Measures of Center

(Also available in CODAP)

Students are introduced to mean, median and mode(s) and consider which of these measures of center best describes various quantitative data.

Lesson Goals	Students will be able to… • Students explore the concept of center of a distribution, learning how to compute the mean, median and mode(s) of a dataset • Students find the mean, median and mode(s) of various columns in the Animals table
Student-facing Lesson Goals	• Let's use mean, median, and modes to describe our data.
Prerequisites	• Simple Data Types • Introduction to Data Science • Contracts: Making Tables and Displays • Histograms • Visualizing the "Shape" of Data
Materials	• PDF of all Handouts and Page • Data Exploration Project Slide Template • Lesson Slides • Printable Lesson Plan (a PDF of this web page)
Supplemental Materials	• Additional Printable Pages for Scaffolding and Practice • *Mode(s) (Desmos)*

Preparation	• There are two optional activities in this lesson plan. One requires a ruler and 4-8 pennies for each group of 3. The other involves an extra datacycle. Decide whether or not you will engage your students with these activities and prepare accordingly. (You may want to adjust the slide deck.)

Glossary

mean :: a representation of the center, or 'typical' value in a set of numbers, calculated as the sum of those numbers divided by the number of values.
median :: the middle element of a quantitative dataset
mode :: the most commonly appearing categorical or quantitative value or values in a dataset
outlier :: observations whose values are very different from the other observations in the same dataset, perhaps due to experimental error. Outliers can also be indicative of data belonging to a different population from the rest of the established samples.
quantitative data :: number values for which arithmetic makes sense
skew :: lack of balance in a dataset's shape, arising from more values that are unusually low or high. Such values tend to trail off, rather than be separated by a gap (as with outliers).

Mean *15 minutes*

Overview

Students learn about ***mean*** (or "average"), and how it is one way (among others!) to summarize a ***quantitative*** column.

Launch

According to the Animal Shelter Bureau, the average pet weighs almost 40 pounds.

Some medicines are dosed by weight: heavier animals need a larger dose that could be dangerous for smaller animals. If someone from the shelter needs to give a dose of medicine to the animals, is the “average” the best estimate we can use?

“The average pet weighs almost 40 pounds” is a statement about the entire dataset, which summarizes a whole column of values with a single number. Summarizing a big dataset means that some information gets lost, so it’s important to pick an appropriate summary. Picking the wrong summary can have serious implications! Here are just a few examples of summary data being used for important things. Do you think these summaries are appropriate or not?

- Students are sometimes summarized by two numbers — their GPA and SAT scores — which can impact where they go to college or how much financial aid they get.
- Schools are sometimes summarized by a few numbers — student pass rates and attendance, for example — which can determine whether or not a school gets shut down.
- Adults are often summarized by a single number — like their credit score — which determines their ability to get a job or a home loan.
- When buying uniforms for a sports team, a coach might look for the most common size that the players wear.

Can you think of other examples where someone uses a number or two to summarize something complex?

Every kind of summary has situations in which it does a good job of reporting what’s typical, and others where it doesn’t really do justice to the data. In fact, the shape of the data can play a huge role in whether or not one kind of summary is appropriate!

One of the ways that Data Scientists summarize quantitative data is by talking about its *center* - literally asking "what is a typical value in this sample?", in the hopes of inferring something about a larger population. But there are many different ways to define "center", and each method has strengths and weaknesses. Let's check the "40 pounds" claim and see if it's an appropriate measure of center. Later on, you'll have a chance to apply what you've learned to your own dataset, to find the best way to provide an overall summary of the data.

Kinesthetic Activity

Divide the class into groups of three. Supply each group with a ruler and 4-8 pennies. Make sure every group has at least one pen or pencil.

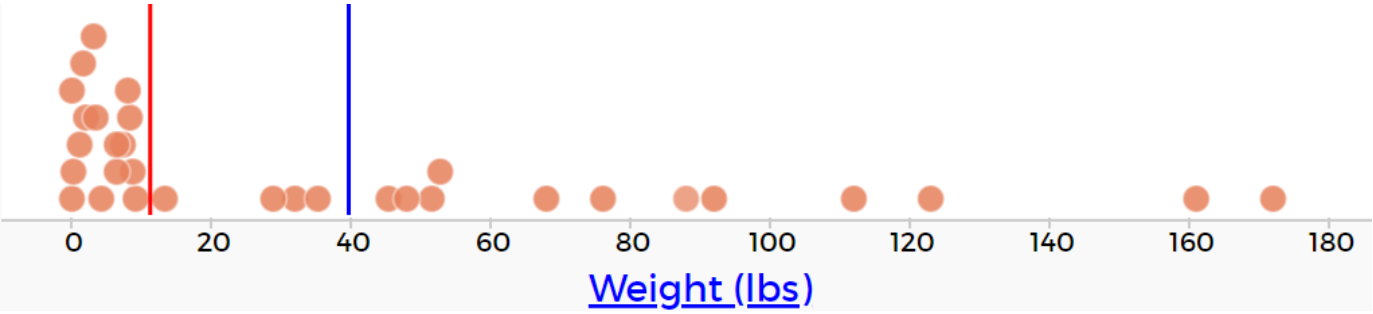
The arithmetic mean is the number that "balances" all the other numbers in the sample. So let's do some real balancing!

1. The ruler represents a number line with values (weight) distributed equally across the line. If there's values at every inch from 0 to 12, where should the pencil be placed in order to balance the ruler on top of it?
2. Place a penny at 1 and 11. Where must the pencil be placed to balance those two values? What is the mean of the values [1, 11]?
3. Place pennies at 1, 9 and 11. Where must the pencil be placed to balance those two values? What is the mean of of the values [1, 9, 11]?
4. Suppose you were to place two pennies at 2, and a third penny at 8. Can you *predict* where the pencil should be placed?

Investigate

Before digging into a discussion of mean, let's look at a visual.

If we plotted all the animals' weights as points on a number line, it would look something like this:



- Do you think there is a midpoint?
  - *There are 32 animals - meaning that there is not one point in the middle.*
- Is there a point that shows up most often?
  - *Since we see that dots are stacked up, it seems likely that there is some repetition in the animals' weights.*
- What do you think the red line represents? How about the blue one? How do you know?
  - *Be sure to solicit students' answers, which may vary. Red is the median, blue is the mean. Uncertainty at this point is okay! The remainder of the lesson is an exploration of these concepts.*

Each of these are different ways of "measuring center".

The Animal Shelter Bureau used one method of summary, called the ***mean***, or "average". The mean of a dataset is the sum of values divided by the number of values. To take the average of a column, we add all the numbers in that column and divide by the number of rows.

Pyret has a way for us to compute the mean of any ***quantitative*** column in a Table. It consumes a Table and the name of the column you want to measure, and produces the mean — or average — of the numbers in that column.

```
# mean :: Table, String -> Number
```

- What is the function's name? Domain? Range?
  - *The function's name is* `mean`*. The function consumes a table and string (domain), and produces a number (range).*

Notice that calculating the mean requires being able to add and divide, so the mean only makes sense for quantitative data. For example, the mean of a list of Presidents doesn't make sense. Same thing for a list of zip codes: even though we can divide a sum of zip codes, the output doesn't correspond to some "center" zip code.

- Type `mean(animals-table, "pounds")`. What does this give us?

- *39.715625.*

- Does this support the Bureau's claims?
  - *No, the mean is less than 40 pounds.*
- Now, turn to Summarizing Columns in the Animals Dataset. In the "measures of center" section, fill in the computed mean.

Synthesize

- Three animals weighing 5, 5, 10, and 100 pounds will have an average mean of 30 pounds. Can you think of another set of four animals that would have the same average? How many sets can you come up with?
- If you heard that the mean age of students in a kindergarten class was 21, would you be surprised? Why or why not?

Median *15 minutes*

Overview

Students learn a second measure of center: the ***median***. They learn the algorithm and the code to find the median, as well as situations where taking the median is more appropriate than the mean.

Launch

You computed the mean of that column to be almost exactly 40 pounds. That IS the average, but if we scan the dataset we'll quickly see that most of the animals weigh less than 40 pounds! In fact, more than half of the animals weigh less than just 15 pounds. What is throwing off the average so much?

Kujo and Mr. Peanutbutter!

In this case, the mean is being thrown off by a few extreme data points. These extreme points are called ***outliers***, because they fall far outside of the rest of the dataset. Calculating the mean is great when all the points are fairly balanced on either side of the middle, but it distorts things for datasets with extreme outliers. The mean may also be thrown off by the presence of ***skewness***: a lopsided shape due to values trailing off to the left or right.

- Make a `histogram` of the `pounds` column, and try different bin sizes.
- Can you see the huge number of animals clumped to the left, with Kujo and Mr. Peanutbutter as outliers skewed to the right?

A different way to measure center is to line up all of the data points — in order — and find a point in the center where half of the values are smaller and the other half are larger. This is the ***median***, or "middle" value of a list.

As an example, consider this list of ACT scores:

```
25, 26, 28, 28, 28, 29, 29, 30, 30, 31, 32
```

Here 29 is the ***median***, because it separates the "bottom half" (5 values below it) from the top half" (5 values above it).

The algorithm for finding the median of a quantitative column is:

1. Sort the numbers
2. Cross out the highest and lowest number
3. Repeat until there is only one number left…

4. When there are an even number of numbers in the list, as in the example below, there will be two numbers left at the end. Take the *mean* of those two numbers.

```
3, 7, 9, 21
```

The median of this list is 8, because 8 is the mean of the two middle numbers, 7 and 9. To find their mean, we added 7 and 9 to get 16 and split 16 in half.

Investigate

- Pyret has a function to compute the median of a list as well. Find the Contract for `median` on the Contracts Page.
- Compute the median for the `pounds` column in the Animals Dataset, and add this to Summarizing Columns in the Animals Dataset.
  - *The median is 11.3.*
- Is it different than the mean?
  - *Yes, it is very different!*
- What can we conclude when the mean is so much greater than the median?
  - *There are some very heavy animals that are causing the mean to be higher.*
- For practice, compute the mean and median for the weeks and age columns.
  - *Weeks: mean - 5.75; median - 4. Age: mean - 4.359375; median - 3.*

Synthesize

Looking at the shape of the data (via a histogram, for example), helps us determine whether it's probably better to use the mean or median.

Strong left skewness and/or low outliers can pull the mean down below the median, while right skewness and/or high outliers can pull it up above the median.

Mean is generally the best measure of center, because it includes information from every single point. But it's misleading for highly-skewed datasets, so statisticians fall back to the median.

Modes *25 minutes*

Overview

Students learn about the mode(s) of a dataset, how to compute them, and when it is appropriate to use them as a measure of center.

Launch

The third measure of center is called the ***modes*** of a dataset. The ***modes*** of a dataset are the values that appear *most often*.

Median and Mean always produce one number and many datasets are what we call "unimodal", having just one mode. But sometimes there are exceptions!

- If two or more values are equally common, there can be more than one mode.
- If all values are equally common, then there is no mode at all!

Consider the following three datasets:

```
1, 2, 3, 4
1, 2, 2, 3, 4
1, 1, 2, 3, 4, 4
```

- The first dataset has *no mode at all!*
- The mode of the second dataset is 2, since 2 appears more than any other number.
- The modes (plural!) of the last dataset are 1 and 4, because 1 and 4 both appear more often than any other element, and because they appear equally often.

Modes are rarely used to summarize quantitative data. It is very common as a summary of *categorical* data, telling us which category occurs most often.

In Pyret, the modes are calculated by the modes function, which consumes a Table and the name of the column you want to measure, and produces a *List* of Numbers.

```
# modes :: Table, String -> List<Number>
```

Investigate

- Compute the `modes` of the `pounds` column, and add it to Summarizing Columns in the Animals Dataset. What did you get?

- *0.1 and 6.5*

Synthesize

The most common animal weights are 0.1 and 6.5! That's well below our mean and even our median, which is further evidence of outliers or skewness.

At this point, we have a lot of evidence that suggests the Bureau's use of "mean" to summarize animal weights isn't ideal. We have three reasons to suspect that ***mean*** isn't the best value to use:

- The median is only 11.3 pounds.
- The modes of our dataset are 6.5 pounds and 0.1 pounds, which suggests clusters of animals that weigh mere fractions of the mean.
- When viewed as a histogram, we can see the right skewness and high outliers in the dataset. Mean is sensitive to datasets with skewness and/or outliers.

"In 2003, the average American family earned $43,000 a year — well above the poverty line! Therefore very few Americans were living in poverty."

- Do you trust this statement? Why or why not?
  - *Sample response: The mean is sensitive to outliers, and billionaires like Elon Musk, Jeff Bezos, etc. pull the mean is heavily to the right. This makes it appear that the "average" American family earns far more than they actually do. That's why the conclusion "very few Americans were living in poverty" cannot be drawn based on the mean.*

Consider how many policies or laws are informed by statistics like this! Knowing about measures of center helps us see through misleading statements.

You now have three different ways to measure center in a dataset. But how do you know which one to use? Depending on the shape of the dataset, a measure could be really useful or totally misleading! Here are some guidelines for when to use one measurement over the other:

- If the data doesn't show much skewness or have outliers, ***mean*** is the best summary because it incorporates information from every value.
- If the data has noticeable outliers or skewness, ***median*** gives a better summary of center than the mean.
- If there are very few possible values, such as AP Scores (1–5), ***modes*** could be a useful way to summarize the dataset.

Optional: We strongly recommend having students practice the Data Cycle with measures of center, using Data Cycle: Measures of Center. Sometimes what's created isn't a table *or* a display, and this

activity demonstrates that. It also drives home an important difference between Arithmetic and Statistical Questions.

Data Exploration Project (Measures of Center*flexible*

Overview

Students apply what they have learned about measures of center to their chosen dataset. In their Data Exploration Project Slide Template, they will complete the first four rows of the "Measures of Center and Spread" table. They will also interpret those measures of center, and record any interesting questions that emerge. To learn more about the sequence and scope of the Exploration Project, visit Project: Dataset Exploration. For teachers with time and interest, Project: Create a Research Project is an extension of the Dataset Exploration, where students select a single question to investigate via data analysis.

Launch

Let's review what we have learned about computing and interpreting three measures of center - mean, median, and modes.

- Describe how to compute mean, median, and modes.
- When ***mean*** provide the best summary?
  - *It includes information from every single point, so it is useful when the data doesn't show much skewness or have outliers.*
- When does ***median*** provide the best summary?
  - *Statisticians fall back to the median when working with highly skewed datasets.*
- When are ***mode(s)*** a useful way to summarize a dataset?
  - *Mode(s) are most useful when a dataset has very few values.*

Investigate

Let's connect what we know about measures of center to your chosen dataset.

- Open your chosen dataset starter file in Pyret.** *Teachers: Students have the opportunity to choose a dataset that interests them from our List of Datasets in the Choosing Your Dataset lesson.*
- Choose two quantitative columns that you'd like to analyze.
- Use Pyret to compute the mean, median and modes of one of them.
- **It's time to add to your Data Exploration Project Slide Template.**

- Locate the "Measures of Center and Spread" section of your Exploration Project and, in the slide following the example, replace `Column A` with the title of the column you just investigated.
- Then type in the mean, median and modes that you just identified. Leave the other rows blank. We will come back to them another day.
- On the next slide, repeat with `Column B` using the second column you're interested in.

Invite students to discuss their results and consider how to interpret them.

Add your interpretations to the two "Measures of Center and Spread" slides and record any questions that emerged in the "My Questions" section at the end of the slide deck.

Synthesize

Share your findings!

Did you discover anything surprising or interesting about your dataset?

Which measures of center do you think were the most useful for the quantitative columns you chose?

What questions did the measures of center inspire you to ask about your dataset?

When you compared your findings with other students, did you make any interesting discoveries? (For instance: Did everyone find mode(s)? Did anyone have a measure of center that was dramatically influenced by an outlier?)

Additional Exercises

- Mode(s) (Desmos)
- Critiquing Written Findings
- Data Cycle: Measures of Center