Defining Linear Functions

(Also available in WeScheme)

Students explore the concept of slope and y-intercept in linear relationships, using function definitions as a third representation (alongside tables and graphs).

Lesson Goals	Students will be able to… • define linear functions using slope and y-intercept • match different representations of the same linear relationship
Student-facing Lesson Goals	• Let's learn to connect different representations of the same linear relationship. • Let's learn to identify the slope and y-intercept in a function definition. • Let's define functions expressing linear relationships.
Prerequisites	• Simple Data Types • Contracts • Functions Can Be Linear • Functions: Contracts, Examples & Definitions
Materials	• PDF of all Handouts and Page • Lesson Slides • Printable Lesson Plan (a PDF of this web page)
Supplemental Materials	• *Exploring Linearity in Definitions Starter File* • *Exploring Linearity in Tables Starter File* • *Exploring Linearity in Graphs Starter File* • *Card Sort Activities focused on Linear Functions: Tables, Graphs, and Definitions (Desmos)*

Key Points for the Facilitator	
Key Points for the Facilitator	• Lines are made of points (try to avoid referring to "lines" as much as "collections of points") • Linear functions can be represented as straight lines on a graph or as sequences that change at a constant rate in a table. • Stress that Graphs, Definitions, and Tables are all *different ways of viewing the same function*. The misconception to fight against is the idea that $f(x) = 3x + 2$ is somehow the "real" function, with the graph being "the thing the function draws."

Glossary

coordinate pair :: a set of numbers describing an object's location on the coordinate plane

dependent variable :: When modeling a relationship between an input and an output (e.g. - distance over time), we are curious about how a change in the input (typically graphed on the x-axis) impacts the output (y). When the output is entirely dependent on the input, we refer to the output as the "dependent variable".

domain :: the type or set of inputs a function expects, i.e., the independent variable(s) that govern the output of the function

independent variable :: When modeling a relationship between an input and an output (e.g. - distance over time), we are curious about how a change in the input (typically graphed on the x-axis) impacts the output (y). When the output is entirely dependent on the input, we refer to the input as the "independent variable".

range :: the type or set of outputs that a function produces, i.e., the dependent variable(s)

slope :: the steepness of a straight line on a graph reported as a number which tells how much y changes for every unit increase in x

y-intercept :: the point where a line or curve crosses the y-axis of a graph

Defining Linear Functions *35 minutes*

Overview

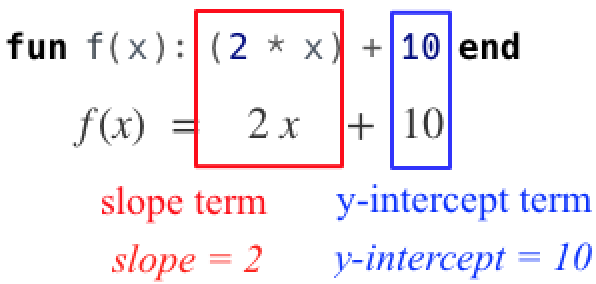
Students explore function definitions as a way of expressing linear relationships, and construct tables and graphs from those definitions.

Launch

As you've seen, a **function definition** is a way of summarizing a relationship between an ***independent variable*** (x) and a ***dependent variable*** (y). You've seen how a linear relationship can be expressed as a table or graph. But what do these kinds of relationships look like as a definition?

> Linear functions are defined by their ***slope*** and ***y-intercept***.

Here we see a function definition written using pyret notation and using function notation.



The **slope-intercept form of the line** includes the slope as the coefficient of x, and the y-intercept as the numerical term. You will hear people describe this form as $y = mx + b$, where m stands for slope and b stands for the y-intercept.

While it is common to write the x-term first and the y-intercept second, they can be written in any order!

Function Notation	Pyret Code
$f(x) = 6x - 10$	`fun f(x): (6 * x) - 10 end`
$f(x) = -10 + 6x$	`fun f(x): -10 + (6 * x) end`

When the slope is zero (and the line is horizontal)… we may choose whether or not to write the slope term.

"Visible" Slope	"Invisible" Slope
$f(x) = 0x + 22$	$f(x) = 22$

When the y-intercept is 0 (and the line crosses the y-axis at the origin)… we may choose whether or not to write the slope term.

"Visible" y-intercept	"Invisible" y-intercept
$f(x) = 3.2x + 0$	$f(x) = 3.2x$

To check our work, we can apply the function to the x-value from any ***coordinate pair*** on our table or graph, and it should produce the y-value!

As with tables and graphs, a function definition can also reveal whether or not the function is linear. Functions that are not linear will follow other forms, for example they may include exponents or absolute values.

Investigate

Let's start by identifying the slope and y-intercept from function definitions.

- Identifying Slope and y-intercept in Definitions

Let's connect definitions to tables and graphs.

Writing down the slope and y-intercept beneath each representation will help!

- Matching Tables to Function Definitions
- Matching Graphs to Function Definitions

Let's write our own definitions from tables and graphs!

- Summarizing Tables with Function Definitions
- Summarizing Graphs with Function Definitions

Common Misconceptions

It is common to think of the graph as the "output" of the function, rather than the function itself. Most math textbooks will use language like "matching the graph to the function", suggesting that the graph is somehow not the function! Since this language is pervasive, it's important to actively push against it.

Synthesize

What strategies did you use?

Function definitions are a way of talking about relationships between an ***independent variable*** and a ***dependent variable***: milk costs $0.59/gallon, a stone falls at $9.8m/s^2$, or there are 30 students for every teacher at a school. If we can figure out the relationship between a small sample of data, we can *make predictions* about what happens next. We can see these relationships as tables, graphs, or symbols in a definition. We can even think about them as a mapping between ***Domain*** and ***Range***!

When we talk about functions, it's helpful to be able to switch between representations, and see the connections between them.

Finding the y-intercept from the Slope and a Point *20 minutes*

Launch

Consider the function $f(x) = 3x$.

x	0	1	2	3
y	0	3	6	9

- What is the slope?
  - *3*
- What is the y-intercept?
  - *0*
- What is the y-value when x = 2?
  - *6*

Anytime the y-intercept is 0, we can multiply any x-value by the slope to get its corresponding y-value.

But if the y-intercept isn't zero… there is another step to finding the y-value.

Consider the function $f(x) = 3x - 2$.

x	0	1	2	3

y	-2	1	4	7

- What is the slope?
  - *3. Same as for the previous function.*
- What is the y-intercept?
  - *-2*
- What is the y-value when x = 2?
  - *4. Two less than the y-value for $x=3$ in the previous function, where the y-intercept was 0.*

The y-intercept always gets added to / subtracted from the product of the slope and the x-value to find the corresponding y-value.

Investigate

As discussed above, the relationship between the x-values and the y-values can be described using $y = mx + b$, where m stands for ***slope*** and b stands for the ***y-intercept***.

If we solve that for the y-intercept…

$b = y - mx$

In other words, *the y-intercept can be calculated by subtracting the product of the slope and any x-value from the corrseponding y-value.*

Let's say the slope is 3. And we know that the line passes through the point (7,9).

- $b = y - mx$
- $m = 3$
- $x = 7$
- $y = 9$

To find the y-intercept, subtract 9 *(the y-value of the point)* minus 3×7 *(the product of the slope and the x-value of the point).*

- $b = 9 - 3(7)$
- $b = 9 - 21$
- $b = -12$… we found our y-intercept!

We can now use the slope and y-intercept to write our function definition:

- $f(x) = 3x - 12$

Consider the table below.

x	80	81	82	83
y	150	155	160	165

- What is the slope?
  - *5*
- Calculate the y-intercept using the first coordinate pair.
  - $b = y - mx$
  - $b = 150 - 5(80)$
  - $b = 150 - 400$
  - $b = -250$
- Do you get the same y-intercept if you use another pair?
  - *Yes.*

Additional Practice

- Card Sort Activities focused on Linear Functions: Tables, Graphs, and Definitions (Desmos)

We also have three Pyret starter files for additional practice with:

- Exploring Linearity in Definitions Starter File.
- Exploring Linearity in Tables Starter File
- Exploring Linearity in Graphs Starter File