Problem Decomposition

(Also available in WeScheme)

Students take a closer look at how functions can work together by investigating the relationship between revenue, cost, and profit.

Lesson Goals	Students will be able to: • Write a ***function*** that explicitly uses another function. • Explain the benefits and drawbacks of functions that depend on each other. • Explain the difference between bottom-up and top-down strategies.
Student-Facing Lesson Goals	• Let's write a function that uses another function and consider the advantages and disadvantages of doing so.
Prerequisites	• Simple Data Types • Contracts • Functions: Contracts, Examples & Definitions • Solving Word Problems with the Design Recipe
Materials	• PDF of all Handouts and Page • Sally's Lemonade Starter File • Lesson Slides • Printable Lesson Plan (a PDF of this web page)
Supplemental Materials	• Additional Printable Pages for Scaffolding and Practice

Key Points for the Facilitator	
	• There are several ways to write the `profit` function - use this opportunity for discussion and to promote higher-order critical thinking. • If students are struggling with understanding the basics of the problem, start by coming up with examples of `cost` and `revenue`. If Sally sells one glass, what is her total revenue? How much does it cost her to produce that one glass? • Ensure students understand the difference between "revenue" and "profit", and that "cost" refers to the cost of *making* the lemonade, not the amount Sally is charging.

Glossary

function :: a relation from a set of inputs to a set of possible outputs, where each input is related to exactly one output

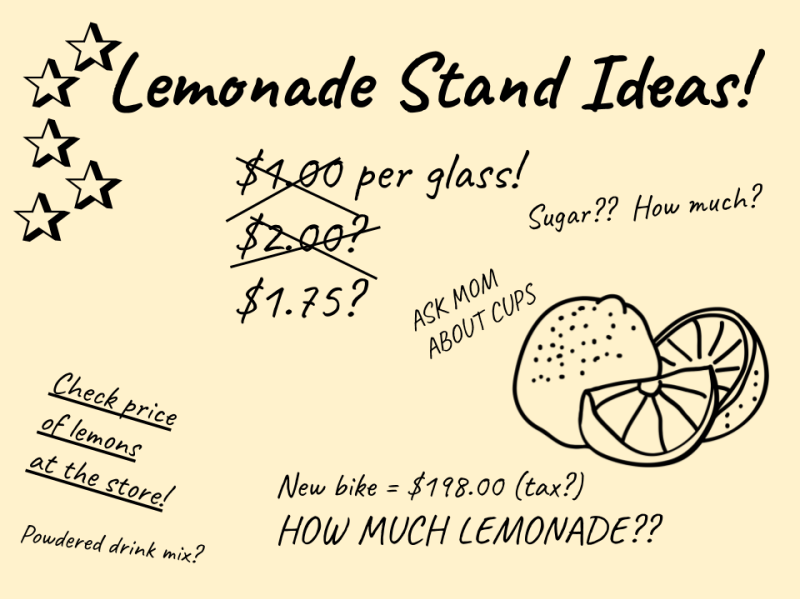
Problem Decomposition *30 minutes*

Overview

Students are introduced to word problems that can be broken down into *multiple* problems, the solutions to which can be composed to solve other problems. They adapt the Design Recipe to handle this situation.

Launch

Display the following image, then have students share everything they Notice about the image. Then, have them share what they Wonder.



- One example of a *relationship* we can find in this situation is that every glass sold incurs **costs**. Sally will need lemons, sugar, powdered drink mix, cups, and water.
- What other relationships can you find? *Below are some possible observations students might share; it's okay if students are not confident using terms such as cost, revenue, and profit!*
  - *Every glass sold brings in $1.75 in **revenue**.*
  - *Every glass sold brings in some amount of **profit**: it costs a certain amount to make, but it brings in another amount in revenue.*
  - *The total cost of the bike will be depend on the tax rate.*
  - *In order to figure out how many lemonade sales it will take to pay for the bike, we'd need to to divide the cost (with tax) by the profit per glass.*

Investigate

- Define functions for `revenue` and `cost` on Word Problems: revenue, cost. *Note: The information you need to write the* `cost` *function is provided in the Design Recipe word problem!*
- Once you've defined the functions, open Sally's Lemonade Starter File.
- Enter your code for `revenue` (including all examples and definitions) below the first prompt. Enter your code for `cost` below the second prompt. Click "Run" and make sure your tests pass.

- What is the difference between *revenue* and *profit*?
  - *Revenue is the total amount of money that comes in, profit is the remaining money* ***after cost has been subtracted****.*
- How could Sally *increase* her profits?
  - *By decreasing her costs, raising her prices (which increases revenue), by selling more lemonade.*
- What is the *relationship* between profit, cost, and revenue?
  - *Profit = Revenue - Cost*

Ensure that students have correctly defined the functions for `revenue` and `cost` before moving onto the next task - using the Design Recipe to define `profit`.

- Complete Word Problem: profit, using the Design Recipe. (There are multiple correct solutions!)
- Add your code for `profit` in Sally's Lemonade Starter File below the third prompt; be sure to type all examples and definitions. Click "Run". Do all your tests pass?

While students are working, walk the room and gauge student understanding. There is more than one way to write the `profit` function! Encourage discussion between students and push them to develop their thinking on the advantages and disadvantages of each correct solution.

Optional: As students finish, or for homework, you may also want them to figure out how many cups of lemonade Sally would have to sell in order to buy her bike using Sally's Bike.

Synthesize

This activity started with a situation, and you modeled that situation with functions. One part of the model was *profit*, which can be written several ways.

Turn to Profit - More than one Way! and reflect on the four function definitions presented.

The four definitions of `profit` from this page are shown here:

```
fun profit(g): (1.75 * g) - (0.3 * g) end
```

```
fun profit(g): (1.75 - 0.3) * g end
```

```
fun profit(g): 1.45 * g end
```

```
fun profit(g): revenue(g) - cost(g) end
```

- Which of these four `profit` definitions do you think is "best", and why?
  - *Possible arguments: The first one is "closest to the word problem". The third one is "fastest", with the computation already completed. The last one is the most readable.*
- Did anyone have additional ideas for how to define a `profit` function?
- If lemons get more expensive, which definitions of `profit` need to be changed?
  - *Every definition except the last one would need to be changed.*
- If Sally raises her prices, which definitions of `profit` need to be changed?
  - *Every definition except the last one would need to be changed.*
- Which definition of `profit` is the most flexible? Why?
  - *The last definition is the most flexible; it can be used with any revenue and cost functions.*

`profit` can be *decomposed* into a simpler function that uses the `cost` and `revenue` functions.

Decomposing a problem allows us to solve it in smaller pieces

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So what's the big deal?

1. Smaller pieces are *easier to think about*, and to test!
2. These pieces can also be *re-used*! Like lego pieces, smaller functions can be used to build all kinds of things.
3. Re-using code means *less code* overall. Less code means fewer places to make mistakes.
4. Re-using code means *less duplicate code*. When code needs to be changed, that change only needs to made in one place, instead of in multiple places.

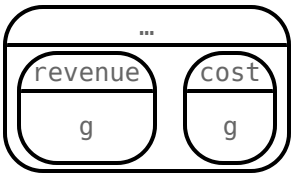
Top-Down vs. Bottom-Up *20 minutes*

Overview

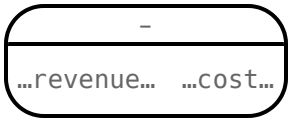
Students explore problem decomposition as an explicit strategy, and learn about two ways of decomposing.

Launch

Top-Down and *Bottom-Up* design are two different strategies for problem decomposition.



Bottom-Up: start with the small, easy relationships like `revenue` and `cost` first. How are they connected with the outer circle? You'll get there eventually, but *we can leave it blank for now (* `...` *)*. In the Lemonade Stand, you defined `cost` and `revenue` first, and then put them together in `profit`. *This is the same approach as building your Circle of Evaluation inside-out!*



Top-Down: start with the "big picture" and then worry about the details later. We could have started with `profit` as `revenue - cost`, and *fill in the details of* `revenue` *and* `cost` *later (thus the* `...` *). This is the same approach as building your Circle of Evaluation outside-in!*

Investigate

Consider the following situation:

Jamal's trip requires him to drive 20mi to the airport, fly 2300mi, and then take a bus 6mi to his hotel. His average speed driving to the airport is 40mph, the average speed of an airplane is 575mph, and the average speed of his bus is 15mph. Aside from time waiting for the plane or bus, how long is Jamal in transit?

Take a moment to think: What would your first step be if you were trying to figure out how long Jamal would be transit? What circles would you draw or functions would you define to solve this? Would you work top-down or bottom-up?

Then turn to Top Down or Bottom Up.

Synthesize

- Whose strategy was top-down? How do you know?
- Do you have questions about either of these strategies?

- Which strategy to do you prefer? Why?

Make sure that students see *both* strategies, and have them discuss which they prefer and why.