Compound Inequalities: Solutions & Non-Solutions

(Also available in Pyret)

Students build upon their understanding of Booleans and simple inequalities to compose compound inequalities using the concepts of union and intersection.

Lesson Goals	Students will be able to: • Understand how the conjunctions `and` and `or` differ • Describe how functions can work together • Describe the solution set of a ***compound inequality***
Student-Facing Lesson Goals	• Let's use two or more inequalities together. • Let's learn the difference between how `and` and `or` are used in programming. • Let's find solutions and non-solutions for compound inequalities.
Prerequisites	• Simple Data Types • Contracts • Simple Inequalities
Materials	• PDF of all Handouts and Page • Compound Inequalities Starter File • Lesson Slides • Printable Lesson Plan (a PDF of this web page)
Supplemental Materials	• Additional Printable Pages for Scaffolding and Practice

Glossary

compound inequality :: an inequality that combines two simple inequalities using *and* or *or*

function :: a relation from a set of inputs to a set of possible outputs, where each input is related to exactly one output

union :: the set of values that makes either or both of a set of inequalities true

Introducing Compound Inequalities

Overview

Students consider the need to *compose* inequalities, and think about how to write them.

Launch

We use inequalities for lots of things:

- Is it hot out? ($temperature > 80°$)
- Did I get paid enough for painting that fence? ($paid \geq \$100$)
- Are the cookies finished baking? ($timer = 0$)

What other examples can you come up with?

Many times we need to *combine* inequalities:

- Should I go to the beach? ($temperature > 80°$ and $weather = "sunny"$)
- Was this burrito worth the price? ($taste = "delicious"$ and $price$ leq $\$15$)

Can you think of examples of when we might want to combine inequalities?

Guide students through other examples of `and` and `or` with various statements, such as:

- "I'm wearing a red shirt AND I'm a math teacher, true or false?"
- "I'm an NBA basketball star OR I'm having pizza for lunch, true or false?"

This can make for a good sit-down, stand-up activity, where students take turns saying compound Boolean statements and everyone stands if that statement is true for them.

Investigate

Both mathematics and programming have ways of combining - or *composing* - inequalities.

Complete Converting Circles of Evaluation to Code and Compound Inequalities — Practice.

Synthesize

Be really careful to check for understanding here.

- Expressions using `and` only produce `true` if both of their sub-expressions are `true`.
- Expressions using `or` produce `true` if **either** of their sub-expressions are `true`.

Strategies for English Language Learners

When describing compound inequalities, be careful not to use "English shortcuts". For example, we might say "I am holding a marker *and* an eraser" instead of "I am holding a marker *and* I am holding an eraser." These sentences mean the same thing, but the first one obscures the fact that "and" joins two complete phrases. For ELL/ESL students, this is unecessarily adds to cognitive load!

Solutions and Non-Solutions of Compound Inequalities

Launch

Complete Compound Inequality Warmup.

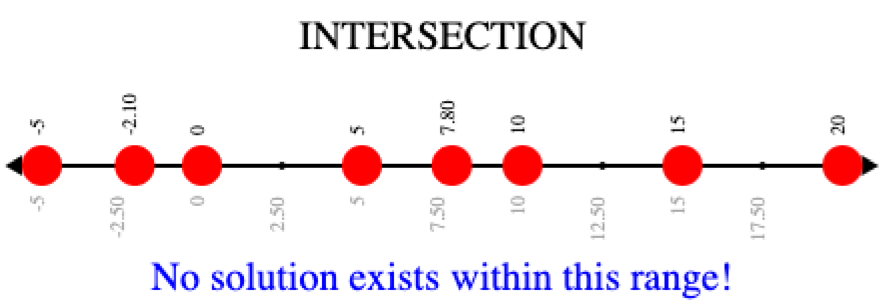
Investigate

- Open the Compound Inequalities Starter File.
- Click "Run" to see graphs of the inequalities you’ve just considered.

When students click "Run", four graphs will appear. The top two are the simple inequalities they’ve just discussed. Encourage students to verify that their solutions and non-solutions are correct.

- The bottom two graphs are produced by the special functions `and-intersection` and `or-union`.
- Read the comments in the Definitions area with your partner to learn how these functions are supposed to work.
- Then complete Exploring Compound Inequality.

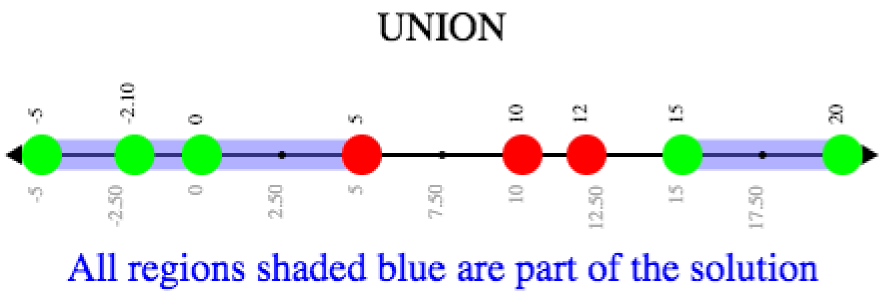
- Why is the circle on 5 red and the circle on 15 green?
  - *The circle on 5 is red because 5 is not part of the solution - it is not bigger than itself.*
  - *The circle on 15 is green because 15 is part of the solution - it is less than or equal to 15.*



- Why isn’t there a solution to $x < 5$ *and* $x \geq 15$
  - *There aren’t any numbers that are both smaller than 5 and greater than or equal to 15, so there is no solution!*

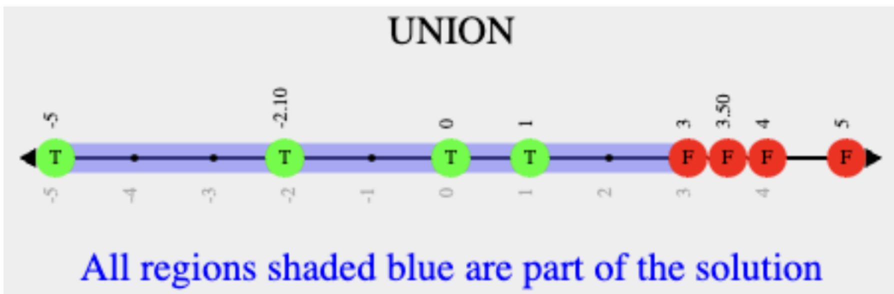
`or-union` takes in two functions and a list of numbers and produces a graph with the points and the shaded ***union*** of values that make either or both of the inequalities true.

In order to make an `or` statement true, a value only has to make one of the inequalities true.



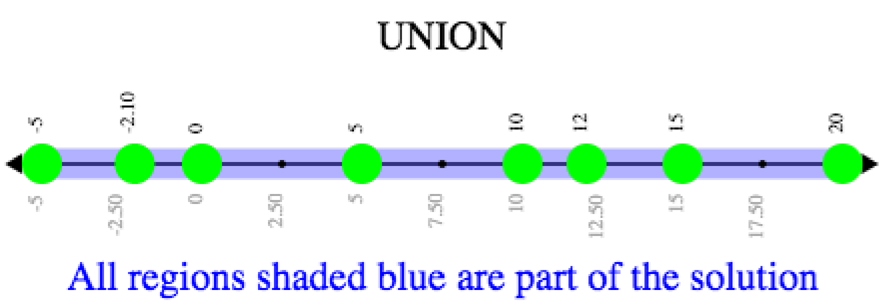
x < 5 or x ≥ 15

Sometimes unions are represented by two separate arrows pointing in opposite directions with a gap between their starting points.



x < 1 or x < 3

Sometimes unions overlap and appear as a single arrow pointing in one direction.



x > 5 or x ≤ 15

Sometimes unions overlap and cover the entire number line!

- Why is the whole graph of $x > 5 \text{ or } x \leq 15$ shaded blue?
  - *Because every number in the universe is either greater than 5 or less than or equal to 15, so there aren't any non-solutions!*

Once students are familiar with the starter file, they are ready to use it as they practice identifying solutions and non-solutions for compound inequalities.

- Turn to Compound Inequalities: Solutions & Non-Solutions.
- Explore the compound inequalities listed using the Compound Inequalities Starter File, identifying solutions and non-solutions for each.

Instead of defining two functions as simple inequalities, we can produce an inequality graph by defining one function to be a ***compound inequality***!

In the following activity, we'll analyze inequality graphs and define a *single* function that produces the graph.

Walk students through the completed first example before they attempt to write this code on their own.

- Turn to Compound Inequality Functions.
- Write code to describe the compound inequalities pictured.

Synthesize

- How did the graphs of intersections and unions differ?

Additional Exercises

- Converting Circles of Evaluation with Booleans to Code 2