The Additive Inverse Property

(Also available in Pyret)

Students develop a more nuanced interpretation of the Commutative and Associative Properties as a result of their exploration of the inverse relationship between addition and subtraction.

Lesson Goals	Students will be able to… • Recognize that subtracting x is the same as adding $-x$. • Acknowledge *flexibility* in the Order of Operations.
Student-facing Lesson Goals	• Let's explore the inverse relationship between addition and subtraction.
Prerequisites	• Translating Between Words and Math • Simple Data Types • Contracts • The Commutative Property • Equivalence • The Associative Property
Materials	• PDF of all Handouts and Page • Printable Lesson Plan (a PDF of this web page)

Key Points For The Facilitator	
Key Points For The Facilitator	• This lesson is all about *mathematical flexibility*, and may challenge preexisting ideas about arithmetic. For students who've had a mostly procedural introduction to mathematics - or who have high math anxiety - the flexibility introduced by this lesson can be intimidating! • Embrace an open mind and expect the same of your students, and be attentive to their emotional response as the lesson progresses. If students are getting anxious, they will not be able to concentrate and focus on the content. • There is no expectation that they will master operations with integers. Rather, the lesson is designed to get students thinking about how every number has an additive inverse.

Glossary

Additive Inverse Property :: Adding a number and its opposite always produces zero.
Associative Property :: When adding three numbers or multiplying three numbers, it does not matter whether you start with the first pair or the last. The same is true when either adding or multiplying four numbers, five numbers, etc.
Commutative Property :: For any expression involving only addition or only multiplication, changing the order of the numbers will not change the result.
equivalent :: expressions are equivalent when they simplify to the same value, no matter what value is assigned to their variables (if there are any)
opposite :: Two numbers are opposites when they are the same distance from zero on the number line.

The Additive Inverse Property *10 minutes*

Overview

Students discover the Additive Inverse Property, which tells us that adding a number and its opposite always produces zero.

Launch

We've learned that the ***Commutative Property*** and ***Associative Property*** apply to addition… but not division. What if there was a way to rewrite subtraction as addition? Then we could apply the Commutative and Associative Properties to subtraction expressions, too! Let's explore this idea.

Let's play a game. Draw a two-column table on the board to record student responses. Label the left-hand side "Starting Value," and label the right-hand side "?" (See example below).

Starting Values	?

I'm going to write a number in the left-hand column. You are going to tell me what value I should *add* to that number, to get a sum of zero. I'll record your response in the right-hand column.

- The first number is 5. What do I need to add to 5 to get 0?
  - *-5. If this is students' first exposure to negative numbers, consider modeling a few examples.*
- The next number is 20. What do I need to add to 20 to get 0?
- How about $^1/_2$?
- How about -45?
- Can someone offer me another pair of numbers that add up to 0?
  - *Allow a variety of students to share. Record responses on the table.*

Explain to students that these number pairs all represent ***opposites***.

The ***Additive Inverse Property*** tells us that adding a number and its opposite always produces zero.

Every number has an additive inverse. Ensure that students are comfortable with this concept before moving onto the next activity.

Investigate

- Turn to The Additive Inverse Property.
- In the first section, determine the additive inverses and write them in the spaces provided.
- Then, fill in the missing number to complete the equations. Some equations use mathematical notation and some use Circles of Evaluation.

Synthesize

- Can you think of a way to visually represent that adding a number and its opposite always produces zero?
- How would you explain the concept of the Additive Inverse Property to another student?
- Looking forward: Can you predict what effect the Multiplicative Inverse Property might have?

Addition and Subtraction: Inverse Operations *20 minutes*

Overview

Students rewrite addition expressions as subtraction, and subtraction expressions as addition by applying their knowledge of the ***Additive Inverse Property***.

Launch

Now that we understand what the additive inverse is, we are ready to think about how we can put it to use… perhaps it can make some computations simpler?

- Complete Discover Inverse Operations: Addition & Subtraction.
- When you're done, complete Discover Inverse Operations: Addition & Subtraction (2).
- What did you observe about the additive inverse?

Emphasize to students that two main ideas should have emerged during the previous exploration:

- Subtraction is equivalent to adding the ***opposite***.
- We can rewrite any subtraction expression as addition, and any addition expression as subtraction!

One powerful advantage emerges when we write subtraction as addition: we now can apply the Commutative and Associative Properties to a much broader set of expressions!

Investigate

Now, students are ready to continue their exploration of subtraction as the inverse of addition. Note: To complete this worksheet, students do not need to be fluent at integer addition and subtraction. We just want students thinking about when expressions are ***equivalent*** based on what they have learned about the additive inverse.

Complete Which One Doesn't Belong?.

Have students share strategies for determining equivalence. What are the different ways that they thought about the Additive Inverse Property?

Synthesize

- Claire and Soraya want to write an equivalent expression for $22 - 30$. Claire studies the expression and announces that, because it involves subtraction, the Commutative Property cannot be applied. Is she correct?
- Soraya grabs a pencil and writes the following: $22 + -30$. She says, "There! I fixed it. Now we can apply the Commutative Property." Explain what Soraya did. Is she correct?
- Use the Additive Inverse Property to simplify this expression using mental computation: $3 + 96.8 - 42.74 - 96.8 + 7 - 3 + 42.74$

The "Left-to-Right" Rule *25 minutes*

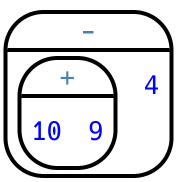
Overview

Students examine whether rigid adherance to the "left-to-right" rule is needed when adding and subtracting.

Launch

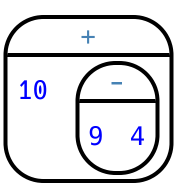
- Consider this expression: $10 + 9 - 4$
- What do we get when we simplify it to a single value?
  - *15*
- How did you arrive at your answer?

Invite students to share their responses. Students will likely note the presence of addition and subtraction. They will also likely conclude that they must work from left to right to arrive at a correct result. This solving strategy can be represented by the Circle of Evaluation, below.



But is it essential to solve from left to right?

Ask if anyone opted to subtract *before* adding. If so, invite them to share their method and then invite other students to weigh in. Have students evaluate the Circle of Evaluation below and then assess if it is equivalent to the Circle of Evaluation, above.



We've learned that the Associative Property applies for expressions with only addition… not addition *and* subtraction. Many of us have also learned that when an expression includes addition and subtraction, we must work from left to right. **So… what's going on!?** It appears that we get the same result regardless of how we simplify this expression.

Investigate

Does subtracting *first* work every time? Can we rearrange the groupings of any expression with both addition and subtraction? Let's investigate.

- Turn to Subtract First… or Solve Left-to-Right?.
- There, you will test out the this algorithm on several different expressions to see if subtracting and then adding produces the correct result every time.
- What do you Notice? What do you Wonder?
- Why are we able to change the groupings for an expression like $10 + 9 - 4$ … but *not* for an expression like $10 - 9 - 4$?
- Describe why the this "subtraction first" algorithm works. (Hint: Think about the ***Additive Inverse Property***!)

Encourage students to think deeply about why this algorithm works – and if you'd like, invite them to consider and discuss why students all across the country are typically taught just one algorithm when, typically, there are an abundance to choose from!

Now, let's put our new knowledge to use! Project the problems below one at a time, and invite students to solve using mental math.

Scan each problem to identify any additive inverses, then solve using mental computation.

- $4 + 5 + 97 - 4 + 3$
  - *Solution:* 105
- $9 + 17 + 41 - 17$
  - *Solution:* 50
- $67 - 104 + 937 - 67 + 104$
  - *Solution:* 937

Synthesize

- How did it feel to scan the problem, find any additive inverses, and then solve mentally?
- Did you like this new approach, or do you prefer solving left to right?
- How would you explain to another student why they do *not* always need to solve from left to right when evaluating expressions with addition and subtraction.
- What are some advantages of solving left to right? What are some disadvantages?