Absolute Value and Opposite

(Also available in WeScheme)

Students use Circles of Evaluation to explore the effect of negation and absolute value on arithmetic and algebraic expressions.

Lesson Goals	Students will be able to… • Define absolute value as a number's distance from zero on the number line. • Recognize that two numbers are referred to as "opposites" if they are the same distance from zero. • Use Circles of Evaluation to analyze expressions with opposite and absolute value.
Student-facing Lesson Goals	• Let's use Circles of Evaluation to think about opposites. • Let's use Circles of Evaluation to think about absolute value.
Prerequisites	• Translating Between Words and Math • Simple Data Types • Contracts • The Commutative Property • Equivalence • The Associative Property • Variables
Materials	• PDF of all Handouts and Page • Printable Lesson Plan (a PDF of this web page)
Preparation	• Draw or project a number line of the integers from -10 to 10. This will be the focal point of a lot of the conversation for today's lesson.

Key Points For The Facilitator	• The difference between absolute value and opposite can be subtle and confusing for students. Sometimes, given an input, they produce the same output - but at other times, their outputs differ.

Glossary

absolute value :: the (positive) distance of a number from zero, annotated | x |

algebraic expression :: a mathematical expression that consists of numbers, operators, and variables. The variables' values are sometimes unknown. If we define variables, algebraic expressions can be simplified to a single numeric value.

arithmetic expression :: a mathematical expression that consists of numbers and operators, which does not have an equal sign. When we apply the laws of arithmetic, arithmetic expressions can be simplified to a single numeric value.

equation :: a statement that two expressions are equal

function :: a relation from a set of inputs to a set of possible outputs, where each input is related to exactly one output

operator :: a symbol that manipulates two Numbers and produces a result

opposite :: Two numbers are opposites when they are the same distance from zero on the number line.

variable :: a name or symbol that stands for some value or expression, often a value or expression that changes

Finding Opposites *20 minutes*

Overview

Students are introduced to their first non-operator ***function*** - `negate` . Using this function, they create and evaluate Circles of Evaluation to explore the notion of ***opposites***.

Launch

Instruct students to draw a number line that spans from -10 to 10, or draw/project one at the front of the room.

- What number is the same distance from zero as 8?
  - -8
- What number is the same distance from zero as -3?
  - 3
- The pairs of numbers we've just identified are called ***opposites*** - two numbers that are the same distance from zero on the number line, with one negative and one positive.
- What are some other pairs of opposites that you can identify?
  - *Possible responses:* 6.5 *and* -6.5; 4 *and* -4; $^{7}/_{3}$ *and* $-\,^{7}/_{3}$.

Investigate

Today is an exciting day! Rather than using an ***operator*** at the top of each Circle of Evaluation, we are going to use… ***functions***!

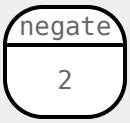
Operators always have two inputs and, in standard mathematical notation, they always go in the middle of their inputs. Functions, however, can have *any amount* of inputs - and all kinds of inputs, not just numbers!

Incorporating ***functions*** into our Circles of Evaluation will expand their utility. With functions, we can diagram and interpret expressions that include opposites, absolute value, exponents, square roots, Booleans… the list goes on.

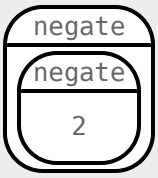
The function used to represent "the opposite" is called `negate` . It can be used like any other operator that we put at the top of a Circle of Evaluation - with one important difference: when we use `negate` , we include just one argument inside the Circle.

- What does the Circle of Evaluation below evaluate to?

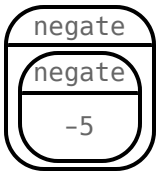
- *The Circle below evaluates to -2.*

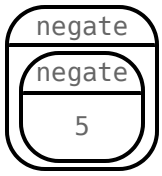


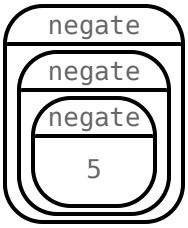
- What happens if we nest a negating Circle of Evaluation *inside of* another negating Circle?
  - *The opposite of the opposite of a number is the number itself.*

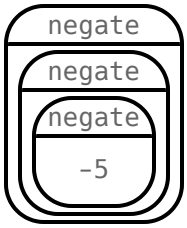


- Try negating a few more times, to make sure you've got the hang of it. What do each of these Circles evaluate to?









- Practice applying `negate` on True or False? Negate. Evaluate each side of the ***equation*** to confirm if it is true or false.
- On True or False? Negate (2), explore Circles of Evaluation with ***Variables***, and review previously explored properties (Identity, Commutative, Associative, and Distributive).

When we want to show negation of a number or an expression, we just use a negative sign! So, while we can read -45 by simply saying "negative 45," another way to read that value's name is by saying, "the opposite of 45." Similarly, $-(x + m)$ can be translated as "the opposite of the sum of x and m."

Pedagogy Note: Can we call it "minus 45"?!

Precise language is crucial when dealing with this symbol: `-`.

In math we want students to read for *comprehension*, not just mechanically sounding out each symbol in order. We read the ***arithmetic expression*** $12 - 9$ as "12 minus 9" *because subtraction is the "verb"*, not because the minus sign happens to be in the middle!

When we encounter a value such as -20, however, it is confusing and misleading to name that value "minus 5." Subtraction is not happening here - negation is! But if the teacher uses the "symbols in order" wording, it reinforces the lower level of reading comprehension for students.

Synthesize

- Two opposites are 8 units apart from each other. What are they?
  - 4 and -4
- Two opposites are 42.5 units apart from each other. What are they?
  - 21.5 and -21.5
- Cia says that opposites are always negative. Are they right? Why?
  - *No, opposites are not always negative. Positive numbers always have negative opposites - but negative numbers have positive opposites!*
- We have learned that opposites are two numbers that are the same distance from zero on the number line, with one negative and one positive. Knowing this, what do you think is the opposite of zero?
  - *Zero is its own opposite!*

Absolute Value *30 minutes*

Overview

Students consider the meaning of ***absolute value***, and apply the concept to Circles of Evaluation using `abs`.

Launch

What is the distance between these two points on the number line: -8 and 5?

Give students a minute to contemplate, and then invite them to verbally share their strategies. Record students' thinking on the board, annotating the number line. All strategies are welcome, with a special interest in any discussion that hones in on the idea of *the distance of a number from zero.*

Explain to students that we have a term for *the distance of a number from zero* - it's ***absolute value***.

Absolute value is the (positive) distance of a number from zero.

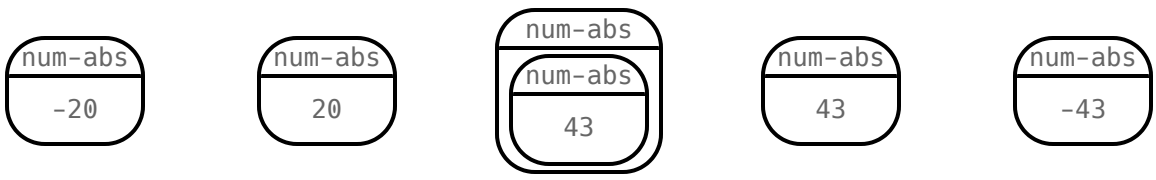
We annotate absolute value like this: $|x|$, with x being any given number. When we encounter an expression like $|x|$, we say "the absolute value of x."

Because *opposites* are the same distance away from zero, they will always have the same absolute value. So, $|4| = 4$ and $|-4| = 4$.

Investigate

The ***function*** that we will use to represent absolute value is `abs`. It can be used like any other operator that we put at the top of a Circle of Evaluation. As with `negate`, when we use `abs`, we include just one argument inside the Circle of Evaluation.

- Let's try evaluating some Circles of Evaluation with `abs`! What does each of the Circles below evaluate to?
  - $20, 20, 43, 43, 43$



- On True or False? Absolute Value & Negate, you will compare expressions with `abs` to expressions with `negate`
- On the bottom half of the page, determine whether variable equations featuring `negate` and `abs` are always, sometimes, or never true. Be sure to explain your response.
- Examine the Circles of Evaluation to determine Which One Doesn't Belong? Absolute Value & Negate. The page starts with numeric values and then integrates variables. Place a check mark by each Circle that meets the condition stated on the left.

Check in with students to ensure that they have a solid understanding of absolute value before moving forward.

Synthesize

Think about the ***algebraic expressions*** $|h|$ and $-h$.

- What do we know about the outcomes of each of these expressions?
  - *$|h|$ is always positive or zero, while $-h$ can be negative, zero, or positive.*
- When do they produce the same outcome?
  - *$-h$ is positive when h is negative, and $-h$ is negative when h is positive. As a result, $|h|$ and $-h$ produce the same outcome only when h is negative or zero.*
- When do they produce different outcomes?
  - *$|h|$ and $-h$ produce different outcomes when h is positive.*