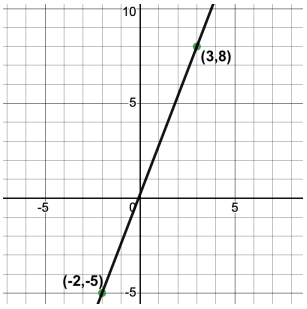
Each of the graphs below represents a line of best fit derived from some data. **Match each definition below to the linear model it describes.**

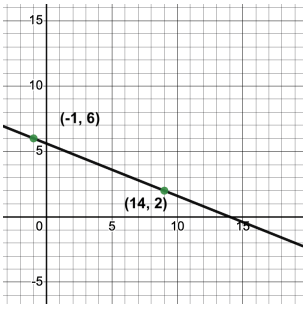
$2x + 5y = 28$	**1**	**A**
$x + 3y = 3$	**2**	**B**
$-13x + 5y = 1$	**3**	**C**
$-9x + 15y = 6$	**4**	**D**
$-6x + 5y = 13$	**5**	**E**









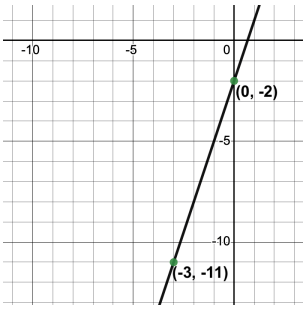


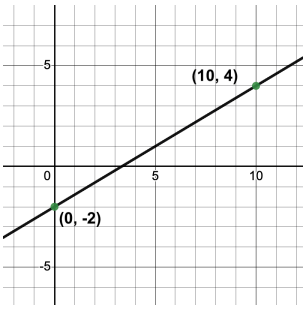
Matching Slope-Intercept Form to Graphs

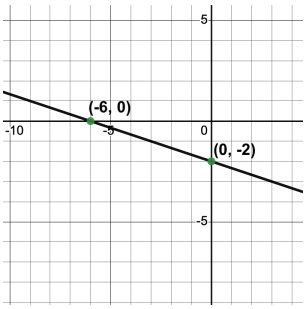
Slope-intercept form: $y = mx + b$ m: *slope* b: *y-intercept*

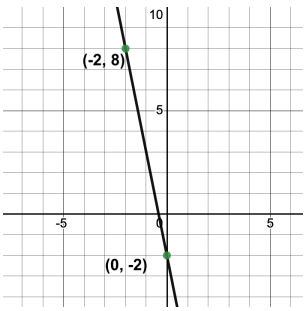
Each of the graphs below represents a line of best fit derived from some data. **Match each definition below to the linear model describes.**

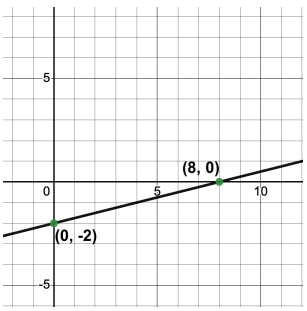
$f(x) = -5x - 2$	**1**	**A**
$g(x) = \frac{1}{4}x - 2$	**2**	**B**
$h(x) = 3x - 2$	**3**	**C**
$j(x) = -\frac{1}{3}x - 2$	**4**	**D**
$k(x) = 0.6x - 2$	**5**	**E**







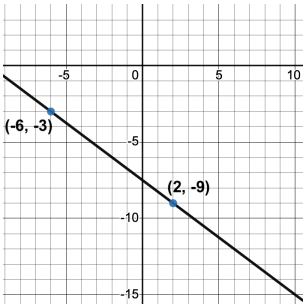


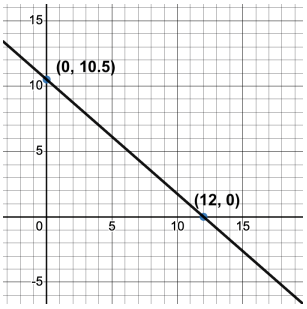


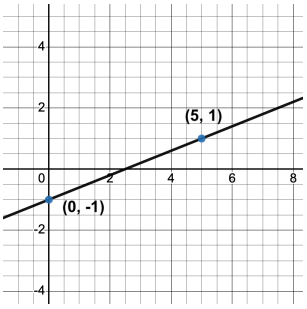
Mixed Practice: Matching Graphs of Linear Functions to their Definitions

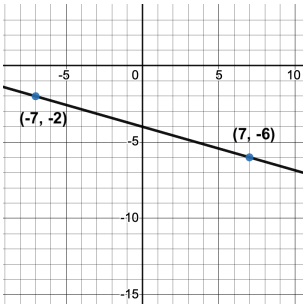
Each of the graphs below represents a line of best fit derived from some data. **Match each equation on the left to its graphical representation on the right.**

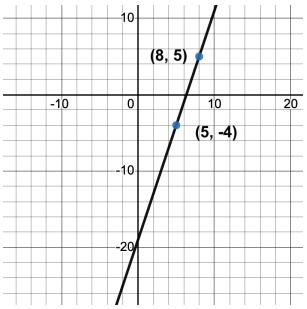
$7x + 8y = 84$	**1**	**A**
$y = \frac{2}{5}x - 1$	**2**	**B**
$y - 5 = 3(x - 8)$	**3**	**C**
$f(x) = \frac{-2}{7}x - 4$	**4**	**D**
$y + 9 = -\frac{3}{4}(x - 2)$	**5**	**E**











Other Forms of Linear Models

For this page, you'll need to have the Fitting a Model: State Demographics Starter File open on your computer. If you haven't already, select ***Save a Copy*** *from the "File" menu to make a copy of the file that's just for you.*

1) Evaluate `lr-plot(states-table, "state", "college-or-higher", "median-income")`. What is the S? _____

2) Interpret the results - what does that S tell us? _____

_____

_____

3) Write the **Slope-Intercept Form** of the optimal linear model below.

$y =$ _____ (slope) x + _____ (y-intercept)

Linear models can be expressed in several forms. In addition to Slope Intercept Form, there is also Standard (a.k.a "General Linear") Form and "Point-Slope" Form. Depending on what you want to *do* with a model, it can be more convenient to use one form instead of another!

Standard Form	Point-Slope Form
$Ax + By = C$	$y - y_1 = m(x - x_1)$

4) Write the Standard Form and Point-Slope Form of the **optimal** linear model below.

Standard Form	Point-Slope Form
_____ (A) $x +$ _____ (B) $=$ _____ (C)	$y -$ _____ (y1) $= m(x -$ _____ (x1) $)$

5) Choose **another** linear model you came up with. Which states did you use to build the model? _____

6) Write the **Slope-Intercept Form** of the optimal linear model below.

$y =$ _____ (slope) x + _____ (y-intercept)

7) Write the Standard Form and Point-Slope Form of that linear model below.

Standard Form	Point-Slope Form
_____ (A) $x +$ _____ (B) $=$ _____ (C)	$y -$ _____ (y1) $= m(x -$ _____ (x1) $)$

8) Which form is most useful to YOU, and why? _____

_____

_____

_____

_____

Exploring the Fuel Efficiency Dataset

*For this page, you'll need to open the Fuel Efficiency Starter File on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. **Read the comments at the top of the file**, which describe what each column in the dataset means.*

Fitting Linear Models

1) Evaluate `A15`, `A45` and `A75` in the Interactions Area. What **model** of car is used in all three rows? _____

2) At what three **speeds** is this model being tested in these rows? _____

3) Does there appear to be a relationship between speed and miles-per-gallon? _____.

4) *Looking at the numbers in the* `mpg-table`, describe its **form** (e.g. - linear, non-linear, or none) and **strength** (strong, moderate, or weak). If it appears to be linear, what is the **direction**? If it does *not* appear to be linear, describe its shape.

_____

_____

5) Use `lr-plot(mpg-table, "model", "speed", "mpg")` to find the optimal **linear** model. What is S for this model? _____
Write the model below, in both math and Pyret notation.

$y =$ _____ (slope) $x +$ _____ (y-intercept / vertical shift)

```
fun y(x): (_____ * x) + _____ end
```

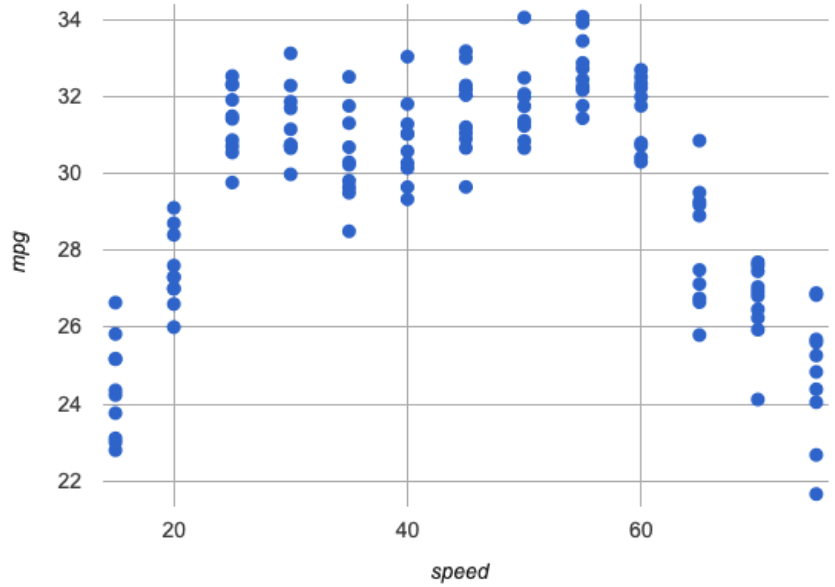
6) Is the best-possible linear model a good fit? _____. Why or why not? _____

_____

_____

Fitting Curves

7) Sketch your lr-plot in the space below, showing the relationship between `speed` and `mpg`. Be sure to label your axes, and draw the linear model!



8) What do you **Notice?** _____

_____

_____

_____

9) What do you **Wonder?** _____

_____

_____

_____

10) Do you think a **curve** would fit better?

_____

11) Draw a **curve** on your scatter-plot, which shows the overall shape in the data. At what speed is the "peak"? _____

12) Based on your best-guess curve, what do you predict `mpg` would be for a new test run at 25mph _____? 60mph _____? 90mph _____?

What Kind of Model? (Descriptions)

Decide whether each situation sounds like it would be better modeled by a linear or quadratic function, and circle your answer.

1) A car is 50 miles away, traveling at 65mph. How far away is the car after each hour?

Linear Quadratic

---

2) A ball is dropped from the top of the Empire State Building, and it keeps dropping faster and faster. **How far has the ball dropped** after x seconds?

Linear Quadratic

---

3) The data plan for a cell phone bill costs $5/gb, plus $15/mo. How much is the bill for a given month, after x number of gigabytes?

Linear Quadratic

---

4) A ball is dropped from the top of the Empire State Building, and it keeps dropping faster and faster. **How fast is the ball moving** after x seconds?

Linear Quadratic

---

5) A cannonball is fired from the deck of the S.S. Parabola, and arcs through the sky before hitting its target, 17 miles away.

Linear Quadratic

---

6) The area of a circle, as its radius increases.

Linear Quadratic

---

7) The circumference of a circle, as its radius increases.

Linear Quadratic

What Kind of Model? (Tables)

Decide whether each representation is best described by a linear model, a quadratic model or neither! Show any work that you feel is useful.

For Class Discussion:

1

x	0	1	2	3	4	5	6
y	5	6	9	14	21	30	41

Linear
Quadratic
Neither

2

x	0	1	2	3	4	5	6
y	0	3	6	9	12	15	18

Linear
Quadratic
Neither

For Independent Practice:

3

x	1	2	3	4	5	6	7
y	1	3	5	7	9	11	13

Linear
Quadratic
Neither

4

x	-3	-2	-1	0	1	2	3
y	-23	-38	-47	-50	-47	-38	-23

Linear
Quadratic
Neither

5

x	-3	-2	-1	0	1	2	3
y	1	2	1	2	1	1	1

Linear
Quadratic
Neither

6

x	1	2	3	4	5	6	7
y	2	5	10	17	26	37	50

Linear
Quadratic
Neither

7

x	-3	-2	-1	0	1	2	3
y	12	7	2	-3	-8	-13	-18

Linear
Quadratic
Neither

8

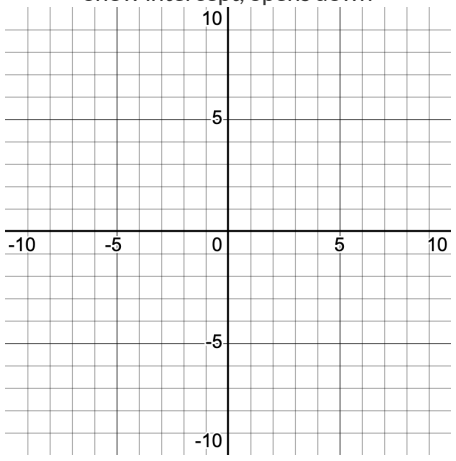
x	1	2	3	4	5	6	7
y	100	102	105	109	114	120	127

Linear
Quadratic
Neither

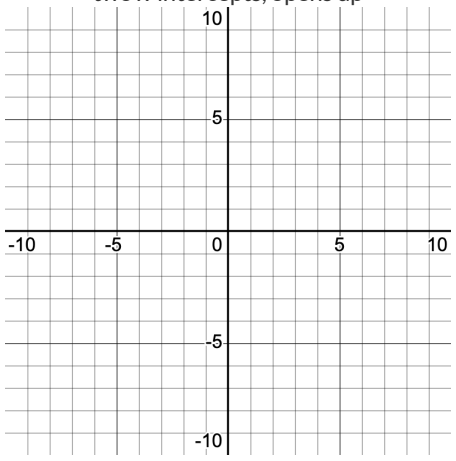
Parabolas

1) Sketch a ***parabola*** on each of the grids below that matches the description.

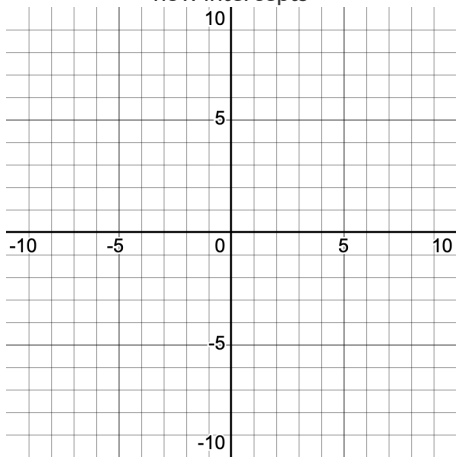
one x-intercept, opens down



two x-intercepts, opens up



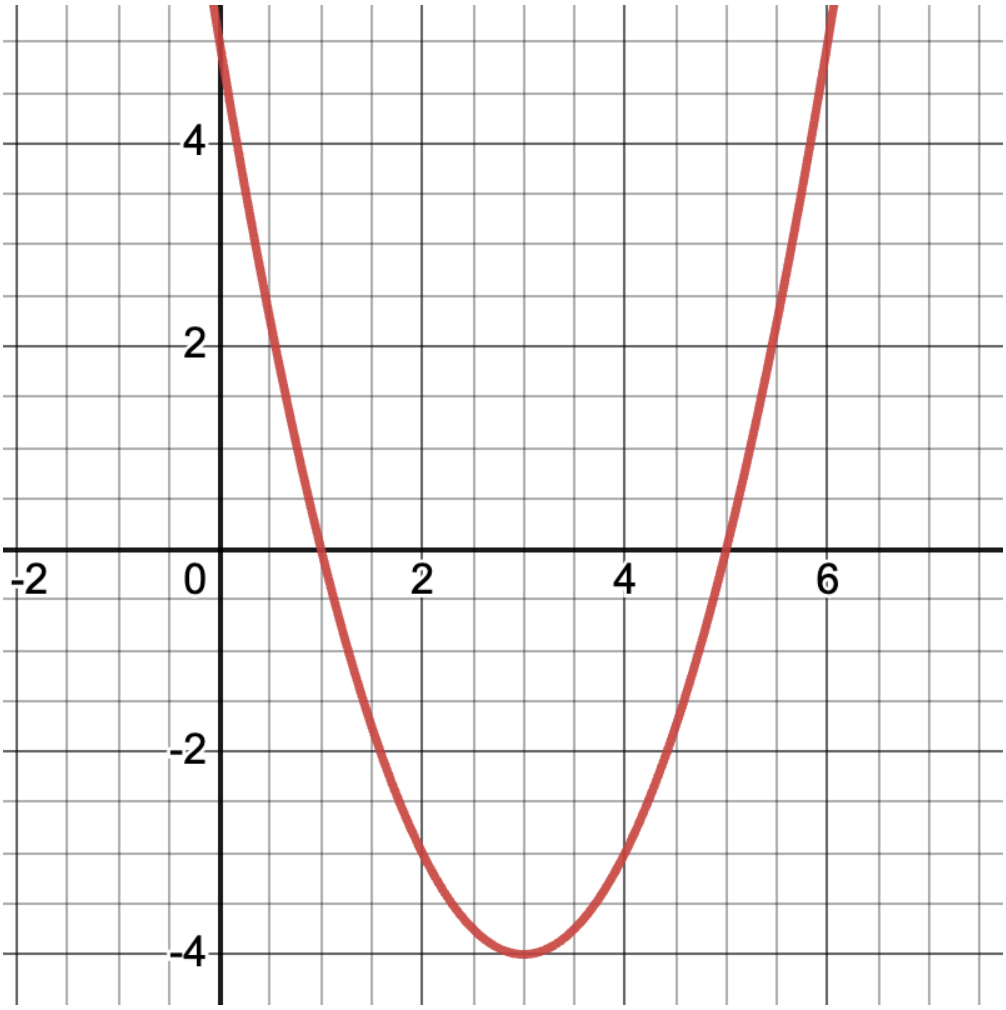
no x-intercepts



2) Label the ***vertex***, ***root(s)***, and ***y-intercept*** of the parabola below with:

A) their coordinates

B) the vocabulary word (above) that describes each



3) Draw a dotted line representing the ***axis of symmetry*** and label it with the equation that defines it.

Graphing Quadratic Models

*For this page, you'll need to have **Exploring Quadratic Functions(Desmos)** open on your computer.*
The parabola you'll see is the graph of $g(x) = x^2$. Another, **identical** parabola is hiding behind it.
This second parabola is written in Vertex Form: $f(x) = a(x - h)^2 + k$. Each coefficient starts at values to make $f(x)$ equivalent to $g(x)$.

1) Using the starting values of a, h, and k you see for $f(x)$ in Desmos, rewrite $g(x) = x^2$ in Vertex Form. $g(x) =$ _____

Magnitude a

2) Try changing the value of a to -4, 0, and 2, graphing each parabola in the squares below. **Label the vertex "V" and any roots with "R"!**

$a = -4$ y x

$a = 0$ y x

$a = 2$ y x

3) What does a tell us about a parabola? _____

_____

Horizontal Translation h

4) Set a back to 1. Change the value of h to -5, 0, and 5, graphing each parabola in the squares below. **Label the vertex "V" and any roots "R"!**

$h = -5$ y x

$h = 0$ y x

$h = 5$ y x

5) What does h tell us about a parabola? _____

Vertical Translation k

6) Set h back to 0. Change the value of k to -5, 0, and 5, graphing each parabola in the squares below. **Label the vertex "V" and any roots "R"!**

$k = -5$ y x

$k = 0$ y x

$k = 5$ y x

7) What does k tell us about a parabola? _____

Modeling Fuel Efficiency v. Speed

Open your copy of the Fuel Efficiency Starter File and click "Run".

num-sqr

Before we try to model our fuel-efficiency data, we need to learn a new Pyret function!

1) Can you predict what the output of the `num-sqr` expressions below will be?

Test them out in the Interactions Area, and record the results. `num-sqr(4)` _____ `num-sqr(6 - 2)` _____

2) What is the Contract for `num-sqr` ? _____

3) What does `num-sqr` do? _____

Interpreting a Quadratic Model

In the Definitions Area of your Fuel Efficiency Starter File, you'll find the definition of a quadratic model `quad1` .

4) In `quad1` , the value of a is _____, the value of h is _____, and the value of k is _____

5) Fit this model to your dataset, using `fit-model` . What S-value did you get? _____

Hint: If you forgot the contract for `fit-model` *, look it up in the contracts pages!*

6) In your own words, describe what needs to change about this model to fit the data. _____

_____

Modeling Fuel Efficiency

Vertex Form: $f(x) = a(x-h)^2 + k$	• a: determines whether the parabola opens up or down and how steep the curve is • h: horizontal shift (also the x-coordinate of the vertex! h is often 0) • k: vertical shift (also the y-coordinate of the vertex!)

7) We've determined that peak fuel efficiency is around 45 mph. What variable in the equation should we replace with 45? _____

Update the definition of `quad1` , click "Run" and re-fit the model. What S-value did you get? _____

8) What y-coordinate of the vertex (*vertical shift*) would best match the shape of the curve? _____

Update the definition of `quad1` , click "Run" and re-fit the model. What S-value did you get? _____

9) What value of a best matches the shape of the curve? _____

Update the definition of `quad1` , click "Run" and re-fit the model. What S-value did you get? _____

10) Make any small changes you'd like, trying to get S as low as you can. Write your final definition below.

`fun` `f(x) :` _____ **`end`** S: _____

What does this model actually mean?

After experimenting, I came up with a quadratic model for this dataset showing that _____ (x-variable) is correlated to _____ (y-variable). The error in the model is described by an ***S-value*** of about _____ (S) _____ (units), which is _____ (insignificant, moderate, significant, extreme) considering that _____ (y-variable) in this dataset range from _____ (lowest y-value) to _____ (highest y-value). The vertex of the parabola drawn by this model is a _____ (minima or maxima?) at about _____ (x, y) which means that _____

Before this point, as speed increases, `mpg` _____. *After* this point, as speed increases `mpg` _____

What Kind of Model? (Definitions)

Decide whether each representation describes a **linear** function, a **quadratic** function, or neither. If the function is quadratic, identify whether the **form** used is Vertex, Standard, or Factored.

1) $f(x) = 3x^2 + 22$

Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

2) $g(x) = 2(x - 11)(x - 243)$

Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

3) $h(y) = 100 - 4y$

Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

4) $z(x) = \frac{3}{5}x + 7$

Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

5)

```
fun graph(x): 12 * x end
```

Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

6)

```
fun m(p): 2 * ((p - 5) * (p - 16)) end
```

Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

7) $r(s) = 42(s - 10)^2 - 3$

Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

8)

```
fun f(x): (2 * num-sqr(x - 1)) + 15 end
```

Linear Quadratic Neither

If Quadratic, is it Vertex, Standard, or Factored?

If Quadratic, what does the form tell you?

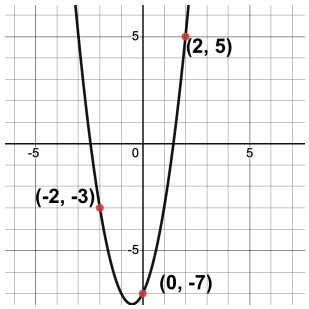
Matching Standard Form to Parabolas

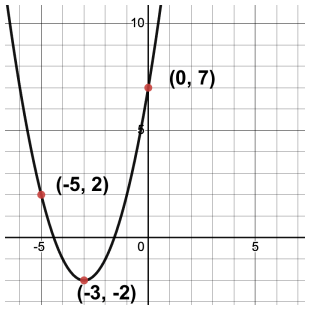
Factored Form: $y = ax^2 + bx + c$

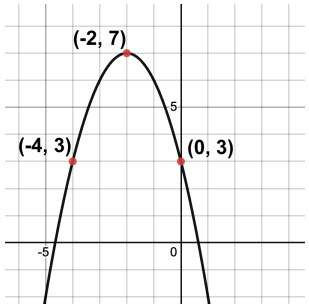
- a: determines whether the parabola opens up or down and how steep the curve is
- c: y-intercept

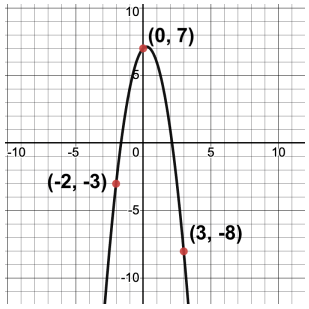
Match each definition below to the graph it describes.

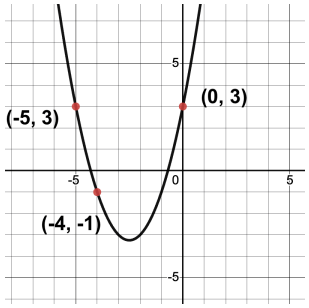
$y = -x^2 - 4x - 3$	1	A
$y = 2x^2 + 2x - 7$	2	B
$y = x^2 + 5x + 3$	3	C
$y = x^2 + 6x + 7$	4	D
$y = -2x^2 + x + 7$	5	E











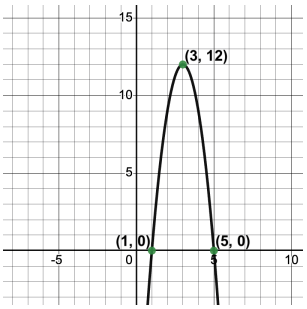
Matching Factored Form to Graphs

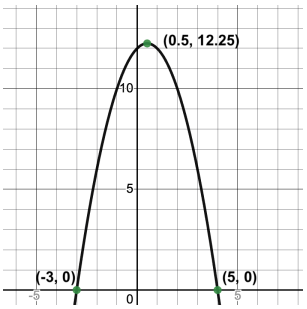
Factored Form: $y = a(x - r_1)(x - r_2)$

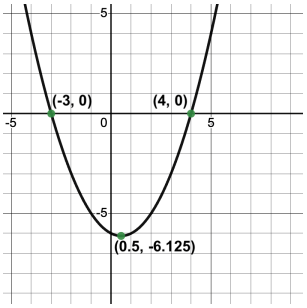
- a: determines whether the parabola opens up or down and how steep the curve is
- r_1 and r_2: roots, x-intercepts

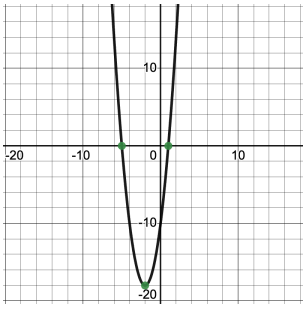
Match each definition below to the graph it describes.

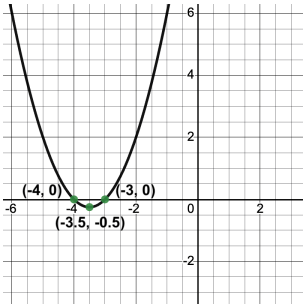
$y = 2(x - 1)(x + 5)$	**1**	**A**
$y = (x + 3)(x + 4)$	**2**	**B**
$y = -3(x - 1)(x - 5)$	**3**	**C**
$y = \frac{1}{2}(x + 3)(x - 4)$	**4**	**D**
$y = -(x - 5)(x + 3)$	**5**	**E**











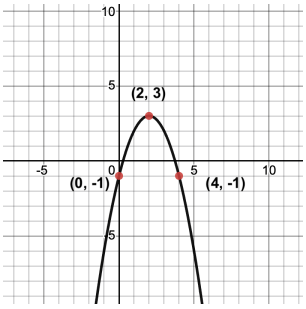
Matching Vertex Form to Graphs

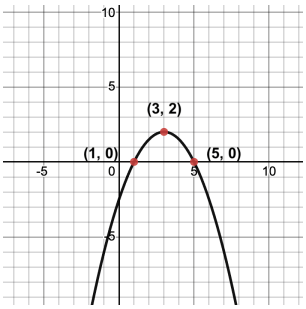
Vertex Form: $y = a(x - h)^2 + k$

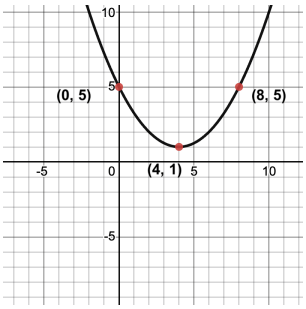
- a: determines whether the parabola opens up or down and how steep the curve is
- h: x-coordinate of the vertex
- k: y-coordinate of the vertex

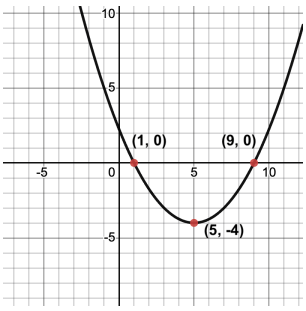
Match each definition below to the graph it describes.

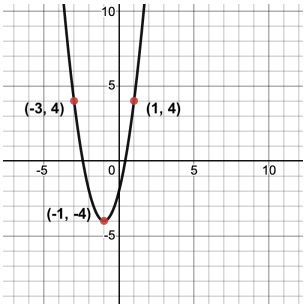
$f(x) = -0.5(x - 3)^2 + 2$	**1**	**A**
$g(x) = 2(x + 1)^2 - 4$	**2**	**B**
$h(x) = -(x - 2)^2 + 3$	**3**	**C**
$j(x) = 0.25(x - 5)^2 - 4$	**4**	**D**
$k(x) = \frac{1}{4}(x - 4)^2 + 1)$	**5**	**E**







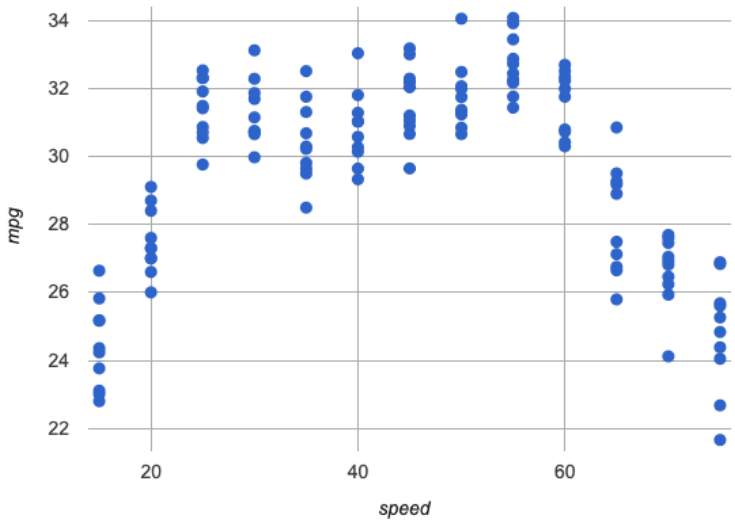




Build a Model from Samples

For this page, you'll need to open the Fuel Efficiency Starter File on your computer. If you haven't already, select ***Save a Copy*** *from the "File" menu to make a copy of the file that's just for you.* ***Read the comments at the top of the file****, which describe what each column in the dataset means.*

The **standard form** of a quadratic equation is $y = Ax^2 + Bx + C$



1) Choose a point from the **left-most column** of dots, and fill in the **standard form** equation below:

$$\underset{\text{y (mpg)}}{\_\_\_\_\_} = A(\underset{\text{x (speed)}}{\_\_\_\_\_})^2 + B(\underset{\text{x (speed)}}{\_\_\_\_\_}) + C$$

2) Choose a point from the **center-most column** of dots, and fill in the **standard form** equation below:

$$\underset{\text{y (mpg)}}{\_\_\_\_\_} = A(\underset{\text{x (speed)}}{\_\_\_\_\_})^2 + B(\underset{\text{x (speed)}}{\_\_\_\_\_}) + C$$

3) Choose a point from the **right-most column** of dots, and fill in the **standard form** equation below:

$$\underset{\text{y (mpg)}}{\_\_\_\_\_} = A(\underset{\text{x (speed)}}{\_\_\_\_\_})^2 + B(\underset{\text{x (speed)}}{\_\_\_\_\_}) + C$$

4) In the space below - or on another sheet of paper - solve this series of equations for A, B, and C:

5) Write your finished model in function and Pyret notation below, then define the function `f` in Pyret and try it out using `fit-model`!

Function Notation	Pyret Notation
$f(x) = A(\underset{\text{x (speed)}}{\_\_\_\_\_})^2 + B(\underset{\text{x (speed)}}{\_\_\_\_\_}) + C$	`fun f(x): ((_____ * num-sqr) + (_____ * x)) + _____ end`

Looking up Rows and Columns

We can define names for values in Pyret, the same way we do in math:

```
name = "Shanti"
age = 16
logo = star(50, "solid", "red")
```

When **looking up a data Row** from a Table, programmers use the `row-n` function. This function takes a Table and a Number as its inputs. The numbers tell the computer which Row we want from the Table. *Note: Rows are numbered starting at zero!*
For example:

```
sasha   = row-n(animals-table, 0)  # define Sasha to be the first row
mittens = row-n(animals-table, 2)  # define Mittens to be the third row
```

When we define these rows, it's more useful to name them based on their *properties* , rather than their identifiers:

```
cat-row = row-n(animals-table,  0) # Sasha is a cat
dog-row = row-n(animals-table, 10) # Toggle is a dog
```

When **looking up a column** from a Row, programmers use square brackets and the name of the column they want.
For example:

```
# these two lines do the same thing! We can use the defined name to simplify our code
row-n(animals-table, 0)["age"] # look up Sasha's age (in row 0)
cat-row["species"]             # look up Sasha's age (using the defined name)
dog-row["age"]                 # look up Toggle's age (using the defined name)
```

Lookup Questions

The table below represents four pets at an animal shelter:

`pets-table`

name	sex	age	pounds
`"Toggle"`	`"female"`	3	48
`"Fritz"`	`"male"`	4	92
`"Nori"`	`"female"`	6	35.3
`"Maple"`	`"female"`	3	51.6

1) *Match* each Lookup Question (left) to the code that will give the answer (right).

"How much does Maple weigh?"	**1**	**A**	`row-n(pets-table, 3)`
"Which is the last row in the table?	**2**	**B**	`row-n(pets-table, 2)["name"]`
"What is Fritz's sex?"	**3**	**C**	`row-n(pets-table, 1)["sex"]`
"What's the third animal's name?"	**4**	**D**	`row-n(pets-table, 3)["age"]`
"How much does Nori weigh?"	**5**	**E**	`row-n(pets-table, 3)["pounds"]`
"How old is Maple?"	**6**	**F**	`row-n(pets-table, 0)`
"What is Toggle's sex?"	**7**	**G**	`row-n(pets-table, 2)["pounds"]`
"What is the first row in the table?"	**8**	**H**	`row-n(pets-table, 0)["sex"]`

2) For each value on the left, write the Pyret expression that will produce that value on the right. The first one has been completed for you.

a.	`"Maple"`	row-n(pets-table, 3)["name"]
b.	`"male"`	
c.	4	
d.	48	
e.	`"Nori"`	

More Practice with Lookups

Consider `shapes-table` below, and the four value definitions that follow.

name	corners	is-round
`"triangle"`	`3`	`false`
`"square"`	`4`	`false`
`"rectangle"`	`4`	`false`
`"circle"`	`0`	`true`

```
shapeA = row-n(shapes-table, 0)
shapeB = row-n(shapes-table, 1)
shapeC = row-n(shapes-table, 2)
shapeD = row-n(shapes-table, 3)
```

1) *Match* each Pyret expression (left) to the description of what it evaluates to (right).

`shapeD`	**1**	**A**	Evaluates to 4
`shapeA`	**2**	**B**	Evaluates to the last row in the table
`shapeB["corners"]`	**3**	**C**	Evaluates to `"square"`
`shapeC["is-round"]`	**4**	**D**	Evaluates to `true`
`shapeB["name"]`	**5**	**E**	Evaluates to `false`
`shapeA["corners"]`	**6**	**F**	Evaluates to 3
`shapeD["name"] == "circle"`	**7**	**G**	Evaluates to the first row in the table

2) For each value on the left, write the Pyret expression that will produce that value on the right. The first one has been completed for you.

a.	`"rectangle"`	`shapeC["name"]`
b.	`"square"`	
c.	`4`	
d.	`0`	
e.	`true`	

Defining Rows

Remember: rows start at index zero!
We've already given you two row definitions, `cat-row` and `dog-row`:

```
cat-row = row-n(animals-table,  0)  # Sasha is a cat
dog-row = row-n(animals-table, 10)  # Toggle is a dog
```

1) Use the Animals Table to identify the index of a row containing…

a lizard _____

a rabbit _____

a fixed animal _____

a male animal _____

a female animal _____

a hermaphroditic animal _____

an unfixed animal _____

a young animal (<2 years) _____

an old animal (>10 years) _____

2) What code would you write to define `lizard-row`?

3) What code would you write to define `rabbit-row`?

4) What code would you write to define `fixed-row`?

5) What code would you write to define `male-row`?

6) What code would you write to define `female-row`?

7) What code would you write to define `hermaphrodite-row`?

8) What code would you write to define `young-row`?

9) What code would you write to define `old-row`?

Add this code to your Animals Starter File! You'll want these rows for later!

Exploring the Covid Dataset

*For this page, you'll need to have the Covid Spread Starter File open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

1) Take a look at the Definitions Area and find the "Notes on Columns". What is the start date for the data in this table? _____

2) Click "Run", and evaluate `covid-table` in the Interactions Area. What do you notice? _____

_____

3) Evaluate `MA1` in the Interactions Area. What does it return? _____

4) Evaluate `CT1`. What information do you learn? _____

5) Evaluate `NH1`. Why is it "unbound" and how could we make it work? _____

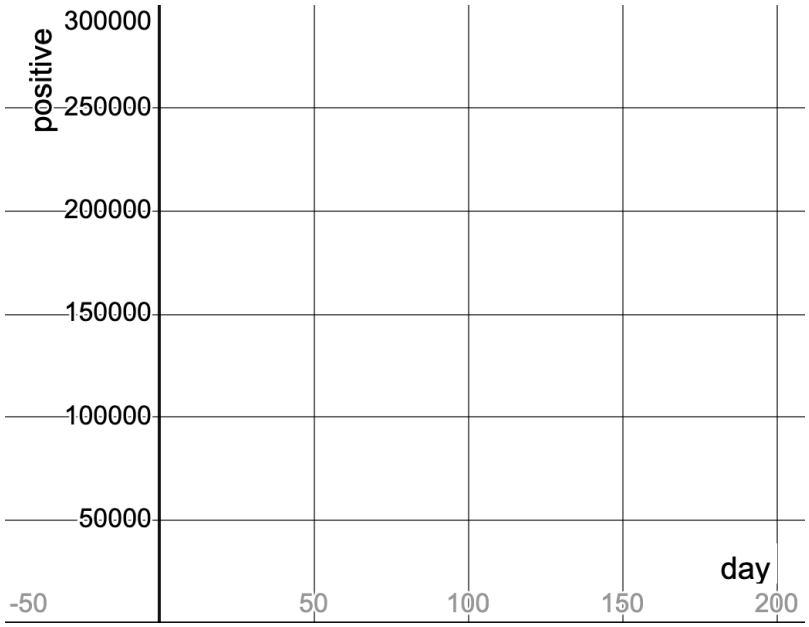
6) Define three new Rows called `NH1`, `RI1` and `VT1`, for New Hampshire, Rhode Island and Vermont. Click "Run" and test them out.

a. How many people in **Vermont** had tested positive by June 10th, 2020? _____

b. How many people in **New Hampshire** tested positive by June 10th, 2020? _____

c. How many people in **Rhode Island** tested positive by June 10th, 2020? _____

7) In Pyret, make a scatter plot showing the relationship between `day` and `positive`, using `state` as your labels, then sketch the resulting scatter plot below.



8) In which state did the number of cases grow *fastest*?

_____

9) In which state did the number of cases grow *slowest*?

_____

10) Are these strong or weak relationship(s)?

_____

11) What do you **Notice?** _____

_____

_____

12) What do you **Wonder?** _____

_____

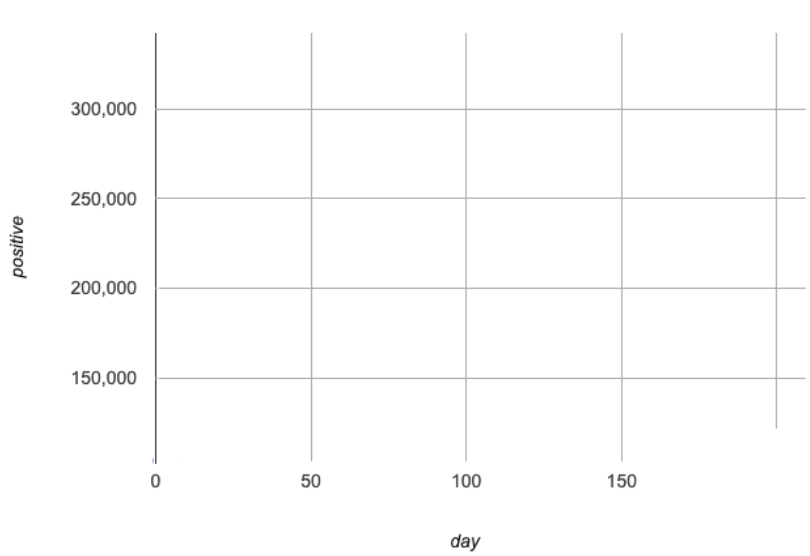
_____

Linear Models for MA-table

*For this page, you'll need to have the Covid Spread Starter File open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

This starter file defines a table *just for MA data*, called `MA-table`: `MA-table = filter(covid-table, is-MA)`

1) Make a scatter plot from `MA-table` showing the relationship between `day` and `positive`, using `state` as the labels. Sketch the plot on the right.



As we've seen, it's easy to fit a linear model to any dataset in Pyret, so let's start by testing how well a linear function could model this data.

2) Use `lr-plot` to obtain the best-possible **linear model** for the MA Covid dataset, and write it below:

y = _____ $S =$ _____

Note: Pyret uses `e` for scientific notation. For example: $2.46e3 = 2.46 \times 10^3 = 2460$

3) The optimized linear model for this dataset predicts an _____ (increase / decrease) of about _____ (slope) _____ (y-variable) per _____ (x-variable).

The error in the model is described by an ***S-value*** of about _____ (S) _____ (units), which is a _____ (poor, ok, good) fit considering that

_____ (y-variable) in this dataset range from _____ (lowest y-value) to _____ (highest y-value).

4) Change the definition of the `linear` function in the Covid Spread Starter File to match the model produced by `lr-plot` and "Save".

5) Do you think a linear function is a good model for this dataset? Why or why not? _____

_____

_____

_____

★ What do you think the code that defines `MA-table` is actually doing? _____

_____

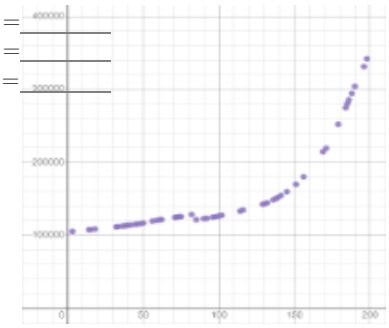
Quadratic Models for MA-table

Fitting the Model Visually $f(x) = a(x - h)^2 + k$

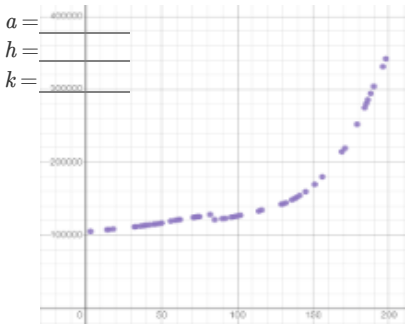
*For this section, you'll need to have **Slide 1: Quadratic Model for MA** of **Modeling Covid Spread (Desmos)** open on your computer.*

1) Try changing the values of a, h and k to find three promising quadratic models, graphing each one and labeling your values in the grids below.

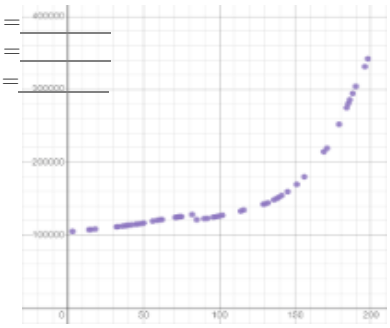
$a =$ _____
$h =$ _____
$k =$ _____



$a =$ _____
$h =$ _____
$k =$ _____



$a =$ _____
$h =$ _____
$k =$ _____



2) Do your quadratic models open up or down? _____. What does that tell us about a? _____.

3) Describe one of your models: Where is the vertex? (_____, _____) What is the horizontal shift? _____ The vertical shift? _____
(x, y) (h, k)

4) Which quadratic form would be the easiest to fit to this data? standard □ factored □ vertex□

Fitting the Model Programmatically $f(x) = a(x - h)^2 + k$

For this section, open your copy of the Covid Spread Starter File.

5) In the space below, define `quadratic1` to be the first model you fit in Desmos.

```
fun quadratic1(x): ( _____ * (num-sqr( x - _____ ))) + _____ end
                     a                  h              k
```

6) Return to Covid Spread Starter File and update the definitions for `quadratic1`, `quadratic2`, and `quadratic3`. Then click "Run" to load your updated definition.

7) Use `fit-model` to determine the S-value of each model using the `MA-table`.
Hint: If you forgot the contract for `fit-model`, look it up in the contracts pages!

S for `quadratic1`: _____ S for `quadratic2`: _____ S for `quadratic3`: _____

What does this model actually mean?

After experimenting, the best quadratic model I came up with for this dataset shows that _____ are correlated to _____.
x-variable y-variable

The vertex of the parabola drawn by this model is a _____ at about _____, which predicts that _____
minima or maxima? (x, y)

_____.

The error in the model is described by an ***S-value*** of about _____, which is a _____
S units bad, ok, good

fit considering that _____ in this dataset range from _____ to _____.
y-variable lowest y-value highest y-value

Are Quadratic Models a Good Fit for This Data?

8) Would you feel good about making predictions based on these models? Why or why not? _____

_____

_____

What Kind of Model? (Tables)

Decide whether each table is best described by a linear, quadratic, or exponential model. **If the model is exponential:** What is the growth factor? Doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?
HINT: Can you draw the arrows to calculate the first difference? The second? *What does it mean if neither one is constant?*

x	y
1	5
2	10
3	15
4	20
5	25
6	30
7	35

1) Linear Quadratic Exponential _____ factor

x	y
0	10
1	100
2	1000
3	10000
4	100000
5	1000000
6	10000000

2) Linear Quadratic Exponential _____ factor

x	y
70	-169
71	-126
72	-81
73	-34
74	15
75	66
76	119

3) Linear Quadratic Exponential _____ factor

x	y
-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

4) Linear Quadratic Exponential _____ factor

x	y
0	3
1	6
2	12
3	24
4	48
5	96
6	192

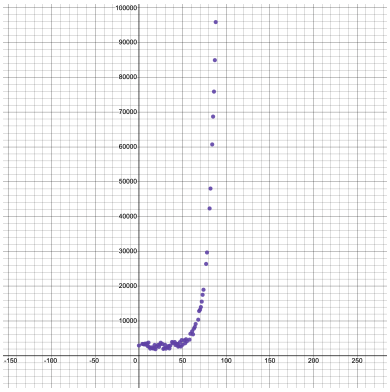
5) Linear Quadratic Exponential _____ factor

x	y
-5	466656
-4	7776
-3	1296
-2	216
-1	36
0	6
1	1

★ Linear Quadratic Exponential _____ factor

What Kind of Model? (Graphs & Plots)

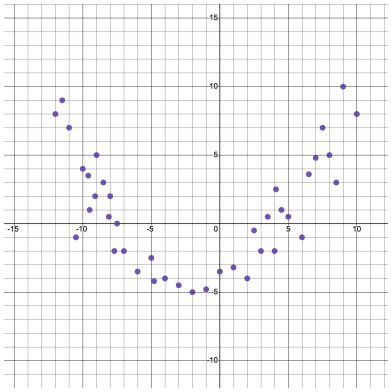
Are these scatter plots best be described by linear, quadratic, or exponential models? **If it's exponential, draw the asymptote!**



1) Linear Quadratic Exponential

How did you know? _____

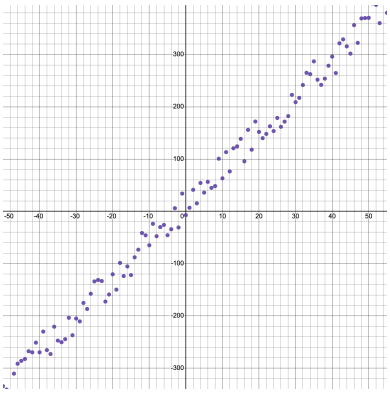
_____



2) Linear Quadratic Exponential

How did you know? _____

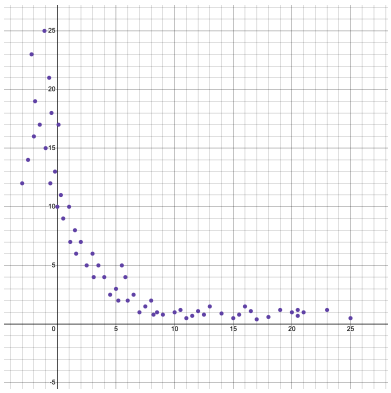
_____



3) Linear Quadratic Exponential

How did you know? _____

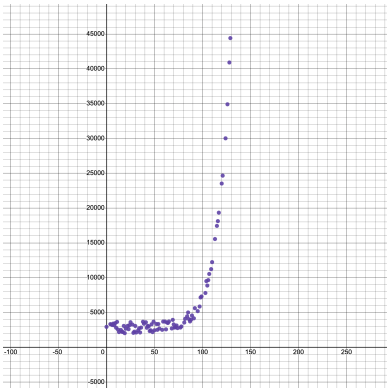
_____



4) Linear Quadratic Exponential

How did you know? _____

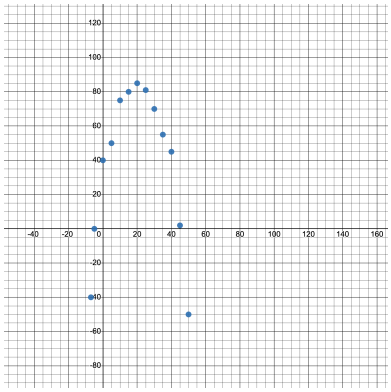
_____



5) Linear Quadratic Exponential

How did you know? _____

_____



6) Linear Quadratic Exponential

How did you know? _____

_____

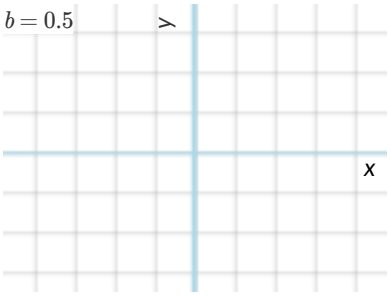
Graphing Exponential Models: $f(x) = ab^x + k$

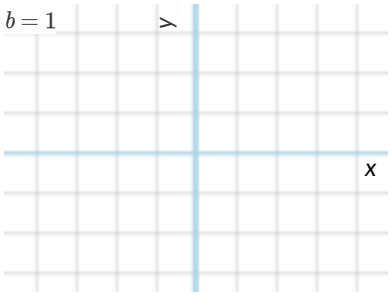
*For this page, you'll need to have **Slide 3: Exploring Exponential Models** of **Modeling Covid Spread (Desmos)** open on your computer.*
The curve you'll see is the graph of $h(x) = 2^x$. Another curve $f(x)$ is hiding behind it with identical coefficients: $k = 0$, $b = 2$ and $a = 1$.

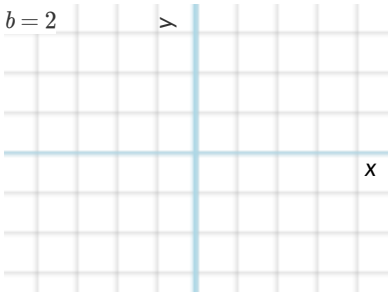
Base b

1) Make sure $k = 0$ and $a = 1$. Experiment with b. **For what values of b is the function *undefined***, with the line disappearing? _____

2) Keeping $a = 1$ and $k = 0$, change b to 0.5, 1, and 2, graphing each curve below. For each curve, **label the coordinates at x=1, 2, and 3.**







3) What does b tell us about an exponential function, when $b > 1$? _____

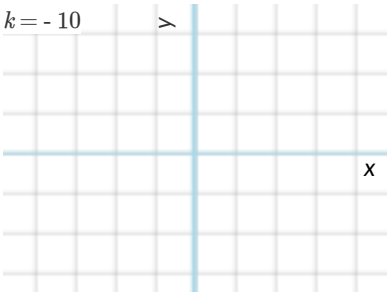
_____

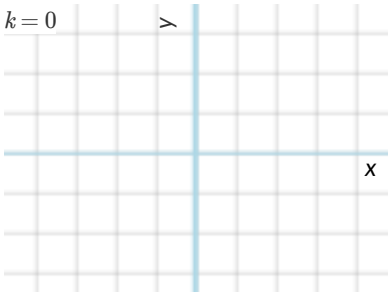
4) What does b tell us about an exponential function, when $0 < b < 1$? _____

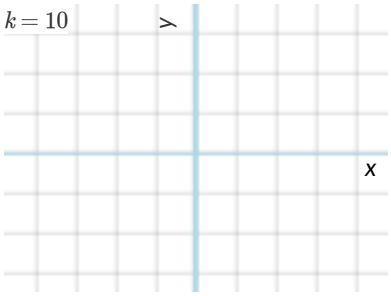
_____

Vertical Shift…and Horizontal Asymptote k

5) Keeping $a = 1$ and $b = 2$, try changing the value of k to -10, 0, and 10, graphing each curve in the squares below. For each curve, find and label the y-value where the curve is "most horizontal", then **draw a horizontal line at that y-value**.





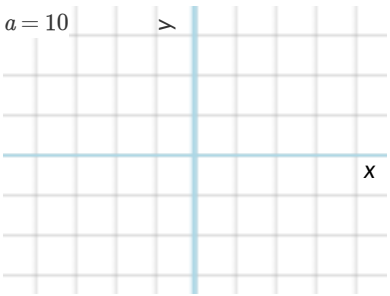


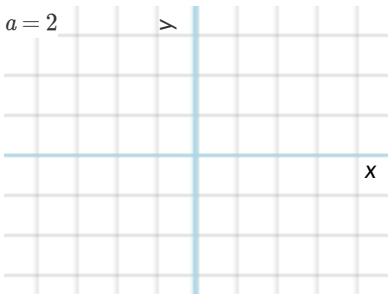
6) What does k tell us about an exponential function? _____

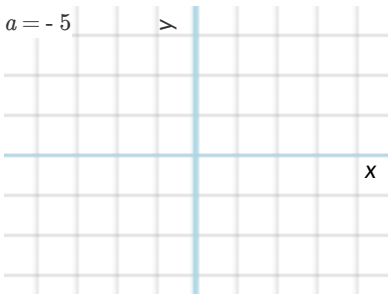
_____

Initial Value a

7) Set $k = 0$ and $b = 2$. Change the value of a to 10, 2, and -5, graphing each curve in the squares below.
For each curve, label the y-intercept (x=0).







8) What does a tell us about an exponential function? _____

What Kind of Model? (Descriptions)

Decide whether each situation is best described by a linear, quadratic, or exponential function.
If the function is exponential: What is the growth factor. Is it doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?

Car Values

A particular kind of car sells for $32,000, and its resale value drops by 12.5% each year.

1) Is the function increasing or decreasing? _____

2) When the car is brand-new (x=0), how much is it worth? _____

3) How much is it worth after…

(1 year) x=1	(2 years) x=2	x=3	x =4

4) What is the **form** of this function? linear □ quadratic □ exponential□

5) **If it's exponential**,
Fill in the coefficients to write a function that shows the value of the car after a given number of years:

$f(x) =$ _____ (initial value a) _____ (growth factor b)^x + _____ (horizontal asymptote k)

Is it exponential *growth*? □ or *decay*? □

Lemonade Stand

Sally is selling lemonade, for $1.25 a glass in hopes of finally be able to get the power drill she's been wanting. She starts with $20 cash.

6) Is the function increasing or decreasing? _____

7) When Sally starts the day (x=0), how many dollars does she have? _____

8) How many dollars will she have after…

(first sale) x=1	(second sale) x=2	x =3	x = 4

9) What is the **form** of this function? □ linear □quadratic □ exponential

10) **If it's exponential**,

Fill in the coefficients to write a function that shows how much Sally has saved after a given number of sales:

$f(x) =$ _____ (initial value a) _____ (growth factor b)^x + _____ (horizontal asymptote k)

Is it exponential *growth*? □ or *decay*? □

What Kind of Model? (Definitions)

Decide whether each representation describes a linear, quadratic, or exponential function.
If the function is exponential: Identify the growth factor and the initial value.

$f(x) = 6x^2 - 5$

1) Linear Quadratic Exponential

How did you know? _____

_____

If it's exponential, what's the _____ (growth factor) _____ (initial value)?

$\text{miles}(\text{hours}) = \dfrac{22 \times \text{hours} + 14}{12 - 9}$

2) Linear Quadratic Exponential

How did you know? _____

_____

If it's exponential, what's the _____ (growth factor) _____ (initial value)?

$\text{cost}(w) = 5(1.2^w) + 16$

3) Linear Quadratic Exponential

How did you know? _____

_____

If it's exponential, what's the _____ (growth factor) _____ (initial value)?

$t(g) = 42 - 2g^2$

4) Linear Quadratic Exponential

How did you know? _____

_____

If it's exponential, what's the _____ (growth factor) _____ (initial value)?

$\text{price}(d) = d^2 + 6d$

5) Linear Quadratic Exponential

How did you know? _____

_____

If it's exponential, what's the _____ (growth factor) _____ (initial value)?

$j(x) = \left(\dfrac{1}{2}\right)^x + 22$

6) Linear Quadratic Exponential

How did you know? _____

_____

If it's exponential, what's the _____ (growth factor) _____ (initial value)?

$f(a) = 20000 - 4.1^a$

7) Linear Quadratic Exponential

How did you know? _____

_____

If it's exponential, what's the _____ (growth factor) _____ (initial value)?

$g(x) = 8(3^{-4x})$

8) Linear Quadratic Exponential

How did you know? _____

_____

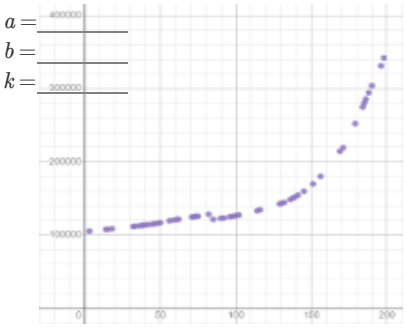
If it's exponential, what's the _____ (growth factor) _____ (initial value)?

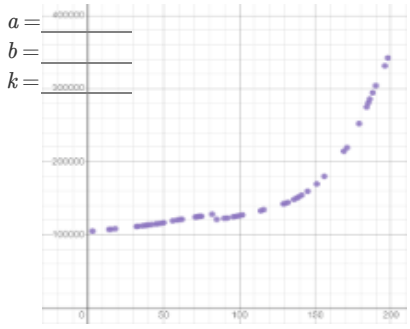
Exponential Models: $f(x) = ab^x + k$

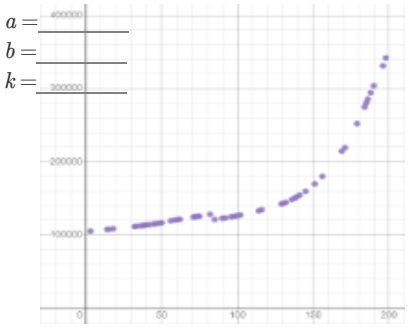
Fitting the Model Visually - MA

*For this section, you'll need to have **Slide 4: Exponential Model for MA** of **Modeling Covid Spread (Desmos)** open on your computer.*

1) Try changing the value of k, then a, then b to find three promising exponential models, graphing each one and labeling your values on the grids below.







Fitting the Model Programmatically - MA

For this section, open your copy of the Covid Spread Starter File.

2) In the space below, define `exponential` for one of the models you fit in Desmos.

`fun exponential(x): (` ____a____ `* num-expt(` ____b____ `, (~1 * x))) +` ____k____ `end`

Two Notes on this function definition:

- `num-expt` *is the function that we use for exponents. It takes in 2 numbers: the base and the power, in this case* b *and* x.
- `(~1 * x)` *at first it may appear that* x *is being multiplied by negative 1, but it is actually being multiplied by* **~1** *(literally the value "roughly 1"). This tells Pyret to round off the calculation, prioritizing **speed** over **precision** to get a result that is "roughly accurate". We've added this to the function definition so that you won't have to wait for several minutes for Pyret to run* `fit-model` *to get an answer for question 4.*

3) Update the definition for `exponential` in the Definitions Area and click "Run" to reload it.
Then use `fit-model` to determine how closely `exponential` fits the `MA-table` and fill in the blanks below to interpret the model.
Hint: If you forgot the contract for `fit-model`*, look it up in the contracts pages!*

According to this exponential model, on ____June 9, 2020____ (day zero) there were about ____a____ + ____k____ ____y-units____ in MA, for a total of about ____a + k____. This number grew exponentially, increasing by a factor of ____Growth Factor: b____ or ____Growth Rate: (b - 1) × 100____ % every day.

The error in the model is described by an ***S-value*** of about ____S____ ____units____, which is a(n) ____bad, ok, good____ model considering that ____y-units____ in this dataset range from ____lowest y-value____ to ____highest y-value____.

4) Estimate how many positive cases there will be after X days by **looking at graph with your eyes**, then use your model to find the answer.

Using your...	Eyes	Model	***Using your...***	Eyes	Model	***Using your...***	Eyes	Model
50 days			150 days			250 days		
350 days			450 days			550 days		

★ Rewrite the model to make Pyret do these calculations with extreme precision. (Remove the part where it multiplies by ~1 .)
WARNING: Be sure to save your work first, as there's a good chance this will lock up your browser and require force-quitting!

What changed? _____

Data scientists perform calculations to do things like send satellites to far-away planets, or analyze large populations of a billion or more. You know that the pros of using ~1 involve speed. **What are the potential downsides of using ~1 to speed up a calculation?**

_____

_____

Modeling Other States

*For this page, you'll need to have the Covid Spread Starter File open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

1) Find the function called `is-MA` in the Definitions Area under "Define some helper functions" and read the comments carefully!

a. What is the Domain of `is-MA` ? _____ What is its Range? _____

b. What do you *think* `is-MA(MA1)` will evalute to? _____. `is-MA(CT1)` ? _____. `is-MA(ME1)` ? _____

Try typing each of the `is-MA` expressions into the Interactions Area on the right and confirm you were correct.

2) Find `MA-table` in the Definitions Area under "Define some grouped and/or random samples". What is that code doing? _____

_____

_____

3) **Define a new function** `is-VT` and **create a new grouped sample** called `VT-table` .

Hint: You can use the code for `is-MA` and `MA-table` as a model.

Modeling VT

*For this section, in addition to Pyret, you will need to have **Slide 5: Exponential Model for VT** of **Modeling Covid Spread (Desmos)** open on your computer.*

4) Use `lr-plot` to obtain the best-possible linear model for the relationship between `day` and `positive` in the `VT-table` , then fill in the blanks below:

The optimized linear model for this dataset predicts an _____ (increase / decrease) of about _____ (slope) _____ (y-variable) per _____ (x-variable).

The error in the model is described by an ***S-value*** of about _____ (S) _____ (units), which is _____ (insignificant, moderate, significant, extreme)

considering that _____ (y-variable) in this dataset range from _____ (lowest y-value) to _____ (highest y-value).

5) Use **Slide 5: Exponential Model for VT** of **Modeling Covid Spread (Desmos)** to come up with the best exponential model you can for the Vermont dataset, and write it below:

_____

6) Add a definition for `exponential-VT` to the Definitions area of Covid Spread Starter File using the model you just found.

- Click "Run" to load your definition.
- Then fit the model using `VT-table`

According to this exponential model, on June 9, 2020 (day zero) there were about _____ (a) + _____ (k) _____ (y-units) in VT, for a total

of about _____ (a + k). This number grew exponentially, increasing by a factor of _____ (Growth Factor: b) or _____ (Growth Rate: (b - 1) × 100) % every

day. The error in the model is described by an ***S-value*** of about _____ (S) _____ (units), which is

_____ (insignificant, moderate, significant, extreme) considering that _____ (y-units) in this dataset range from _____ (lowest y-value) to _____ (highest y-value).

7) Are exponential models a good fit for this data? Why or why not? _____

_____

What Kind of Model? (Descriptions 2)

Decide whether each situation is best described by a linear, quadratic, or exponential function.
If the function is exponential: What is the growth factor. Is it doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?

High Fives

Two students started a club. At every meeting, students in attendance must high-five each of the other students. Club membership has been growing steadily by one student each meeting.

1) Is the function increasing or decreasing? _____

2) When the 2 students started the club (x=0), how many high-fives happened? _____

3) How many high-fives happen at the subsequent meetings…

(3 students) x=1	(4 students) x=2	x=3	x=4

4) What is the **form** of this function? linear □ quadratic□ exponential□

5) **If it's exponential**,
Fill in the coefficients to write a function that shows the how many high-fives happen for a given number of students:

$f(x) =$ _____ (initial value a) _____ (growth factor b)^x + _____ (horizontal asymptote k)

Is it exponential *growth* ? □ or *decay* ? □

Going Viral

A student posted their animation of a puppy doing a back flip into a pile of laundry and the meme went viral! Every person that sees the meme falls in love with it and shares it with 10 new friends.

6) Is the function increasing or decreasing? _____

7) When the student posts it (x=0), how many total times has it been shared? _____

8) How many times will it have been shared after…

(the next person shares) x=1	(their friends share) x=2	x=3	x =4

9) What is the **form** of this function? linear □ quadratic □ exponential□

10) **If it's exponential**,
Fill in the coefficients to write a function that shows how many times the meme has been shared after a given number of "share cycles":

$f(x) =$ _____ (initial value a) _____ (growth factor b)^x + _____ (horizontal asymptote k)

Is it exponential *growth* ?□ or *decay* ? □

Campus Housing Data

A college is looking at enrollment and housing data for students who've decided what their major will be, vs. those who are undecided:

	# On Campus	# Off Campus	% On Campus
Undecided	120	80	120/200 = 60%
Decided	80	100	80/180 = 44%

1) According to the table, how many Undecided Majors live *off* -campus? _____

2) According to the table, how many Decided Majors live *on* -campus? _____

3) Who is more likely to live on campus: Decided or Undecided Majors? _____

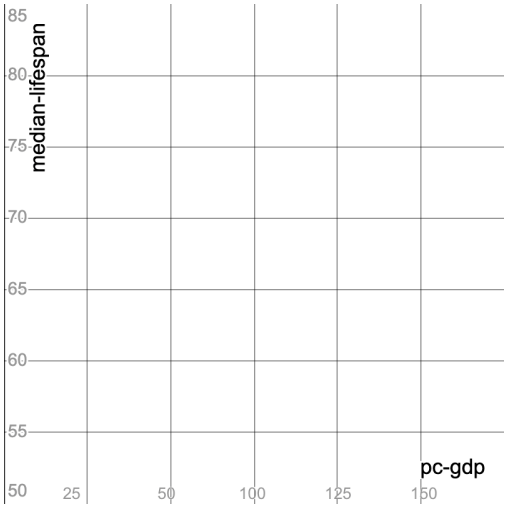
4) Do you think there is a relationship between deciding on a major and living on or off campus? If so, why?

Exploring the Countries Dataset

*For this section, you'll need the [Countries of the World Starter File] open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. The columns in this dataset are described below:*

- **country** - name of the country
- **gdp** - total Gross Domestic Product of the country. GDP is often used to measure the economic health of a country.
- **population** - number of people in the country
- **pc-gdp** - the average GDP *per-person, in thousands* of $US
- **has-univ-healthcare** - indicates if the country has universal healthcare
- **median-lifespan** - the median life expectancy of people in the country

1) Make a scatter plot showing the relationship between `pc-gdp` and `median-lifespan`, and sketch its plot below.



2) What do you **Notice?** _____

3) What do you **Wonder?** _____

4) Are there any countries that stand out? Why or why not? _____

5) Suppose a wealthy country is suffering heavy casualties in a war. Draw a star on the plot, showing where you might expect it to be.

6) Do you think you see a relationship? If so, describe it. Is it linear or nonlinear? Strong or weak?

Fitting Models for the Countries Dataset

For this page you will be working with both the Countries of the World Starter File and the ***Desmos*** *file* ***Fitting Wealth-v-Health and Exploring Logarithmic Models****.*

Find linear, quadratic and exponential models for the relationship between `pc-gdp` and `median-lifespan`. As you find each model:

- update the corresponding definition in the Countries of the World Starter File
- click "Run" to load your new definition
- use `fit-model` to calculate the ***S-value*** *Hint: If you forgot the contract for* `fit-model` *(to calculate S), look it up in the contracts pages!*

1) Find the optimized **linear model** for this data using `lr-plot`.

$linear(x) =$ _____ (slope (m)) x + _____ (y-intercept / vertical shift) _____ (S-value)

The optimized linear model for this dataset predicts that a _____ (x-units) _____ (increase / decrease) in per-capita gdp (x-variable) will increase _____ (y-variable) by _____ (y-units). The error in the model is described by an S - $value$ of about _____ (S) (y-units), which is _____ (insignificant / reasonable / significant / extreme) considering _____ (y-units) in this dataset range from _____ (lowest y-value) to _____ (highest y-value).

2) Find the best **quadratic model** you can, using the second slide *(Wealth-v-Health Quadratic)* in the Desmos activity.

$quadratic(x) =$ _____ (quadratic coefficient (a)) $(x -$ _____ (horizontal shift (h)) $)^2 +$ _____ (vertical shift (k)) _____ (S-value)

The vertex of the parabola drawn by my model is a _____ (minima or maxima?) at about (_____ (x, y)).

- *Before* this point, as _____ (x-variable) increases, _____ (y-variable) _____ (increases or decreases?).
- *After* this point, as _____ (x-variable) increases, _____ (y-variable) _____ (increases or decreases?).

The error in the model is described by an S - $value$ of about _____ (S) (y-units), which is _____ (insignificant / reasonable / significant / extreme) considering _____ (y-units) in this dataset range from _____ (lowest y-value) to _____ (highest y-value).

3) Find the best **exponential model** you can, using the third slide *(Wealth-v-Health Exponential)* in the Desmos activity.

$exponential(x) =$ _____ (initial value (a)) (_____ (growth factor (b)) ^{x}) + _____ (vertical shift (k)) _____ (S-value)

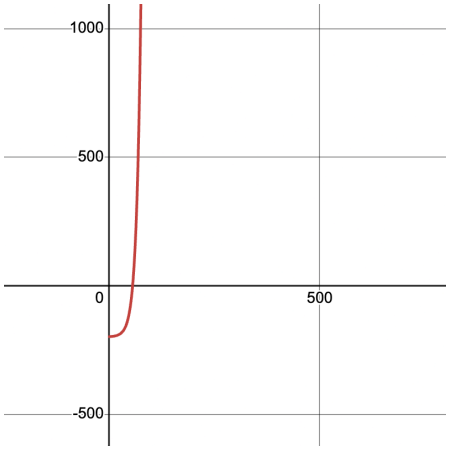
According to this exponential model, a country with a _____ (x-variable) of zero _____ (x-unit) would have a _____ (y-variable) of _____ (a) + _____ (k) _____ (y-units), for a total of about _____ (a + k). This number grows exponentially, increasing by a factor of _____ (Growth Factor: b) or _____ (Growth Rate: (b - 1) × 100) % with every _____ (x-unit) increase in _____ (x-variable).

The error in the model is described by an S - $value$ of about _____ (S) _____ (y-units), which is _____ (insignificant / reasonable / significant / extreme) considering _____ (y-units) in this dataset range from _____ (lowest y-value) to _____ (highest y-value).

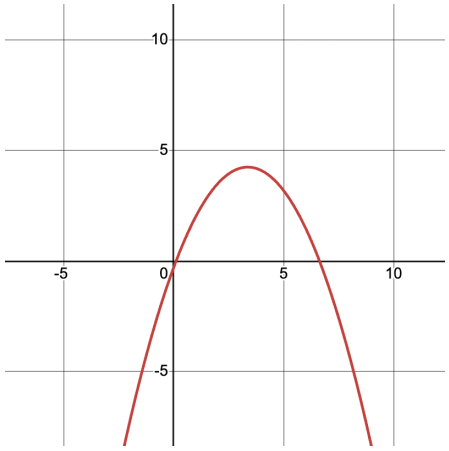
4) Are any of these models a good fit for this data? Why or why not?

What Kind of Model? (Graphs & Plots)

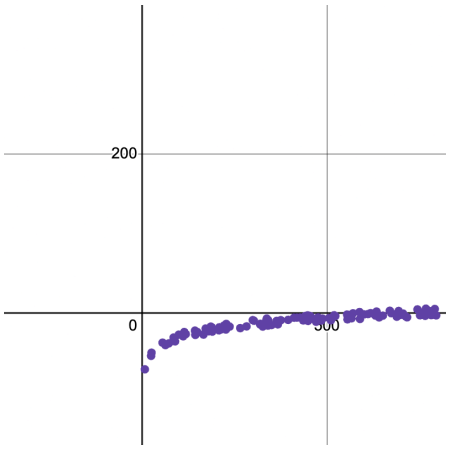
Decide whether each representation is best described by a quadratic, exponential, or logarithmic function. **If you think it's exponential OR logarithmic, draw a diagonal line for $y = x$, and then sketch the reflection of the curve.**



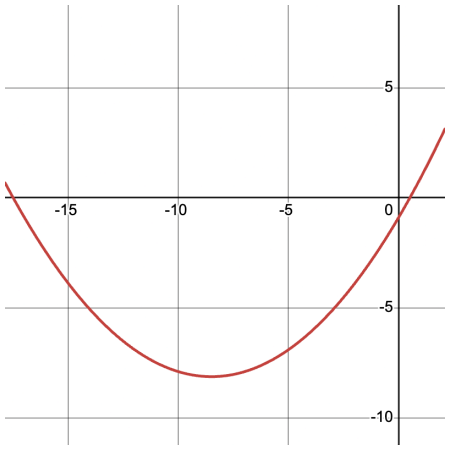
1) Quadratic Exponential Logarithmic



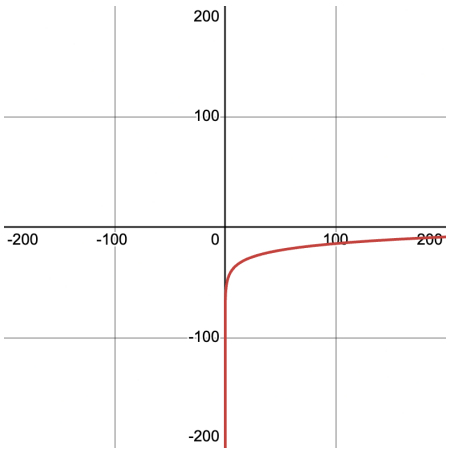
2) Quadratic Exponential Logarithmic



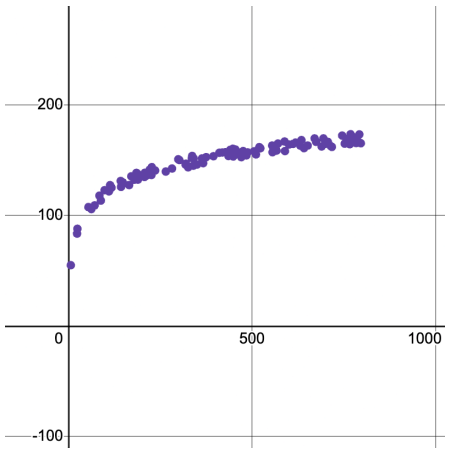
3) Quadratic Exponential Logarithmic



4) Quadratic Exponential Logarithmic



5) Quadratic Exponential Logarithmic



6) Quadratic Exponential Logarithmic

What Kind of Model? (Tables)

Decide whether each representation is best described by a quadratic, exponential, or logarithmic function.
If the function is exponential, find the *base* (also called the *growth factor*): How much does y increase (2x? 10x?) for a single increase in x?
If the function is logarithmic, find the *base*: How much does x need to increase (2x? 10x?) just to get a single increase in y?
HINT: Can you draw the arrows to calculate the first difference? The second? *What does it mean if neither one is constant?*

x	y
1	0
10	1
100	2
1000	3
10000	4
100000	5
1000000	6

1) Quadratic Exponential ___ base Logarithmic ___ base

x	y
0	1
1	10
2	100
3	1000
4	10000
5	100000
6	1000000

2) Quadratic Exponential ___ base Logarithmic ___ base

x	y
70	-169
71	-126
72	-81
73	-34
74	15
75	66
76	119

3) Quadratic Exponential ___ base Logarithmic ___ base

x	y
5	1
10	2
20	3
40	4
80	5
160	6
320	7

4) Quadratic Exponential ___ base Logarithmic ___ base

x	y
-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

5) Quadratic Exponential ___ base Logarithmic ___ base

x	y
1	0
6	1
36	2
216	3
1296	4
7776	5
466656	6

6) Quadratic Exponential ___ base Logarithmic ___ base

Evaluating Logarithmic Expressions

	Expressions	Translation	Evaluates to:
1	$\log_2(8)$	"The power you raise 2 to get 8"	3
2	$\log_2(1)$	"The power you raise 2 to get 1"	0
3	$\log_5(25)$	"The power you raise _____ to get _____"	
4	$\log_5(1)$	"The power you raise _____ to get _____"	
5	$\log_3(81)$	"The power you raise _____ to get _____"	
6	$\log_3(1)$	"The power you raise _____ to get _____"	
7	$\log_2(16)$		
8	$\log_2(32)$		
9	$\log_{10}(1000)$		
10		"The power you raise 0.1 to get 0.01"	
11		"The power you raise 4 to get 64"	
12		"The power you raise 4 to get 1"	

Graphing Logarithmic Models: $f(x) = a \log_b x + k$

*Use this page with **Slide 4: Exploring Logarithmic Functions** of **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)**.*

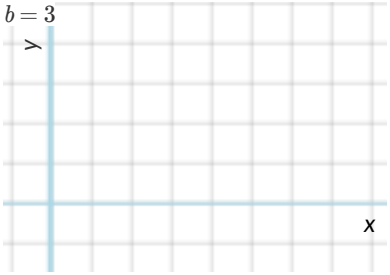
- The **blue curve** is the graph of $h(x) = 1 \log_2 x + 0$. Its constants will remain set at $a = 1$, $b = 2$, and $k = 0$.
- You can modify the **red curve** $g(x)$ (which is hiding behind $h(x)$!) by changing its coefficients: a, b, and k.

Base b

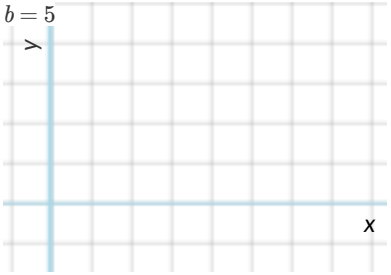
*Keep **k** at 0 and **a** at 1. Change the value of **b** as indicated on each grid below.*

1) Sketch each graph and **label the coordinates where** $x = 1$, $y = 1$, $y = 2$ **and** $y = 3$.

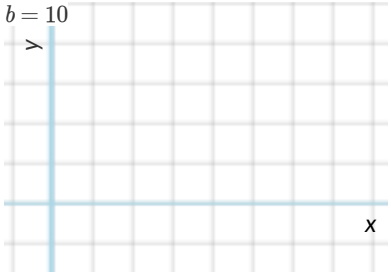
$b = 3$



$b = 5$



$b = 10$



2) How does the value of b impact the shape of a logarithmic function? _____

3) What connections can you draw between the value of b and exponents? _____

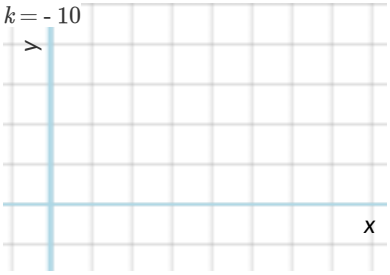
_____

Vertical Shift k

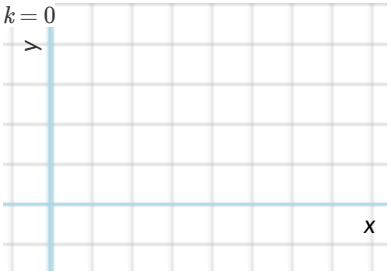
*Set **a** to 1 and **b** to 2. Change the value of **k** as indicated on each grid below.*

4) Sketch each graph and **label the coordinate where** $x = 1$.

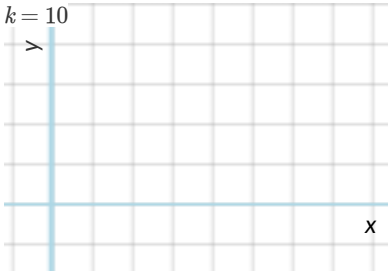
$k = -10$



$k = 0$



$k = 10$



5) How does the value of k impact the shape of a logarithmic function? _____

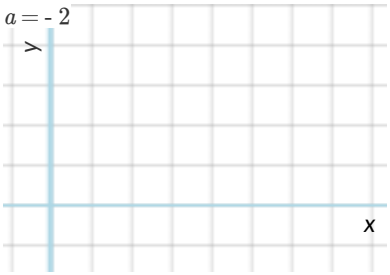
6) Why does $y = k$ when $x = 1$? _____

Logarithmic Coefficient a

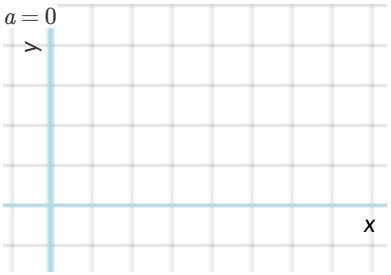
*Set **k** to 0 and **b** to 10, then zoom out out so you can see as far as $x = 1{,}000$.*
Change $h(x)$ to $h(x) = 1 \log_{10}(x) + 0$ so that the blue curve lands on top of the red curve.

7) In each graph, **label the coordinates where** $x = 10$ **and** $x = 100$ **and** $x = 1000$.

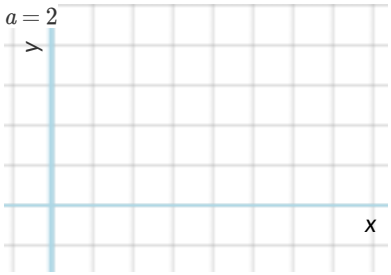
$a = -2$



$a = 0$



$a = 2$



8) What is the value of x when $1 \log_2(x) = 4$? _____ What about when $2 \log_4(x) = 4$? _____ When $3 \log_8(x) = 4$? _____

★ How are a and b related? _____

What Kind of Model? (Descriptions)

Decide whether each situation describes a quadratic, exponential, or logarithmic function. **HINT:** draw a table and plug in some points!

1) Earthquakes release enormous amounts of energy, which we can compare to the energy released by blowing up pounds of dynamite. For example, $\text{richter}(12{,}000) = 4.0$, meaning that the force of blowing up 12,000 pounds of dynamite produces a 4.0 on the Richter scale! $\text{richter}(400{,}000) = 5.0$, $\text{richter}(12{,}540{,}000) = 6.0$, and $\text{richter}(398{,}000{,}000) = 7.0$.

Quadratic Exponential Logarithmic

2) A car accelerates at a constant rate of 5mph/s. After 1 second, $\text{distance}(1) = 2.5\text{miles}$.
$\text{distance}(2) = 10$, $\text{distance}(3) = 22.5$, and $\text{distance}(4) = 40$

Quadratic Exponential Logarithmic

3) Moore's law says that the number of transistors in a microprocessor will double roughly every 1.5 years. Starting with 16 transistors, how many years will it take to reach 4,294,967,296 transistors?

Quadratic Exponential Logarithmic

4) The population of a colony of bacteria can double every 20 minutes, as long as there is enough space and food. Starting with 1 bacteria, $f(20) = 2$, $f(40) = 4$, $f(60) = 8$, $f(80) = 16$…

Quadratic Exponential Logarithmic

5) Sequan puts $100 in a savings account, earning 4% interest. After a year, $\text{savings}(1) = \$104$.
$\text{savings}(2) = \$108.16$, $\text{savings}(2) = \$112.49$…

Quadratic Exponential Logarithmic

6) If the *width and length* of a rectangle doubles, how much does the *area* change?

Quadratic Exponential Logarithmic

Changing the Scale

*For this page, you'll need to have **Slide 5: Wealth-v-Health (Logarithmic)** of **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)** and Countries of the World Starter File open on your computer.*

Fitting a Logarithmic Model $f(x) = a\log_b x + k$

Open the `Data Table` folder by clicking on the triangle (▶)

- x_1 is the per-capita income for each country in thousands of $US, and y_1 is the median lifespan.
- Next to y_1 you'll see a dark circle with spots (∴) inside. If the circle is dark, that means that those points are visible on our graph. Click the circle to "turn off" those dots, then click it again to turn them back on.
- Move the graph by clicking and dragging the background.
- Notice that a magnifying glass (🔍) appears to the bottom left of the table. (You may have to scroll down to see the bottom of the table!) Clicking on the magnifying glass resizes/rescales the graph to fit all the points in the table.

1) Write the numbers you see along the x-axis, from left to right: _____

Continue this pattern - what would the next three numbers be? _____

2) Circle the type of function that describes this pattern: Linear Quadratic Exponential

3) Move the sliders for a and c to create the best-fitting logarithmic model you can find, and write it below.
Note: The Bootstrap Pyret function `log` always uses $b = 10$.

$logarithmic(x) =$ _____ (log coefficient (a)) $\log_{10}(x) +$ _____ (vertical shift (k))

```
fun logarithmic(x): (_____ * log(x)) + _____ end
```

4) Modify `logarithmic(x)` in Countries of the World Starter File to define this model, and fit it using `fit-model`.

The error in the model is described by an S - *value* of about _____ (S) _____ (units), which is _____ (insignificant / reasonable / significant / extreme)

considering _____ (y-variable) in this dataset ranges from _____ (lowest y-value) to _____ (highest y-value).

Scaling the x-Axis

- Click on the wrench button (🔧) in the top-right corner of the Desmos graph to **Open the "Graph Settings" window.**
- **Expand the "More Options" section** by clicking the triangle (▶).
- **Change the x-axis scale** from `Linear` to `Logarithmic`.
- Adjust the view by zooming and dragging the graph to get all of the points in view on the screen and filling most of it.

5) What is the shape of the point cloud *now*, after changing the scale? Linear Quadratic Exponential

6) Write the numbers you see along the x-axis, from left to right: _____

Continue this pattern - what would the next three numbers be? _____

7) Circle the type of function that describes this pattern: Linear Quadratic Exponential

8) Adjust the sliders for a and c to improve the model. *Toggle back and forth between `logarithmic` and `linear` x-axis scales as you work.*
When you are satisfied with your model, record both forms of the definition below.

$logarithmic2(x) =$ _____ (log coefficient (b)) $\log_{10}(x) +$ _____ (vertical shift (k))

```
fun logarithmic2(x): (_____ * log(x)) + _____ end
```

9) Modify the definition of `logarithmic2(x)` in Pyret to match this model. Use the `fit-model` function to find its ***S-value***: _____

10) Why do you think transforming the **x-axis** makes our data look linear? _____

Transforming the Data

*For this page, you'll need to have **Slide 6: Wealth-v-Health (Transformed)** of **Fitting Wealth-v-Health and Exploring Logarithmic Models (Desmos)** open on your computer.*

- **Find the `Wealth vs. Health` folder, which is open at the top of the expression list**
- This is the same table we've seen before, and the "points" circle (∴) shows us that these dots are "on" and visible.
- Underneath the `Wealth vs. Health` folder, you'll see a **function** $g(x)$ and a **list** y_2 defined to be the same as y_1.
- **Open the second folder, called `Log(Wealth) vs. Health`, by clicking on the triangle (▶)**

1) Compare the two tables. (Here is a side by side comparison of how they each begin.)

Wealth vs. Health **Log(Wealth) vs. Health** **Compare the 2 tables. What do you notice? What do you wonder?**

x_1	y_1
1.99051	52.1
11.76559	78.6
15.19295	77.2
6.26897	60.6
24.95776	76.9
20.5888	77.5

$g(x_1)$	y_2
0.29896436	52.1
1.0706137	78.6
1.1816421	77.2
0.79719619	60.6
1.3972056	76.9
1.313631	77.5

2) Read the comments in rows 3 to 6 of the Desmos file. Where do the x-values in the second table come from? _____

_____

3) Why is the second column of both tables the same? _____

- **Turn the points for the first table OFF, then turn the points for our new table ON.**
  Our log transformation is so drastic that it looks like all the black datapoints are smashed against the y-axis!
- **Rescale** the graph (🔍) to see the cloud.

4) What is the shape of this point cloud? linear□ quadratic □ exponential □

5) Why do you think transforming the **x-values** make our data look linear? _____

_____

_____

6) Through trial and error, move the sliders for m and b to create the best-fitting linear model you can find, and write it below.

$$f(x) = \underset{\text{slope (m)}}{\_\_\_\_\_}\,x + \underset{\text{y-intercept / vertical shift}}{\_\_\_\_\_}$$

Let's compare the coefficients from your models.

Linear (From above) slope (m) _____ y-intercept / vertical shift _____

Logarithmic (From Changing the Scale) log coefficient (a) _____ vertical shift (k) _____

7) How are they similar? _____

_____

Logarithmic Models

Open your copy of the Countries of the World Starter File and click "Run".

Transforming: From Logarithmic *Plots* to Linear Ones

1) Find the definition of `g(r)`. What does this function do? _____

_____

2) Find the Contract for `build-column` on the Contracts Page.

What is its **Range**? _____ What is its **Domain**? _____

3) At the end of the program, you'll find this code:

```
countries-transformed = build-column(countries-table, "log(pc-gdp)", g)
```

What do you think it does? _____

_____

4) Click "Run", and evaluate `countries-transformed` in the Interactions Area on the right to test it out!

a. What is different about this Table? *Hint: Find the last column!* _____

b. Where did the column on the right come from? _____

5) Use this new table to make an `lr-plot` comparing `log(pc-gdp)` and `median-lifespan`, with `country` as the label. Record the regression line and S value below:

$y =$ _____ (slope) $x +$ _____ (vertical shift) S: _____

Inverting: From Linear *Models* to Logarithmic Ones

6) Use the coefficients of the *linear* model you just made to complete the *logarithmic* model below:

$logarithmic3(x) =$ _____ (log coefficient (a)) $log_{10}(x) +$ _____ (vertical shift (k))

```
fun logarithmic3(x): ( _____ * log(x)) + _____ end
```

7) Let's interpret this model:

A country where the _____ (x-axis) *is* _____ (base (b)) *times higher than another is also predicted to have a* _____ (y-axis) *that is* _____ (log coefficient (a)) _____ (y-axis units) *longer.*

8) Add the definition of `logarithmic3` to your starter file, and use it with `fit-model` to calculate the value of S: _____

9) Complete the table below, copying your S values from the previous models:

Linear	Quadratic	Exponential	Logarithmic

10) Compare the two smallest S values using percent change. *How much better* is the logarithmic model? _____

_____

Evaluating Logarithmic Expressions

	Expressions	Translation	Evaluates to:
1	$\log_2(8)$	"2 raised to what power gives us 8?"	3
2	$\log_2(1)$	"2 raised to what power gives us 1?"	0
3	$\log_5(25)$	"_____ raised to what power gives us _____?"	
4	$\log_5(1)$	"_____ raised to what power gives us _____?"	
5	$\log_3(81)$	"_____ raised to what power gives us _____?"	
6	$\log_3(1)$	"The power you raise _____ to get _____"	
7	$\log_2(16)$		
8	$\log_2(32)$		
9	$\log_{10}(1000)$		
10		"The power you raise 0.1 to get 0.01"	
11		"The power you raise 4 to get 64"	
12		"The power you raise 4 to get 1"	

Does Wealth impact lifespan equally if there's Universal Healthcare?

*For this page, you'll need the Countries of the World Starter File open on your computer. If you haven't already , select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

1) Add `fun no-universal(r): not(r["has-univ-healthcare"]) end` at the bottom of the Definitions Area.

Read the function definition carefully. What do you think it does? _____

_____

2) Click "Run" and evaluate `no-universal(albania)` in the Interactions Area. What does Pyret return? _____

3) What does that mean? _____

4) Add `countries-wo-univ = filter(countries-transformed, no-universal)` to the Definitions Area and click "Run".

What does evaluating `countries-wo-univ` in the Interactions Area produce? _____

_____

5) Using the two definitions you just added as models:

- **Define a new function** called `yes-universal`, which returns the value in the `has-univ-healthcare` column.
- **Define a new table** called `countries-w-univ`, which shows all the countries with universal healthcare.
- Click "Run" to load these new definitions once you have them both typed into the Definitions Area.

6) Fill in the table below by:

- Building an `lr-plot` for each of these tables with the transformed-column `log(pc-gdp)`.
- Using what you learn from `lr-plot` to write logarithmic models for each table.
- Using `fit-model` to find S for each of your logarithmic models and their corresponding un-transformed `countries-w-univ` and `countries-wo-univ` tables.

	With Universal Healthcare	Without Universal Healthcare
Linear Model	$f(x) =$ _____ (slope) x + _____ (y-intercept)	$f(x) =$ _____ (slope) x + _____ (y-intercept)
Logarithmic Model	$f(x) =$ _____ (log coefficient) $\log_{10}(x) +$ _____ (y-intercept)	$f(x) =$ _____ (log coefficient) $\log_{10}(x) +$ _____ (y-intercept)
S	_____ years	_____ years
What does each model predict the increase in `median-lifespan` to be for each 10x increase in `pc-gdp`?		
predicted increase	_____ years	_____ years

7) Was the relationship stronger for `countries-w-univ` or `countries-wo-univ`? _____

8) For which table is `pc-gdp` expected to have a bigger impact on `median-lifespan`? _____

9) Does the strength of the relationship determine how large of an impact `pc-gdp` has on `median-lifespan`? _____

Exploring Periodic Data

1) According to the table, the ride goes from _____ (lowest point) feet to _____ (highest point) feet in _____ minutes

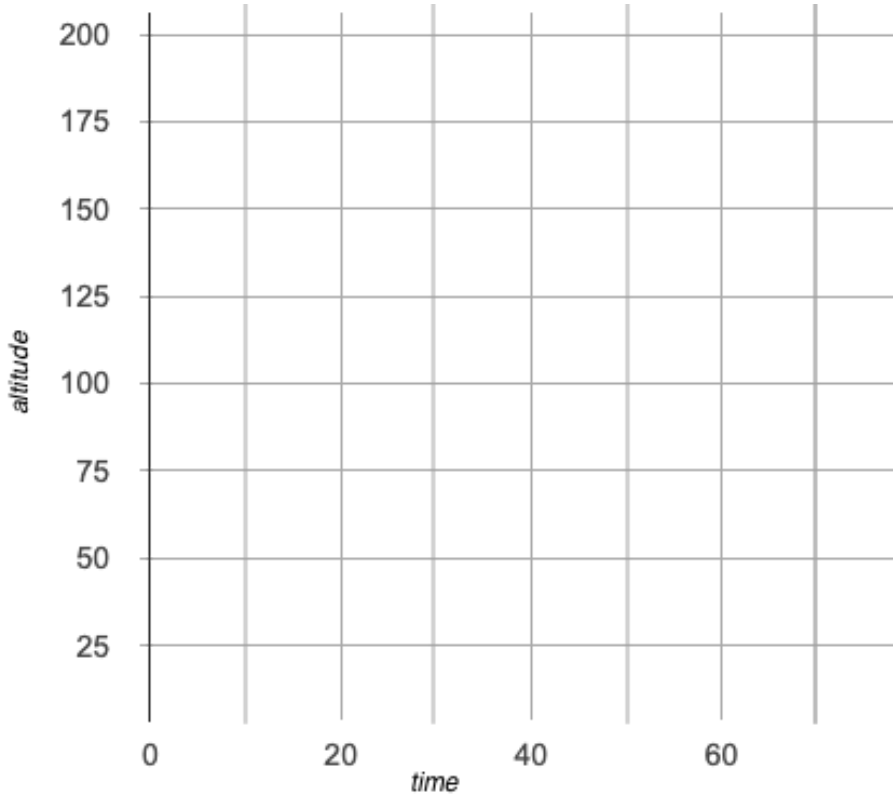
2) It then returns to the lowest point _____ minutes later.

3) Once it gets going, the ride does a **full cycle** from **high-point to high-point** (or low-point to low-point!) in _____ minutes.

4) The altitude **right between** the highest and lowest point is _____ feet. *Draw this as a dashed line on your graph.*

5) Plot each of the points in the table (left) on the coordinate plane (right) to create ***scatter plot***.

`time` (minutes)	`altitude` (feet)
0	5.0
5	55.0
10	154.9
15	205.0
20	155.2
25	55.2
30	5.0
35	54.7
40	154.6
45	205.0
50	155.5
55	55.5
60	5.0



6) What do you **Notice** about the data in the table? _____

_____

_____

7) What do you **Wonder**? _____

_____

8) Working from left to right, *connect the dots* one pair at a time using straight lines. This will create a display known as a **line-graph**.

9) Describe the relationship you see between `time` and `altitude`. (Is it linear, quadratic, exponential, etc.?) _____

_____

10) What kind of ride do you think your teacher was on, and why? _____

_____

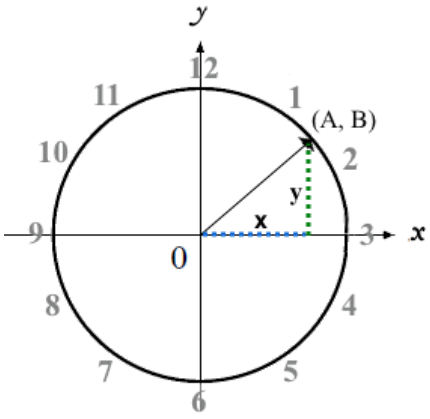
_____

Reasoning about Unit Clocks

A *unit clock* (shown below) is centered at the origin $(0, 0)$. As time passes, the point (A, B) rotates around the circle.

1) The radius r of the circle below has a length of 1. What is the length of the **hypotenuse** of the right-triangle formed by A and B? _____

Time	A
12:00	0
1:30	
3:00	
4:30	
6:00	
7:30	
9:00	
10:30	
12:00	0



Time	B
12:00	1
1:30	
3:00	
4:30	
6:00	
7:30	
9:00	
10:30	
12:00	1

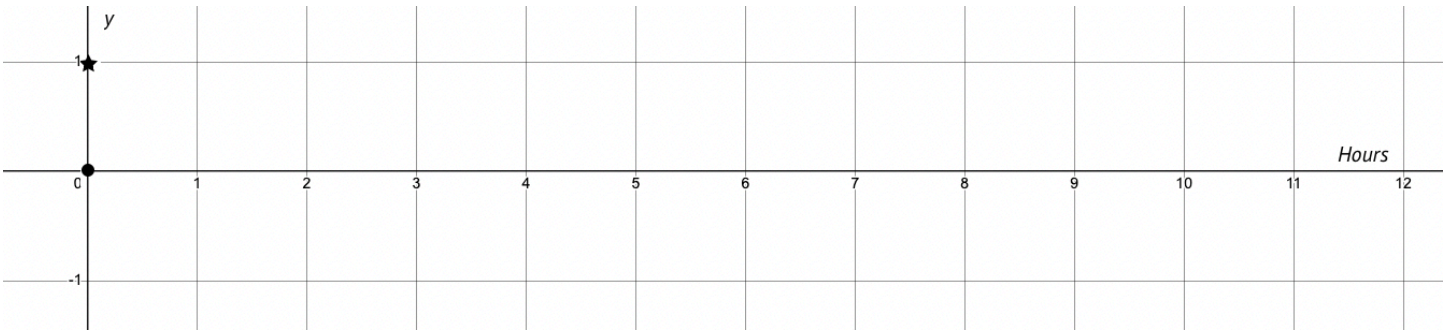
2) The tables above show the values of A and B at 12 o'clock. *Fill in the values of A and B at 3, 6 and 9 o'clock.*

3) In the diagram above, the hand is pointing to (A,B) at **1:30**. At this time, $A = B$. In the space below, (a) draw and label the right triangle, then (2) **fill in the remaining blanks in both tables to show A and B.**

4) Use the values you computed at 1:30 to fill in the rest of the table with values of A and B at 4:30, 7:30, and 10:30.

5) In the graph below, draw a **dot** for the coordinates (*time* , A) in each row of the table. Connect them from left-to-right, to form a curve.

6) In the graph below, draw a **star** for the coordinates (*time* , B) in each row of the table. Connect them from left-to-right, to form a curve.

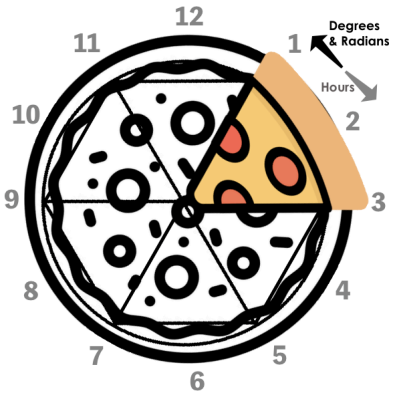


*Open the Desmos File **Exploring Periodic Functions**. You should be on **Slide 1: Unit Clocks**.*

7) "Turn on" the `x(time)` folder, and compare the graph to your own graph of A. Do they match?

8) Turn off that first folder, and turn on the one for `y(time)`. Compare the graph to your own graph of B. Do they match?

Converting Between Angles



The table below lists different angles within a circle.

1) Fill in the rows for 12, 9, and 6 o'clock, converting between hours, degrees, and radians.

2) Fill in the rows for 1:30, 10:30, 7:30, and 4:30, converting between hours, degrees, and radians.

Remember: degrees and radians both start with zero at "3 o'clock", and increase in the opposite direction of the hours!

We've filled in the rows for 12:00 and 3:00, as well as the length columns.

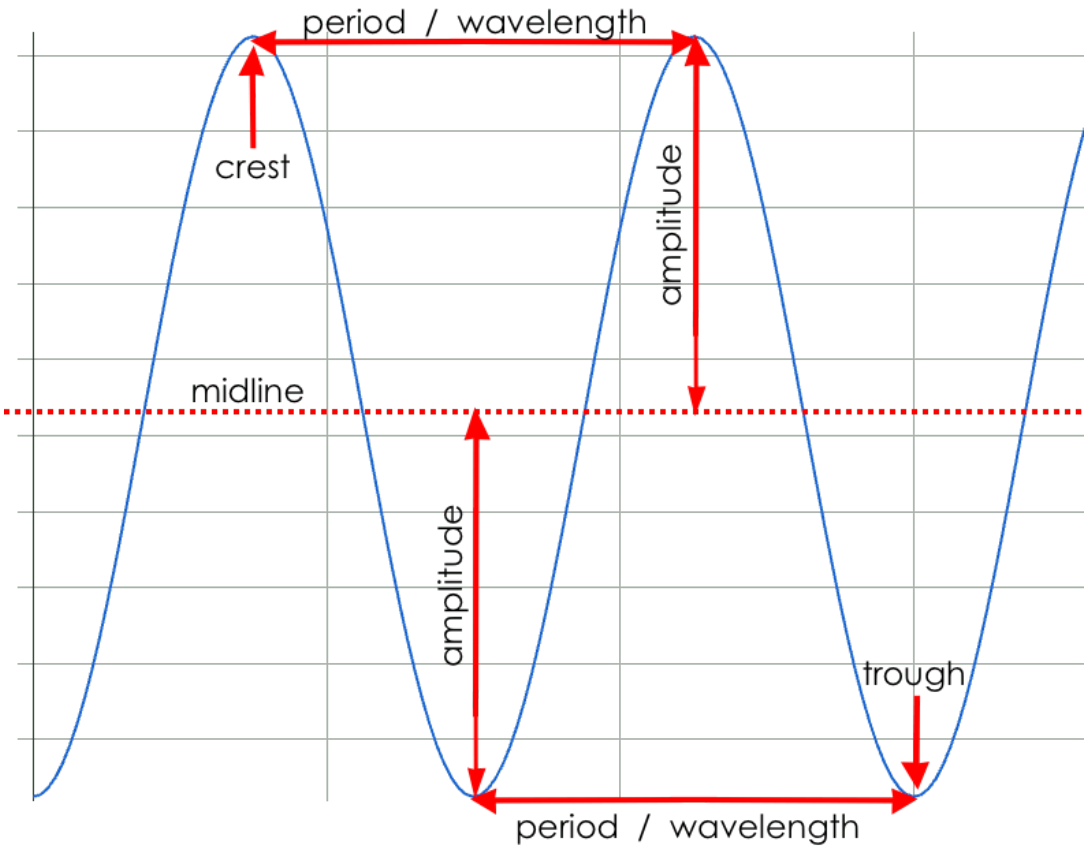
Time	θ Degrees	θ Radians	x	y
3:00	0°	0π	1	0
1:30			$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
12:00	90°	$\frac{2}{4}\pi$	0	1
10:30			$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
9:00			-1	0
7:30			$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
6:00			0	-1
4:30			$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
3:00	360°	$\frac{8}{4}pi$	1	0

3) In Pyret, experiment the functions `sin` and `cos`, passing in different **radian values** from the table above.

a. Which function computes $x(\theta)$? _____

b. Which function computes $y(\theta)$? _____

Words for Describing Periodic Functions



Based on what you can learn from the diagram, describe what each of the terms means in your own words.

Peaks - _____

_____

Troughs - _____

_____

Period - _____

_____

Midline - _____

_____

Amplitude - _____

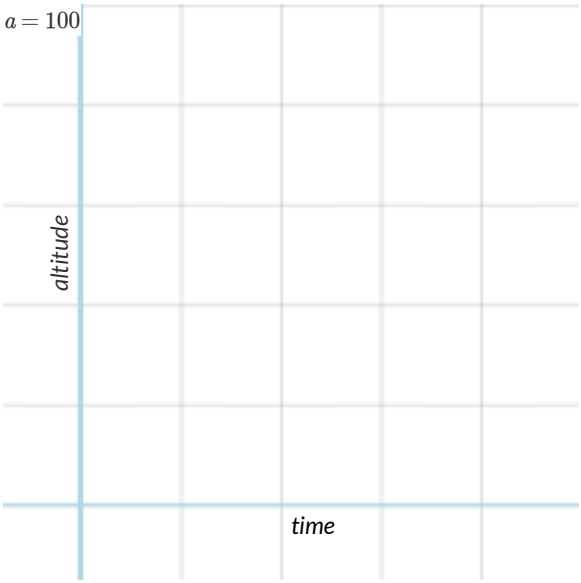
Graphing Periodic Models: Amplitude (a)

The standard form of periodic models is $f(x) = a\ sin(b\cdot(x - h)) + k$. Let's explore the role of ***amplitude*** a in periodic functions! *Open the Desmos File* ***Exploring Periodic Functions*** *to Slide 2:* ***Modeling the Ferris Wheel Dataset (sin)****. You should see four sliders for a, b, h, and k.*

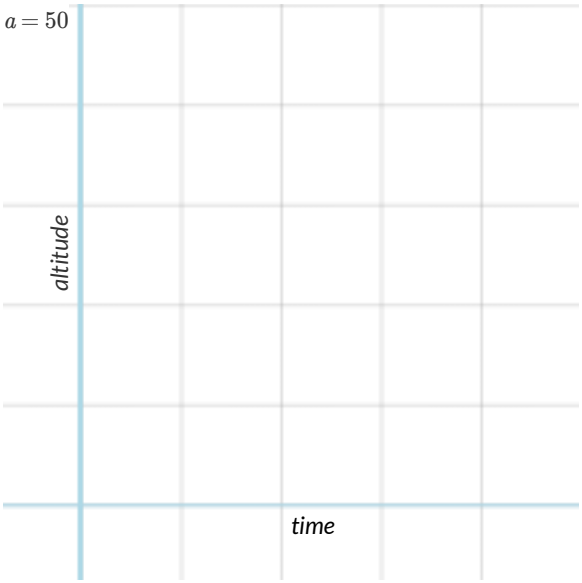
1) Adjust the sliders to fit the data as best you can, and fill in the coefficients: _____ (a), _____ (b), _____ (h) and _____ (k)

2) Change **ONLY** the slider for a, experimenting with values at 100, 50, -50, and 0, graphing each curve below. **For each curve, label the coordinates at time=15, 30, and 45.**

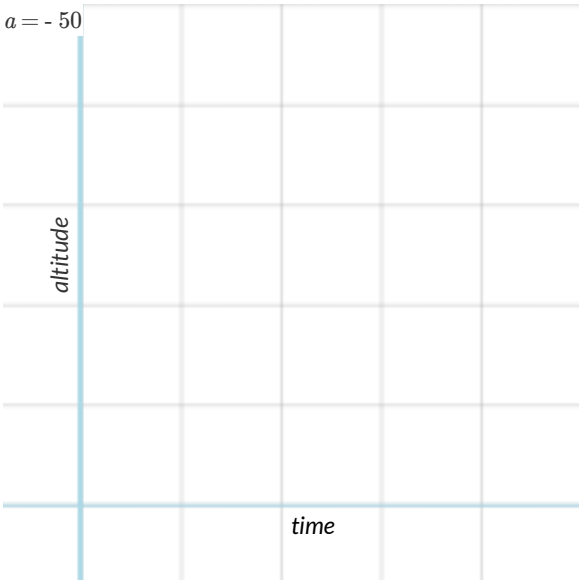
$a = 100$



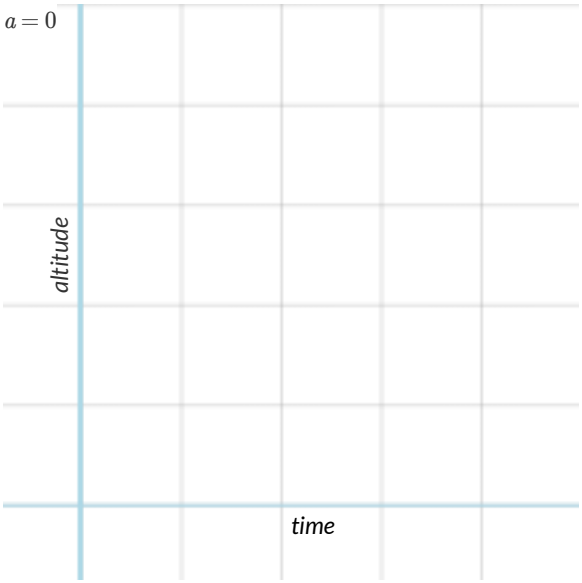
$a = 50$



$a = -50$



$a = 0$



3) What does a tell us about a periodic function? _____

_____

The distance between two adjacent *peaks* or *troughs* is called the *period*: the interval over which the pattern repeats itself.

4) What effect does changing a have on the ***period*** of a periodic function? _____

Graphing Periodic Models: Frequency (b)

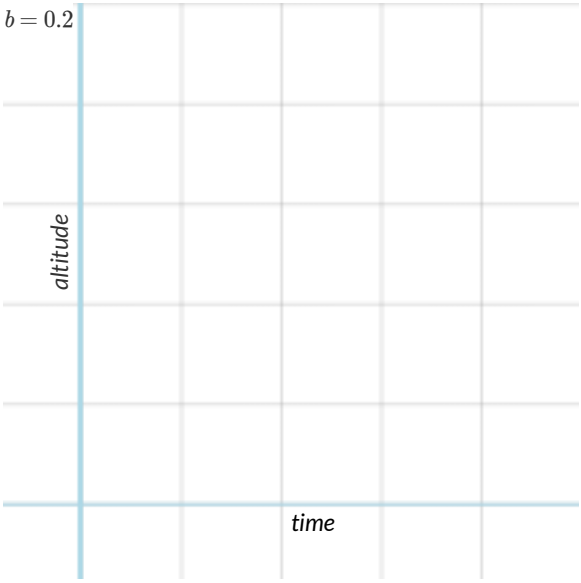
The standard form of a periodic model is $f(x) = a\ sin(b\cdot(x - h)) + k$. On this page, we'll explore the role of ***amplitude*** a in periodic functions. *Open the Desmos File* ***Exploring Periodic Functions****. You should be on* ***Slide 2: Modeling the Ferris Wheel Dataset (sin)*** *and see four sliders for a, b, h, and k.*

1) Adjust the sliders to fit the data as best you can, and fill in the coefficients: _____ (a), _____ (b), _____ (h) and _____ (k)

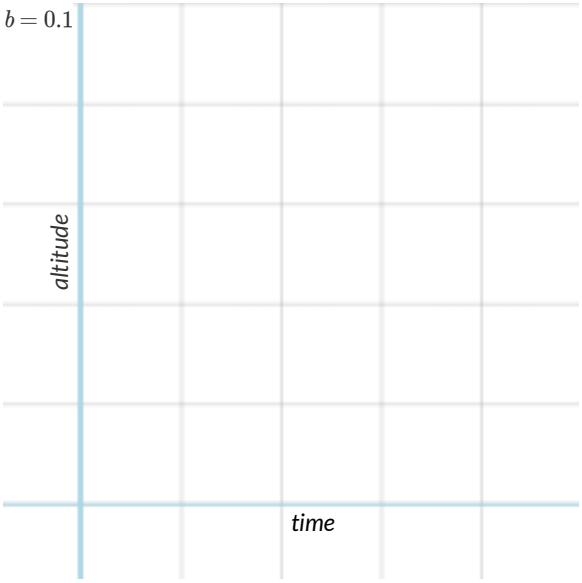
2) Click on one of the ***peaks*** (highest-points) on the graph of your periodic function. Desmos will add a gray dot to *all* of the peaks.

3) Change **ONLY** the slider for b, experimenting with values at $0.2, 0.1, 0.05$, and 0, graphing each curve below.
For each curve, label two adjacent peaks.

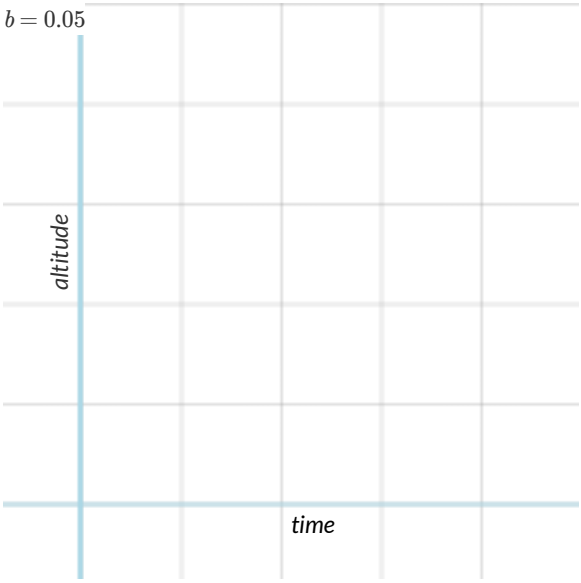
$b = 0.2$



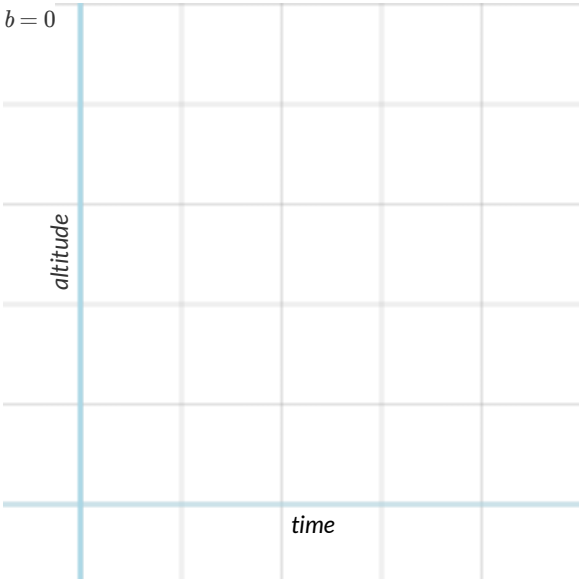
$b = 0.1$



$b = 0.05$



$b = 0$



The distance between two adjacent *peaks* or *troughs* is called the *period*: the interval over which the pattern repeats itself.

4) What is the ***period*** when $b = 0.2$? _____ when $b = 0.1$? _____ When $b = 0.5$? _____ ★ When $b = 0$? _____

5) As the ***frequency*** (b) doubles, the ***period*** _____. As the ***frequency*** (b) gets cut in half, the ***period*** _____

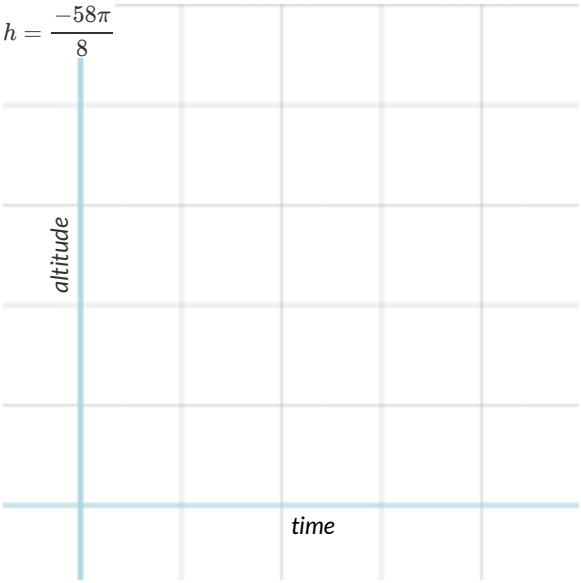
Graphing Periodic Models: Horizontal/Phase Shift (*h*)

The standard form of a periodic model is $f(x) = a\ sin(b \cdot (x - h)) + k$. On this page, we'll explore the role of ***amplitude*** a in periodic functions. *Open the Desmos File* ***Exploring Periodic Functions****. You should be on* ***Slide 2: Modeling the Ferris Wheel Dataset (sin)*** *and see four sliders for* a, b, h, *and* k.

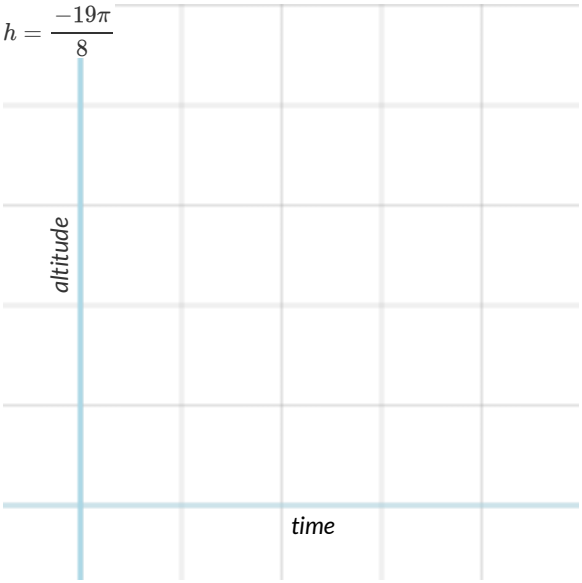
1) Adjust the sliders to fit the data as best you can, and fill in the coefficients: _____ (*a*), _____ (*b*), _____ (*h*) and _____ (*k*)

2) Change **ONLY** the slider for h, experimenting with values at $\frac{-58\pi}{8}$, $\frac{-19\pi}{8}$, $\frac{19\pi}{8}$, and $\frac{58\pi}{8}$, graphing each curve below. **For each curve, label the coordinates at time=15, 30, and 45.**

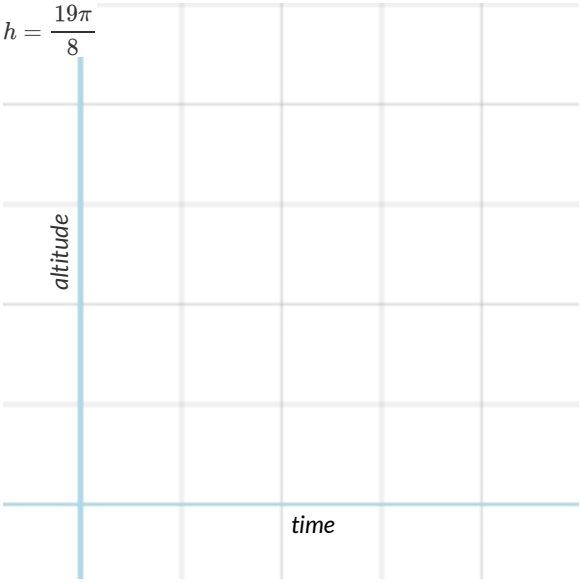
$h = \frac{-58\pi}{8}$



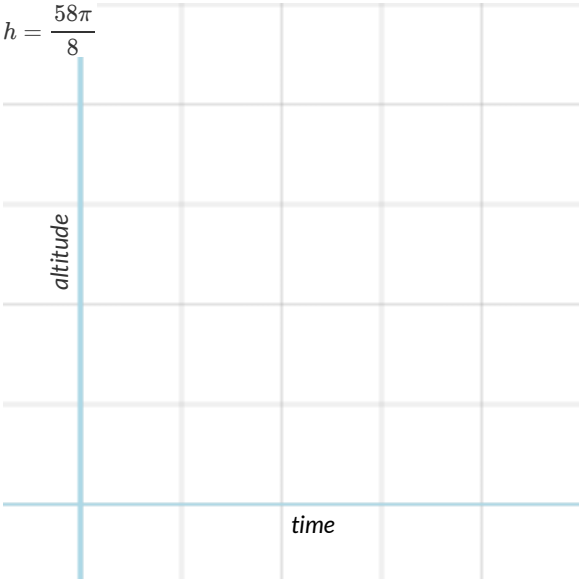
$h = \frac{-19\pi}{8}$



$h = \frac{19\pi}{8}$



$h = \frac{58\pi}{8}$



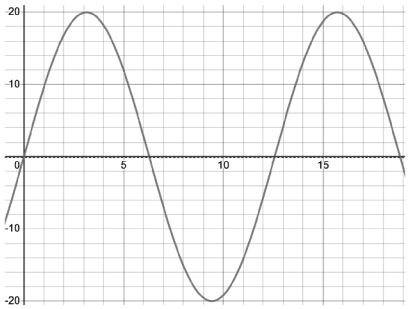
3) Describe the change in the graph when h *increases*: _____

4) Describe the change in the graph when h *decreases*: _____

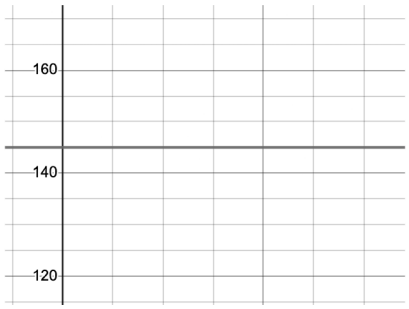
5) The model fits as long as h changes by increments of $\frac{77\pi}{8}$, because _____

_____

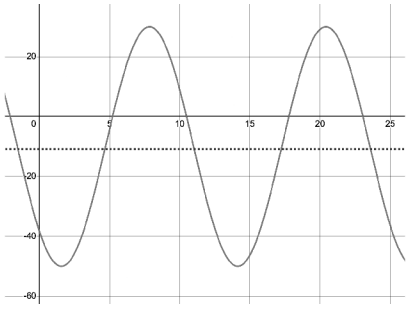
Matching Periodic Descriptions



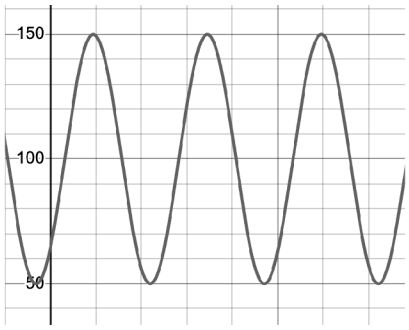
1



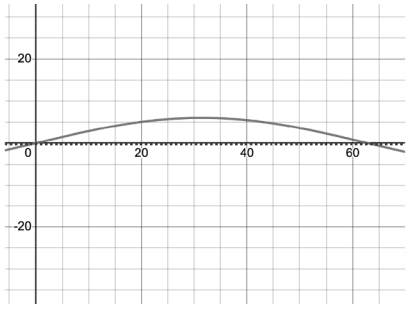
2



3



4



5

A This function has an amplitude of 50

B This function has a midline of -10

C This function has peaks, troughs, and a midline at 145

D This function has a wavelength of more than 60

E This function has peaks at 20 and troughs at -20

Modeling the Ferris Wheel Data

Modeling with sin

*For this section, use **Slide 2: "Modeling the Ferris Wheel Dataset (sin)"** of the **Exploring Periodic Functions** Desmos File.* You'll find **the data from the Ferris Wheel plotted in red**, along with a basic periodic model of the form $f(x) = a\sin(b(x - h)) + k$.

1) Use the sliders to estimate the best periodic fit.

2) The ***peaks*** are at _____ feet, ***troughs*** are at _____ feet, ***midline*** is at _____ feet and the ***amplitude*** is _____ feet

3) The ***period*** of the data is _____ minutes. If $\text{period} = \frac{2\pi}{\text{frequency}}$, what is the ***frequency***? _____ cycles per minute

4) Adjust the slider for horizontal shift to find the best fit, then write your model below in Function and Pyret notation. **Express h in terms of pi.**

Function Notation	$f(x) =$ _____ (amplitude) $\times sin($ _____ (frequency) (x - _____ (horizontal shift))) + _____ (vertical shift)
Pyret Notation	`fun f(x): (` _____ `* sin(` _____ `* (x -` _____ `))) +` _____ `end`

Translating from sin to cos

*For this section, advance to **Slide 3: "Translating from sin to cos"** of the **Exploring Periodic Functions** Desmos File.* You'll see a function f(x) defined here graphed in blue, which uses cos instead of sin.

5) Adjust the sliders so that the function q perfectly overlaps the function p. What is the value of a? _____ b? _____ k? _____

6) What was the value of h, expressed as a decimal? _____ What was the value of h, expressed a fraction of pi?

---

7) Change the definition of p in Desmos to math row 1 below and adjust the definition of q to match the new curve. Complete the second row by changing the definition of p in Desmos again and adjust q again.

Function using sin	Function using cos	Vertical Shift k
$p(x) = 10\ sin(1\cdot(x - 0)) + 2$	$q(x) =$	
$p(x) =$	$q(x) =$	

8) Do you think that all basic cosine functions can be expressed as sine functions? Why or why not? _____

Modeling with cos

*For this section, advance to **Slide 4: "Modeling the Ferris Wheel Dataset (cos)"** of the **Exploring Periodic Functions** Desmos File.*

9) Translate your sin-based model to a cos-based one. Express the horizontal shift in terms of pi.

Function Notation	$g(x) =$ _____ (amplitude) $\times cos($ _____ (frequency) (x - _____ (horizontal shift))) + _____ (vertical shift)
Pyret Notation	`fun g(x): (` _____ `* cos(` _____ `* (x -` _____ `))) +` _____ `end`

Make Your Own Ferris Wheel!

Matching Terms

1) The Ferris Wheel is being upgraded! Match the upgrade on the left to the property that it will change on the right. **NOTE:** some upgrades might change more than one property!

The wheel is being raised *higher*	1	A	***midline***
		B	***vertical shift***
The wheel is being made to spin *faster*	2	C	***frequency***
		D	***amplitude***
The wheel is being made *larger*	3	E	***period***

Design a New Wheel

2) *Design your own Ferris Wheel!* Fill in the table below, then **trade papers with someone else**.

Radius	Altitude of Center	Speed

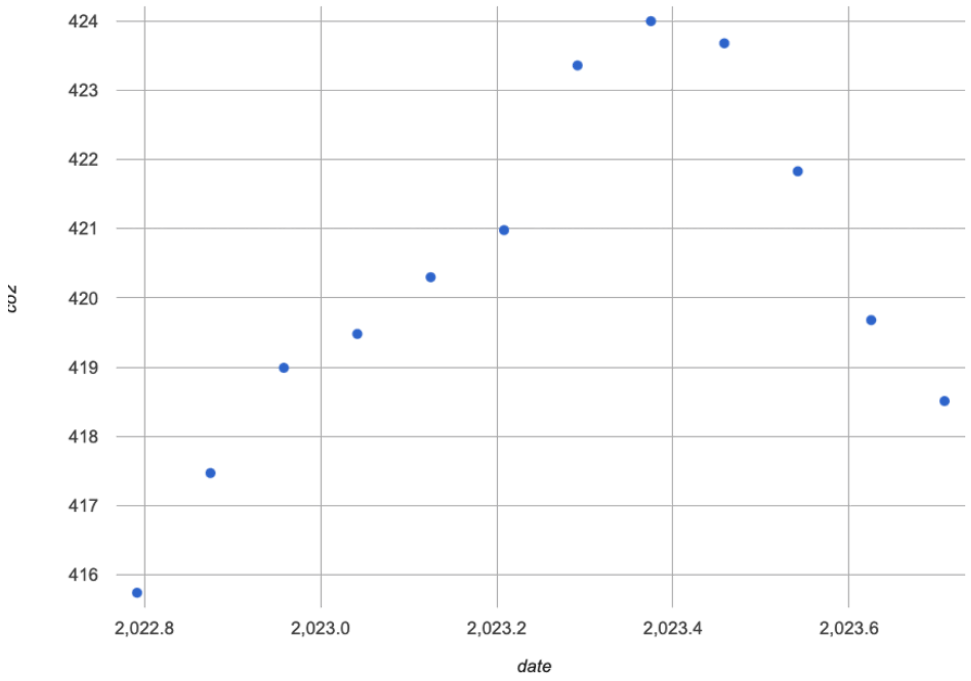
3) Based on the table above, what function will model the height of the wheel over time?

Function Notation	$f(x) =$ _____ (amplitude) $\times sin($ _____ (frequency) $(x -$ _____ (horizontal shift) $)) +$ _____ (vertical shift)
Pyret Notation	`fun f(x): (`_____ `* sin(`_____ `* (x -` _____ `))) +` _____ `end`

Modeling Recent Carbon Dioxide Levels

The data below was generated from the Carbon Dioxide Starter File, showing the amount of CO_2 in the atmosphere (parts per million) on specific dates from December 2022 to November 2023. ***NOTE:*** *the* `date` *column is the* ***decimal year*** *(so "June 15th, 2023" would be 2023.5).*

date	co2 (ppm)
2022.708	415.91
2022.792	415.74
2022.875	417.47
2022.958	418.99
2023.042	419.48
2023.125	420.30
2023.208	420.98
2023.292	423.36
2023.375	424.00
2023.458	423.68
2023.542	421.83
2023.625	419.68
2023.708	418.51



1) **Connect the dots on the scatter plot to form a line-graph.**

2) The distance between the lowest ***trough*** and highest ***peak*** is _____ parts per million, so the ***amplitude*** (a) is _____ parts per million

3) Draw the ***midline*** on your graph. (HINT:look at ***amplitude*** and ***trough***!). What is the ***vertical shift*** (c) of the model? _____ parts per million

4) Estimate the ***phase shift*** by estimating the *decimal year* when the data **first** crosses the ***midline*** (d): _____ years

5) Calculate the ***period*** between the ***troughs*** by subtracting the `dates` for the lowest values in 2022 and 2023: _____ years

6) If $\text{period} = \dfrac{2\pi}{\text{frequency}}$, what is the ***frequency***? _____

7) Using your computed values for a, b, h, and k, define your periodic function below in both Function and Pyret notation.

Function Notation

$$periodic(x) = \underset{\text{amplitude}}{\_\_\_\_} \times sin(\underset{\text{frequency}}{\_\_\_\_}(x - \underset{\text{horizontal shift}}{\_\_\_\_})) + \underset{\text{vertical shift}}{\_\_\_\_}$$

Pyret Notation

```
fun periodic(x): (_____ * sin(_____ * (x - _____))) + _____
                                                  end
```

8) Define this model in Pyret, and fit it to the `recent` data. What S-value do you get? _____

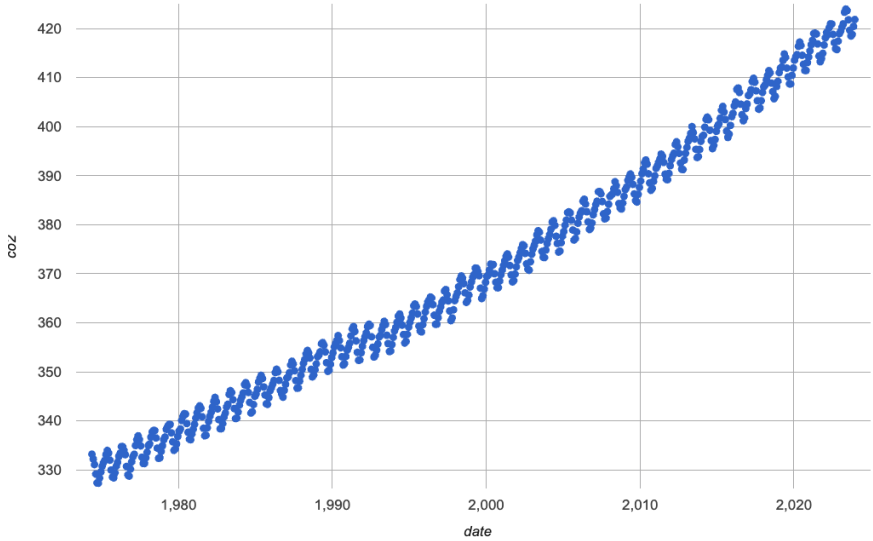
9) **What does this model actually mean?** Fill in the blanks below, and read the completed model aloud with your partner.

Between the end of 2022 and 2023, the amount of CO_2 in the air fluctuated between _____ lowest *and* _____ highest *parts-per-million. This pattern appears to be* ***periodic****, with an amplitude of* _____ amplitude *rising and falling around a* ***midline*** *of* _____ midline*. With 1 year representing a full cycle, we expect this pattern to repeat each year for a frequency of* _____ frequency*.*

Modeling Historical Carbon Dioxide Levels

The data below was generated from Carbon Dioxide Starter File, showing the amount of CO_2 in the atmosphere (parts per million) on specific dates from 1974 to 2023 (the `co2-table`). ***NOTE:*** *the `date` column is the* ***decimal year*** *(so "June 15th, 2023" would be 2023.5).*

Looking for Patterns



1) Use `lr-plot` to find the best linear model for the `co2-table`. What is its S-value? _____ parts per million

2) Write the function below (in Pyret and Function Notation):

Function Notation

linear(x) = _____ (slope) x + _____ (y-intercept / vertical shift)

Pyret Notation

```
fun linear(x): (_____ * x) + _____ end
```

3) Copy your periodic model from Modeling Recent Carbon Dioxide Levels below:

Function Notation

$periodic(x) =$ _____ (amplitude) $\times$ *sin*(_____ (frequency) (x - _____ (horizontal shift))) + _____ (vertical shift)

Pyret Notation

```
fun periodic(x): (_____ * sin(_____ * (x - _____))) + _____ end
```

Creating Hybrid Models

We can think of $f(x) = A\sin(B(x - h)) + k$ as being the sum of *two* functions: $p(x) = A\sin(B(x - h))$ and $q(x) = k$.

4) Which function defines the "up and down" wave (p or q)? _____ Which function defines the line the wave "wraps around"? _____

5) What do you think would happen if q were changed so that k is a *higher* number? _____

6) What do you think would happen if q were changed so that k is a *lower* number? _____

7) What do you think would happen if q were changed to $q(x) = 2x + -3000$ _____

★ Define a NEW function `hybrid` in Pyret, which combines your periodic model with the optimal linear one. Write your new model below, in Function or Pyret notation:

What is the best S-value you can get? _____ parts per million

Contracts for Algebra 2

Contracts tell us how to use a function, by telling us three important things:

1. The **Name**
2. The **Domain** of the function - what kinds of inputs do we need to give the function, and how many?
3. The **Range** of the function - what kind of output will the function give us back?

For example: The contract `triangle :: (Number, String, String) -> Image` tells us that the name of the function is `triangle`, it needs three inputs (a Number and two Strings), and it produces an Image.

With these three pieces of information, we know that typing `triangle(20, "solid", "green")` will evaluate to an Image.

Name	Domain	Range
`# above`	`::` `( Image , Image )` (above, below)	`-> Image`
above(circle(10, "solid", "black"), square(50, "solid", "red"))		
`# bar-chart`	`::` `( Table , String )` (table-name, column)	`-> Image`
bar-chart(animals-table, "species")		
`# box-plot`	`::` `( Table , String )` (table-name, column)	`-> Image`
box-plot(animals-table, "weeks")		
`# build-column`	`::` `( Table , String , (Row -> Value) )` (table-name, column, builder-function)	`-> Table`
build-column(animals-table, "kilos", kilograms)		
`# count`	`::` `( Table , String )` (table-name, column)	`-> Table`
count(animals-table, "species")		
`# first-n-rows`	`::` `( Table , Number )` (table-name, num-rows)	`-> Table`
first-n-rows(animals-table, 15)		
`# fit-model`	`::` `( Table , String , String , String , (Num -> Num) )` (table-name, labels, xs, ys, model-function)	`-> Image`
fit-model(animals-table, "name", "pounds","weeks", f)		
`# histogram`	`::` `( Table , String , String , Number )` (table-name, labels, values, bin-size)	`-> Image`
histogram(animals-table, "species", "weeks", 2)		
`# line-graph`	`::` `( Table , String , String , String )` (table-name, labels, xs, ys)	`-> Image`
line-graph(animals-table, "name", "pounds","weeks")		
`# log`	`::` `( Number )` (n)	`-> Number`
log(4)		
`# log-base`	`::` `( Number , Number )` (base, n)	`-> Number`
log-base(2, 4)		

Name	Domain	Range
# lr-plot	:: (Table (table-name) , String (labels) , String (xs) , String (ys))	-> Image
lr-plot(animals-table, "name", "pounds","weeks")		
# num-sqr	:: (Number)	-> Number
num-sqr(4)		
# overlay	:: (Image (top) , Image (bottom))	-> Image
overlay(circle(10, "solid", "black"), square(50, "solid", "red"))		
# pie-chart	:: (Table (table-name) , String (column))	-> Image
pie-chart(animals-table, "species")		
# put-image	:: (Image (front) , Number (x-coordinate) , Number (y-coordinate) , Image (behind))	-> Image
put-image(circle(10, "solid", "black"), 10, 10, square(50, "solid", "red"))		
# rotate	:: (Number (degrees) , Image (img))	-> Image
rotate(45, star(50, "solid", "dark-blue"))		
# row-n	:: (Table (table-name) , Number (index))	-> Row
row-n(animals-table, 2)		
# S	:: (Table (table-name) , String (xs) , String (ys) , (Num -> Num) (model-function))	-> Number
S(animals-table, "name", "pounds","weeks", f)		
# scale	:: (Number (factor) , Image (img))	-> Image
scale(1/2, star(50, "solid", "light-blue"))		
# scatter-plot	:: (Table (table-name) , String (labels) , String (xs) , String (ys))	-> Image
scatter-plot(animals-table, "name", "pounds","weeks")		
# sort	:: (Table (table-name) , String (column) , Boolean (ascending))	-> Table
sort(animals-table, "species", true)		
# string-contains	:: (String (haystack) , String (needle))	-> Boolean
string-contains("hotdog", "dog")		
	::	->
	::	->
	::	->

These materials were developed partly through support of the National Science Foundation, (awards 1042210, 1535276, 1648684, 1738598, and 1501927), and are licensed under a Creative Commons 4.0 Unported License. Based on a work at www.BootstrapWorld.org. Permissions beyond the scope of this license may be available by contacting contact@BootstrapWorld.org.