Name: ____

Algebra 2 Student Workbook

Fall, 2023

Workbook v0.8-beta

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Introduction to Computational Data Science

Many important questions ("What's the best restaurant in town?", "Is this law good for citizens?", etc.) are answered with *data*. Data Scientists try and answer these questions by writing *programs that ask questions about data*.

Data of all types can be organized into **Tables**.

- Every Table has a **header row** and some number of **data rows**.
- **Quantitative data** is numeric and measures *an amount*, such as a person's height, a score on a test, distance, etc. A list of quantitative data can be ordered from smallest to largest.
- **Categorical data** is data that specifies *qualities*, such as sex, eye color, country of origin, etc. Categorical data is not subject to the laws of arithmetic — for example, we cannot take the "average" of a list of colors.

The Animals Dataset

This is a printed version of the animals spreadsheet. **The numbers on the left side are NOT part of the table!** Instead, they are provided to help you identify the index of each row.

	name	species	sex	age	fixed	legs	pounds	weeks
0	Sasha	cat	female	1	false	4	6.5	3
1	Snuffles	rabbit	female	3	true	4	3.5	8
2	Mittens	cat	female	2	true	4	7.4	1
3	Sunflower	cat	female	5	true	4	8.1	6
4	Felix	cat	male	16	true	4	9.2	5
5	Sheba	cat	female	7	true	4	8.4	6
6	Billie	snail	hermaphrodite	0.5	false	0	0.1	3
7	Snowcone	cat	female	2	true	4	6.5	5
8	Wade	cat	male	1	false	4	3.2	1
9	Hercules	cat	male	3	false	4	13.4	2
10	Toggle	dog	female	3	true	4	48	1
11	Boo-boo	dog	male	11	true	4	123	24
12	Fritz	dog	male	4	true	4	92	3
13	Midnight	dog	female	5	false	4	112	4
14	Rex	dog	male	1	false	4	28.9	9
15	Gir	dog	male	8	false	4	88	5
16	Max	dog	male	3	false	4	52.8	8
17	Nori	dog	female	3	true	4	35.3	1
18	Mr. Peanutbutter	dog	male	10	false	4	161	6
19	Lucky	dog	male	3	true	3	45.4	9
20	Kujo	dog	male	8	false	4	172	30
21	Buddy	lizard	male	2	false	4	0.3	3
22	Gila	lizard	female	3	true	4	1.2	4
23	Bo	dog	male	8	true	4	76.1	10
24	Nibblet	rabbit	male	6	false	4	4.3	2
25	Snuggles	tarantula	female	2	false	8	0.1	1
26	Daisy	dog	female	5	true	4	68	8
27	Ada	dog	female	2	true	4	32	3
28	Miaulis	cat	male	7	false	4	8.8	4
29	Heathcliff	cat	male	1	true	4	2.1	2
30	Tinkles	cat	female	1	true	4	1.7	3
31	Maple	dog	female	3	true	4	51.6	4

Categorical or Quantitative?

- **Quantitative data** measures an *amount* and can be ordered from smallest to largest.
- **Categorical data** specifies *qualities* and is not subject to the laws of arithmetic — for example, we cannot take the “average” of a list of colors.

Note: Numbers can be sometimes be categorical rather than quantitative!

For each piece of data below, circle whether it is **Categorical** or **Quantitative** data.

1	Hair color	categorical	quantitative
2	Age	categorical	quantitative
3	ZIP Code	categorical	quantitative
4	Date	categorical	quantitative
5	Height	categorical	quantitative
6	Sex	categorical	quantitative
7	Street Name	categorical	quantitative

For each question, circle whether it will be answered by **Categorical** or **Quantitative** data.

8	We’d like to find out the average price of cars in a lot.	categorical	quantitative
9	We’d like to find out the most popular color for cars.	categorical	quantitative
10	We’d like to find out which puppy is the youngest.	categorical	quantitative
11	We’d like to find out which cats have been fixed.	categorical	quantitative
12	We want to know which people have a ZIP code of 02907.	categorical	quantitative
13	We’d like to sort a list of phone numbers by area code.	categorical	quantitative

Questions and Column Descriptions

What questions can you ask about the animals dataset? For each question, **can it be answered by this dataset?**
Make sure you have at least two questions that can be answered, and at least one that cannot.

Notice	Wonder	Answered by this dataset?
I notice that	…so I wonder	*Yes* *No*
I notice that	…so I wonder	*Yes* *No*
I notice that	…so I wonder	*Yes* *No*
I notice that	…so I wonder	*Yes* *No*
I notice that	…so I wonder	*Yes* *No*
I notice that	…so I wonder	*Yes* *No*
I notice that	…so I wonder	*Yes* *No*

Describe the table, and two of the columns, by filling in the blanks below.

1. This dataset is ______________________________, which contains __________ data rows.
2. Some of the columns are:

   a. ______________________ (column name), which contains ______________________ (*categorical or quantitative*) data. Some example values are:

   ______________________________.

   b. ______________________ (column name), which contains ______________________ (*categorical or quantitative*) data. Some example values are:

   ______________________________.

Introduction to Programming

The **Editor** is a software program we use to write Code. Our Editor allows us to experiment with Code on the right-hand side, in the **Interactions Area**. For Code that we want to *keep*, we can put it on the left-hand side in the **Definitions Area**. Clicking the "Run" button causes the computer to re-read everything in the Definitions Area and erase anything that was typed into the Interactions Area.

Data Types

Programming languages involve different ***data types***, such as Numbers, Strings, Booleans, and even Images.

- Numbers are values like `1`, `0.4`, `1/3`, and `-8261.003`.
  - Numbers are *usually* used for quantitative data and other values are *usually* used as categorical data.
  - In Pyret, any decimal *must* start with a 0. For example, `0.22` is valid, but `.22` is not.
- Strings are values like `"Emma"`, `"Rosanna"`, `"Jen and Ed"`, or even `"08/28/1980"`.
  - All strings *must* be surrounded in quotation marks.
- Booleans are either `true` or `false`.

All values evaluate to themselves. The program `42` will evaluate to `42`, the String `"Hello"` will evaluate to `"Hello"`, and the Boolean `false` will evaluate to `false`.

Operators

Operators (like `+`, `-`, `*`, `<`, etc.) work the same way in Pyret that they do in math.

- Operators are written between values, for example: `4 + 2`.
- In Pyret, operators must always have a space around them. `4 + 2` is valid, but `4+2` is not.
- If an expression has different operators, parentheses must be used to show order of operations. `4 + 2 + 6` and `4 + (2 * 6)` are valid, but `4 + 2 * 6` is not.

Applying Functions

Applying functions works much the way it does in math. Every function has a name, takes some inputs, and produces some output. The function name is written first, followed by a list of ***arguments*** in parentheses.

- In math this could look like $f(5)$ or $g(10, 4)$.
- In Pyret, these examples would be written as `f(5)` and `g(10, 4)`.
- Applying a function to make images would look like `star(50, "solid", "red")`.
- There are many other functions, for example `num-sqr`, `num-sqrt`, `triangle`, `square`, `string-repeat`, etc.

Functions have ***contracts***, which help explain how a function should be used. Every Contract has three parts:

- The *Name* of the function - literally, what it's called.
- The *Domain* of the function - what *types of values* the function consumes, and in what order.
- The *Range* of the function - what *type of value* the function produces.

Strings and Numbers

Make sure you've loaded the code.pyret.org (CPO), clicked "Run", and are working in the ***Interactions Area***.

Strings

String values are always in quotes.

- Try typing your name *(in quotes!)* .
- Try typing a sentence like "I'm excited to learn to code!" *(in quotes!)* .
- Try typing your name with the opening quote, but *without the closing quote.* Read the error message!
- Now try typing your name *without any quotes.* Read the error message!

1) Explain what you understand about how strings work in this programming language. ______________________________

____

Numbers

2) Try typing `42` into the Interactions Area and hitting "Enter".

3) Is `42` the same as `"42"`? Why or why not? Write your answer below:

____

4) What is the largest number the editor can handle?

____

5) Try typing `0.5`. Then try typing `.5`. Then try clicking on the answer. Experiment with other decimals. Explain what you understand about how decimals work in this programming language. ______________________________

____

____

6) What happens if you try a fraction like `1/3`? ______________________________

____

7) Try writing **negative** integers, fractions and decimals. What do you learn? ______________________________

____

Operators

8) Just like math, Pyret has ***operators*** like `+`, `-`, `*` and `/`. Try typing in `4 + 2`, and then `4+2` (without the spaces). What can you conclude from this?

____

9) Type in the following expressions, **one at a time**: `4 + 2 * 6`, `(4 + 2) * 6`, `4 + (2 * 6)`. What do you notice? ______________________________

____

10) Try typing in `4 + "cat"`, and then `"dog" + "cat"`. What can you conclude from this?

____

____

Booleans

Boolean-producing expressions are yes-or-no questions and will always evaluate to either `true` ("yes") or `false` ("no"). What will each of the expressions below evaluate to? *Write down your prediction in the blanks provided and then type the code into the Interactions Area to see what it returns.*

	Prediction	Result		Prediction	Result
1) `3 <= 4`			2) `"a" > "b"`		
3) `3 == 2`			4) `"a" < "b"`		
5) `2 < 4`			6) `"a" == "b"`		
7) `5 >= 5`			8) `"a" <> "a"`		
9) `4 >= 6`			10) `"a" >= "a"`		
11) `3 <> 3`			12) `"a" <> "b"`		
13) `4 <> 3`			14) `"a" >= "b"`		

15) In your own words, describe what `<` does.

16) In your own words, describe what `>=` does.

17) In your own words, describe what `<>` does.

	Prediction:	Result:
18) `string-contains("catnap", "cat")`		
19) `string-contains("cat", "catnap")`		

20) In your own words, describe what `string-contains` does. Can you generate another expression using `string-contains` that returns true?

21) There are infinite numbers values out there (...-2,-1,0,-1,2...) and infinite string values ("a", "aa", "aaa"...) But how many different *Boolean* values are there?

Functions for Tables

Open the [Animals Starter File] and click "Run".
In the Interactions Window on the right, type `animals-table` and hit "Enter" to see the default view of the table.

sort

Suppose we wanted to see the names of the animals in alphabetical order…

The `sort` function takes in three pieces of information:

1. A table
2. A column we want to sort the table by (declared using a String)
3. The order in which we want the column sorted (declared using a Boolean)

Test out these two expressions in the Interactions Area and record what you learn about ordering below:

- `sort(animals-table, "species", true)`
- `sort(animals-table, "species", false)`

1) `true` sorts the table… ______________________________

2) `false` sorts the table… ______________________________

Suppose we wanted to sort the `animals-table` by the `weeks` column to determine which animals were adopted quickest…

3) Would you use `true` or `false`? Explain. ______________________________

4) Test it out, and write your thinking about ***quantitative*** columns at the end of your explanations of `true` and `false` above.

5) Which animal(s) were adopted the quickest? ______________________________

6) Some functions produce Numbers, some produce Strings, some produce Booleans. What did the `sort` function produce? __________

There are many other functions available to us in Pyret. We can describe them using contracts. The Contract for `sort` is:

```
# sort :: Table, String, Boolean -> Table
```

- Each Contract begins with the function name: *in this case* `sort`
- Lists the data types required to satisfy its Domain: *in this case* Table, String, Boolean
- And then declares the data type of the Range it will return. *in this case* Table

Contracts can also be written with more detail, by adding *variable names* in the Domain:

```
# sort :: ( Table , String , Boolean) -> Table
         table-name column-name order
```

Suppose we wanted to sort the `animals-table` by the `legs` column to determine which animals had the most legs…

7) Fill in the blanks below with the code you'd use (We've put pieces of the Contract below each line to help you!):

__________ (____________________ , ____________________ , __________)
function-name table-name :: Table column-name :: String order :: Boolean

8) Which animal(s) had the most legs? ______________________________

9) Think of another question you might answer quickly by sorting the table.

____

10) What code would you write to answer your question?

__________ (____________________ , ____________________ , __________)
function-name table-name :: Table column-name :: String order :: Boolean

Functions for Tables (continued)

count

```
# count :: Table, String -> Table
```

1) What is the Domain of `count` ? ______________________________

2) What is the Range of `count` ? ______________________________

3) What do you suspect the String in the Domain will describe? ______________________________

Suppose we wanted to know how many animals had 4 legs...
Type `count(animals-table, "legs")` into the Interactions Area and click "Enter"

4) What did the expression produce? ______________________________

5) How many animals had 4 legs? ______________________________

6) Think of another question you might be able to answer with the `count` function.

____

7) Fill in the blanks with the code you'd write.

______________ (______________________ , ______________________)
function-name table-name :: Table column-name :: String

8) Tables that summarize data with a count are commonly used in the real world. Give two examples of where you've seen them before:

- Example 1: ______________________________
- Example 2: ______________________________

9) Newscasters and journalists often incorporate data into their reporting. How else might they display this information, besides using a table?

____

first-n-rows

10) Type `first-n-rows(animals-table, 5)`. What happens? ______________________________

11) If we wanted a table of the first 3 rows of the `animals-table`, what code would you write? ______________________________

12) What is the Contract for `first-n-rows` ? ______________________________

★ What happens when you type `first-n-rows(sort(animals-table, "pounds", true), 5)` ?

____

> *Note: The Domain for* `first-n-rows` *is Table, Number. In this case, the output of* `sort(animals-table, "pounds", true)` *is the Table!*

★ See if you can figure out how to compose the code that would generate a table of the 10 oldest animals!

______________ (______________________________ , ______________)
function-name Table Number

Circles of Evaluation: Count, Sort, First-n-rows

For each scenario below, draw the Circle of Evaluation and then use it to write the code.
When you're done, test your code out in the Animals Starter File and make sure it does what you'd expect it to.

```
# count :: Table, String -> Table
# first-n-rows :: Table, Number -> Table
# sort :: Table, String, Boolean -> Table
```

1) **We want to see the 10 animals who were adopted the quickest.**
Circle of Evaluation:

code: ______________________________

2) **We want to see the heaviest animal**
Circle of Evaluation:

code: ______________________________

3) **We want to take the first 8 animals from the table and put them in alphabetical order (by name).**
Circle of Evaluation:

code: ______________________________

4) **You notice that the lightest 16 animals weigh under 10 pounds and you want to know the count by species of those animals.**
Circle of Evaluation:

code: ______________________________

Exploring Displays

In the Animals Starter File, use the contracts below to make each type of display. Then answer the questions below.

Bar Charts `# bar-chart :: Table, String -> Image`

____________ (____________________ , ________________)
function-name | table-name :: Table | column-name :: String

Sketch a bar chart below.

Bar charts summarize 1 column of ____________ data.
categorical/quantitative

This kind of display tells us...

____

____

____

Pie Charts `# pie-chart :: Table, String -> Image`

____________ (____________________ , ________________)
function-name | table-name :: Table | column-name :: String

Sketch a pie chart below.

Pie charts summarize 1 column of ____________ data.
categorical/quantitative

This kind of display tells us...

____

____

____

Box Plots `# box-plot :: Table, String -> Image`

____________ (____________________ , ________________)
function-name | table-name :: Table | column-name :: String

Sketch a box plot below.

Box plots summarize 1 column of ____________ data.
categorical/quantitative

This kind of display tells us...

____

____

____

Histograms `# histogram :: Table, String, String, Number -> Image`

____________ (____________ , ____________ , ____________ , ____________)
function-name | table-name :: Table | labels :: String | values :: String | bin-width :: Number

Sketch a histogram below.

Histograms summarize 1 column of ____________ data.
categorical/quantitative

This kind of display tells us...

____

____

____

Circles of Evaluation: Composing Functions to Make Displays

Using the Contracts below as a reference, draw the Circle of Evaluation for each prompt.

```
# pie-chart :: Table, String -> Image
# bar-chart :: Table, String -> Image
# histogram :: Table, String, String, Number -> Image
# box-plot :: Table, String -> Image
# first-n-rows :: Table, Number -> Table
# sort :: Table, String, Boolean -> Table
```

1) Make a bar-chart of the lightest 16 animals by `sex`.

★) What other bar chart might you want to compare this to? ______________________________

2) Take the heaviest 20 animals and make a histogram of `weeks` to adoption (use `"species"` for your labels).

★) What other histogram might you want to compare this to? ______________________________

3) Make a box-plot of `age` for the 11 animals who spent the most `weeks` in the shelter.

★) What other box plot might you want to compare this to? ______________________________

4) Make a pie-chart of `species` for the 18 animals who spent the fewest `weeks` in the shelter.

★) What other pie chart might you want to compare this to? ______________________________

Exploring the States Dataset

Open the State Demographics Starter File and **Save a Copy** of the file that's just for you.
Then, click "Run" and type `states-table` into the Interactions Area on the right to see the dataset.

What do you Notice about this dataset?	What do you Wonder about this dataset?

1) What code will produce a table showing the number of states in each region? ______

2) Which states do you **think** have the most people? ______

3) What code will produce a table containing the five states with the largest population in 2020?

______

4) Which states do you **think** have the most poverty? ______

5) What code will produce a table containing the ten states with the highest poverty rate?

______

6) What code will produce a table containing the states with the lowest **median** income?

______

7) What code will produce a table containing the states with the lowest **per-capita** ("average" or "mean") income?

______

★ What does it mean if a state has a higher `per-capita income` than `median-income`? ______

______

The two lines of code under `# Define some rows` *extract rows 0 and 1 from the table, and define them as* `alabama` *and* `alaska`.

8) Type `alabama` into the Interactions Area. What do you get back? ______

9) Underneath the definition of those rows, add a new definition for `california` and click "Run", so that Pyret reads your new definition.

10) Add a definition for your own state, then **click "Run"** and test it out in the Interactions Area!

11) Add any additional Notices or Wonderings you have about this dataset to the table at the top.

Looking for Patterns

Open your copy of the State Demographics Starter File.

Part 1

1) What columns do you think might be related to one another? (e.g. - is the number of veterans related the amount of land-area? Is the population in 2010 related to the population in 2020?) List three possible relationships below.

a. I think that ______________________________ may be related to ______________________

b. I think that ______________________________ may be related to ______________________

c. I think that ______________________________ may be related to ______________________

```
# scatter-plot :: (Table, String, String, String) -> Image
                          labels  explanatory response
```

2) Use the Contract above to make a scatter-plot for the **first relationship** you wrote.

a. What states border your own? Find your state and its neighbors by mousing over the display. How do they compare? __________

____

b. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? ________________

____

3) Make a scatter-plot for the **second relationship** you wrote. Then find your home state, and its neighbors.

a. How does your home state compare to the neighboring ones? ______________________________

____

b. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? ________________

____

4) Make a scatter-plot for the **third relationship** you wrote. Then find your home state, and its neighbors.

a. How do they compare? ____

____

b. If there's a pattern in this scatter-plot, what does that mean? If there isn't, what does *that* mean? ________________

____

Part 2

Wait to complete this until after diving deeper into statistical relationships!

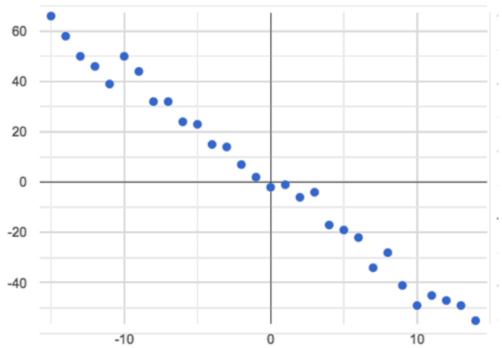
Revisit the three scatter plots you made and add the following labels to the descriptions you wrote in Question 1:

- Place an "L" by any relationships that you think might be linear.
- Place a "P" by any relationships that were positive.
- Place an "N" by any relationships that were negative.
- Place an "S" by the strongest-looking relationship.
- Place a "W" by the weakest-looking relationship.

Identifying Form, Direction and Strength

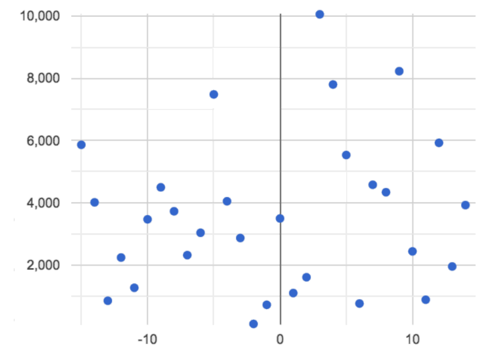
What do your eyes tell you about the Form, Direction, & Strength of these displays?
Note: If the form is nonlinear, we shouldn't report direction - a curve may rise and then fall.

A



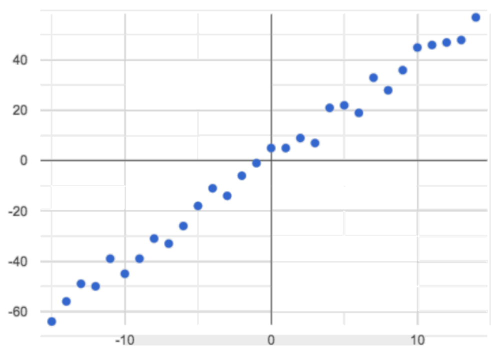
Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

B



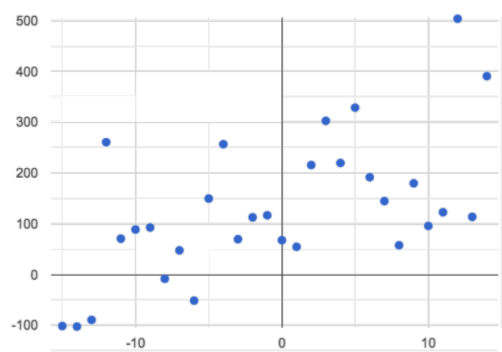
Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

C



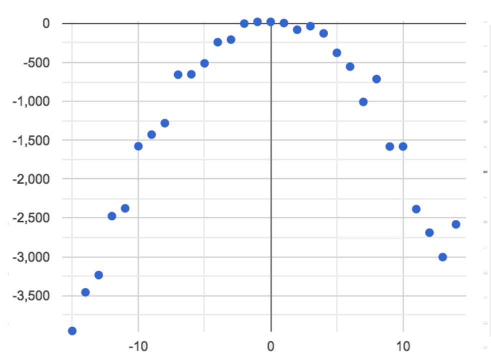
Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

D



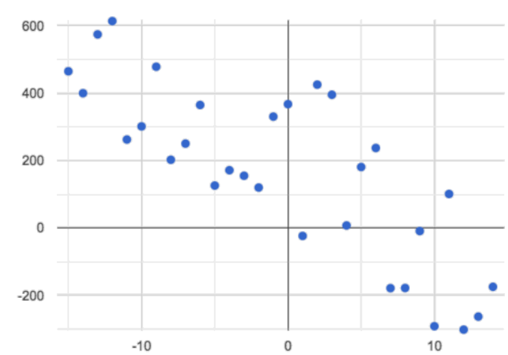
Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

E



Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

F



Form:	Linear	Nonlinear	None
Direction:	Positive	Negative	N/A
Strength:	Strong	Weak	

Identifying Form, Direction and Strength (Matching)

Match the description (left) with the scatter plot (right).

Note: The computer won't tell us if the relationship we see in a scatter plot is linear, so we must train our eyes to decide this ourselves. For linear relationships, we should train our eyes to assess their direction and get a feel for their strength, rather than relying completely on what numbers the computer reports.

The relationship appears to be linear, negative, and of moderate strength. **1**

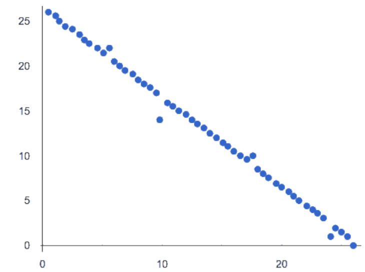
This relationship is nonlinear. **2**

The x and y variables in this dataset do not appear to be related. **3**

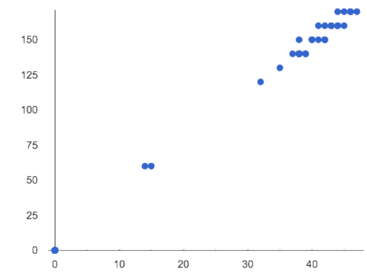
The relationship appears to be linear, positive, and strong. **4**

The relationship appears to be linear, negative, and strong. **5**

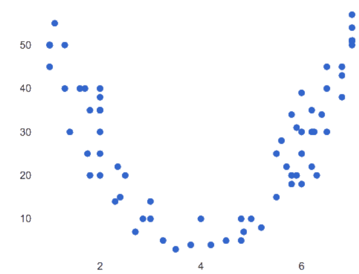
A



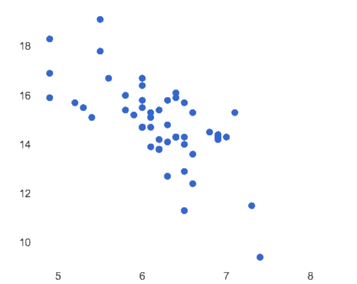
B



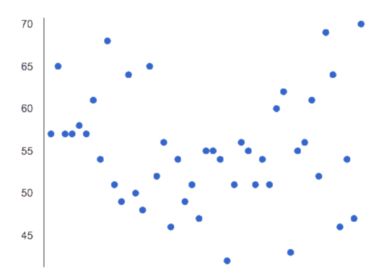
C



D



E



Build a Model from Samples: College Degrees v. Income

*Open your copy of the [State Demographics Starter File](). If you haven't already, **Save a Copy** now.*

1) Record the `pct-college-or-higher` and `median-income` values for the `alabama` and `alaska` rows, as (x,y) pairs below:

(________________, ________________) (________________, ________________)

AL pct-college-or-higher, AL median-income; AK pct-college-or-higher, AK median-income

2) Using the space below, compute the equation of the line passing between these two points. **This line will be your linear model** (also known as the "predictor function", or "line of best fit"), which predicts `median-income` as a function of `pct-college-or-higher`.

3) Write the complete model below (in both Function and Pyret notation):

$f(x) =$ ____________ x + ____________ (slope, y-intercept)

`fun f(x): ( ____________ * x) + ____________ end`

- Type your function into the Definitions Area on the left, modifying the existing function `f(x)`.
- Then click "Run", and make sure you fix any errors or warnings.
- In the Interactions Area, try plugging in the `pct-college-or-higher` value for Alabama by typing `f(22.6)`

4) How well does it predict the correct median income for Alabama? ________________ What about Alaska? ________________

Consider: If it doesn't predict it perfectly, why might that be?

Try different `pct-college-or-higher` values from *other* states, to see how well our Alabama-Alaska model fits the rest of the country.

5) Identify a state for which this model works well: ________________________

6) Identify a state for which this model works poorly: ________________________

7) What median income does this model expect a state without ANY college graduates (0%) to earn? ________________________

Fit a Model: College Degrees v. Income

This page will require you to work with your copy of the State Demographics Starter File in which you should have already defined `f(x)` *based on your work on Fit a Model: College Degrees v. Income.*

Type `fit-model(states-table, "state", "pct-college-or-higher", "median-income", f)` *in the Interactions Area.*

1) Find the points for AL and AK along the predictor line.

2) What do you Notice?

____

____

____

3) What do you Wonder?

____

____

____

4) In the upper left corner, you'll see that it says "R-sqr", followed by a number. What is that number? ______________

5) Change the definition of `f` so that the slope is *less steep* and click "Run". What is the R^2 value now? ______________

6) Change the definition of `f` so that the slope is *negative* and click "Run". What is the R^2 value now? ______________

7) Change the definition of `f` so that it draws a horizontal line and click "Run". What is the R^2 value? ______________

8) Change the y-intercept so that the horizontal line passes through more of the points. What is the R^2 value? ______________

9) What do you think R^2 tells us? ______________________________

____

____

____

Better Modeling: College Degrees v. Income

Open your copy of the State Demographics Starter File.

Build a Model through Trial & Error

In the `# Define some rows` section, look closely at the definitions for `alaska` and `alabama` .
Add two new definitions for `MA` (row 21) and `NV` (row 28).

1) Record the `college-or-higher` and `median-income` values for `MA` and `NV` , as (x,y) pairs below:

(________________ , ________________) (________________ , ________________)
MA college-or-higher, MA median-income; NV college-or-higher, NV median-income

2) Derive the `MA-NV model` (using the same steps you followed to derive the `AL-AK model` on Fit a Model: College Degrees v. Income) and write it below (in both Function and Pyret notation), then fit the model and record the R^2:

$g(x) =$ ____________ x + ____________ (slope, y-intercept) `fun g(x): (` __________ `* x) +` __________ `end` R^2: ______

3) Identify two other states that you think would make a better model: ________________ and ________________.

- *Add two new definitions for these states to your State Demographics Starter File.*

4) Record the `college-or-higher` and `median-income` values for these states, as (x,y) pairs below:

(________________ , ________________) (________________ , ________________)
college-or-higher, median-income; college-or-higher, median-income

5) Derive your model and write it below (in both Function and Pyret notation), then fit the model and record the R^2:

$h(x) =$ ____________ x + ____________ (slope, y-intercept) `fun h(x): (` __________ `* x) +` __________ `end` R^2: ______

6) Adjust the slope and y-intercept of your model to get the **best R^2 possible.** Write the best model (and R^2) below:

$best(x) =$ ____________ x + ____________ (slope, y-intercept) `fun best(x): (` __________ `* x) +` __________ `end` R^2: ______

Build a Model Computationally

`lr-plot` computes the *optimal linear model* using all of the data points.

7) Evaluate `lr-plot(states-table, "state", "college-or-higher", "median-income")` . What is R^2? ______

8) On the line below, write the optimal linear model that was computed through linear regression:

$optimal(x) =$ ____________ x + ____________ (slope, y-intercept) `fun optimal(x): (` __________ `* x) +` __________ `end`

Interpreting Linear Models

Open your copy of the State Demographics Starter File.

We started with a model based on Alabama and Alaska. We can interpret the slope and R^2 value below:

The *Alabama-Alaska* (sensible name) *model predicts that a 1* *percent* ([x-axis units]) *increase in* *percent college degrees* ([x-axis]) *is associated with a* *5613 dollar* ([slope, y-units]) *increase* (increase / decrease) *in* *median household income* ([y-axis]). *Based on the R^2 of* *-15.63* (R^2 value), *this model fits* *really, really poorly* (really well, decently, poorly, etc.).

1) Describe the optimal model YOU created via linear regression:

The linear-regression (sensible name) model predicts that a 1 percent ([x-axis units]) increase in percent college degrees ([x-axis]) is associated with a ______ ([slope, y-units]) ______ (increase / decrease) in median household income ([y-axis]). With an R^2 of ______ (R^2 value), this model fits ______ (really well, decently, poorly, etc.).

---

2) What does the **slope** of this linear model tell us? ______

______

3) What does the **y-intercept** of this linear model tell us? ______

______

4) Suppose a state goes from 10% to 11% college graduation. According to this model, what kind of change would we expect to see in the median household income? ______ What if it goes from 50% to 51%? ______ From 90% to 91%? ______

5) Does this model predict the same increase in income for *every* additional 1% `college-or-higher`? Why or why not? ______

______

6) Use `fit-model` to fit your model to the scatter plot again, but **swap the x- and y-columns**. Do you get the same R^2? Why or why not?

______

______

______

---

7) Describe another model you created:

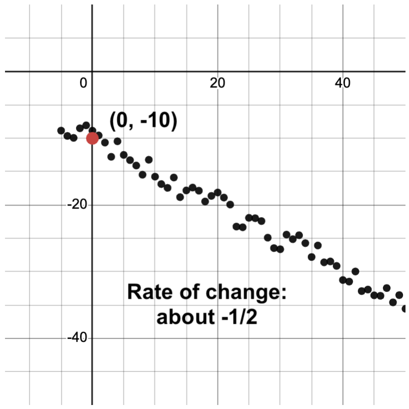
The ______ (sensible name) model predicts that a 1 ______ ([x-axis units]) increase in ______ ([x-axis]) is associated with a ______ ([slope, y-units]) ______ (increase / decrease) in ______ ([y-axis]). With an R^2 of ______ (R^2 value), model fits ______ (really well, decently, poorly, etc.).

Which Form is Best?

For each set of data provided below,

- Decide which form of the line would be the easiest to build from the available information.
- Write a definition of the linear model in that form.
- Translate the definition into Pyret notation.

1

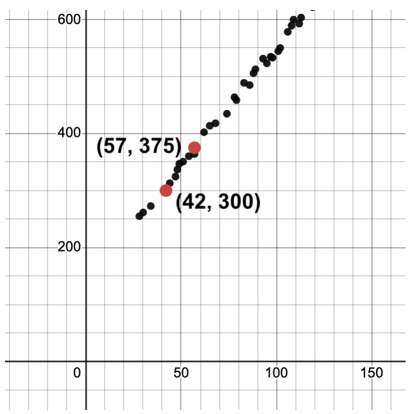


Linear Model: ____________________

slope-intercept form? point-slope form? standard form?

`fun f(x) :` ____________________ `end`

2

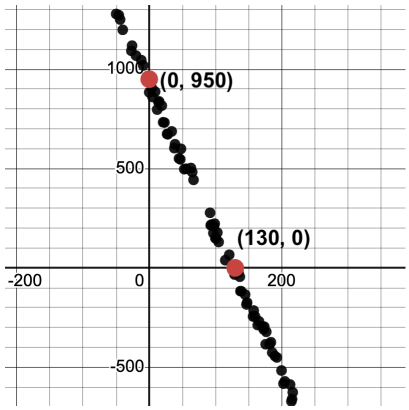


Linear Model: ____________________

slope-intercept form? point-slope form? standard form?

`fun f(x) :` ____________________ `end`

3



Linear Model: ____________________

slope-intercept form? point-slope form? standard form?

`fun f(x) :` ____________________ `end`

Exploring the Fuel Efficiency Dataset

For this page, you'll need to open the Fuel Efficiency Starter File on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. **Read the comments at the top of the file**, which describe what each column in the dataset means.

Fitting Linear Models

1) Evaluate `A15`, `A45` and `A75` in the Interactions Area. What **model** of car is used in all three rows? ____________________

2) At what three **speeds** is this model being tested in these rows? ____________________

3) Does there appear to be a relationship between speed and miles-per-gallon? __________.

4) If so, describe its **form** (e.g. - linear or curved) and **strength** (strong, moderate, or weak).
If it appears to be linear, what is the **direction**? If it does *not* appear to be linear, describe its shape.

____

____

____

5) Use `lr-plot(mpg-table, "model", "speed", "mpg")` to find the optimal **linear** model. What is the R^2? ____________________
Write the model below, in both math and Pyret notation.

$f(x) =$ ______________ x + ______________
slope y-intercept

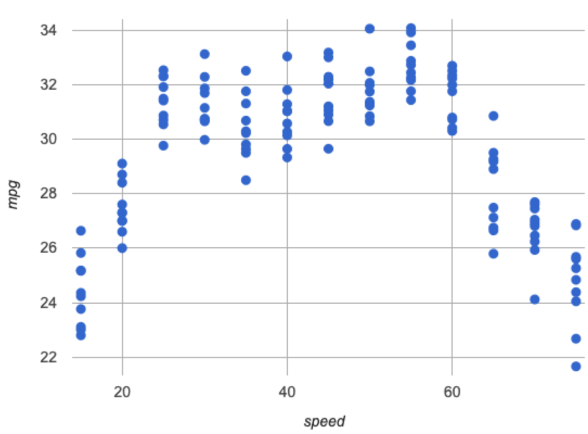
`fun f(x): ( ______________ * x) + ______________ end`

6) Is the best-possible linear model good? __________. Why or why not? ____________________

____

Fitting Curves

7) Sketch your lr-plot in the space below, showing the relationship between `speed` and `mpg`. Be sure to label your axes, and draw the linear model!



8) What do you **Notice?** ____________________

____

____

9) What do you **Wonder?** ____________________

____

____

10) Do you think a **curve** would fit better? ______________

11) Draw a **curve** on your scatter-plot, which shows the overall shape in the data.

12) At what speed does your curve "peak"? ______________

What Kind of Model? (Descriptions)

Decide whether each situation describes a linear or quadratic function, and circle your answer.

1) A ball is dropped from the top of the Empire State Buiding, and it accelarates at just under 10/m/s. How far has the ball dropped after x seconds?

Linear Quadratic

---

2) A car is 50 miles away, traveling at 65mph. How far away is the car after each hour?

Linear Quadratic

---

3) The data plan for a cell phone bill costs $5/gb, plus $15/mo. How much is the bill for a given month, after x number of gigabytes?

Linear Quadratic

---

4) A ball is dropped from the top of the Empire State Buiding, and it accelarates at just under 10/m/s. How fast is the ball moving after x seconds?

Linear Quadratic

---

5) A cannonball is fired from the deck of the S.S. Parabola, and arcs through the sky before hitting its target, 17 miles away.

Linear Quadratic

---

6) The area of a circle, as its radius increases

Linear Quadratic

---

7) The circumference of a circle, as its radius increases

Linear Quadratic

What Kind of Model? (Tables)

Decide whether each representation is best described by a linear model, a quadratic model or neither! Show any work that you feel is useful.

For Class Discussion:

1

x	0	1	2	3	4	5	6
y	5	6	9	14	21	30	41

Linear
Quadratic
Neither

2

x	0	1	2	3	4	5	6
y	0	3	6	9	12	15	18

Linear
Quadratic
Neither

For Independent Practice:

3

x	1	2	3	4	5	6	7
y	1	3	5	7	9	11	13

Linear
Quadratic
Neither

4

x	-3	-2	-1	0	1	2	3
y	-23	-38	-47	-50	-47	-38	-23

Linear
Quadratic
Neither

5

x	-3	-2	-1	0	1	2	3
y	1	2	1	2	1	1	1

Linear
Quadratic
Neither

6

x	1	2	3	4	5	6	7
y	2	5	10	17	26	37	50

Linear
Quadratic
Neither

7

x	-3	-2	-1	0	1	2	3
y	12	7	2	-3	-8	-13	-18

Linear
Quadratic
Neither

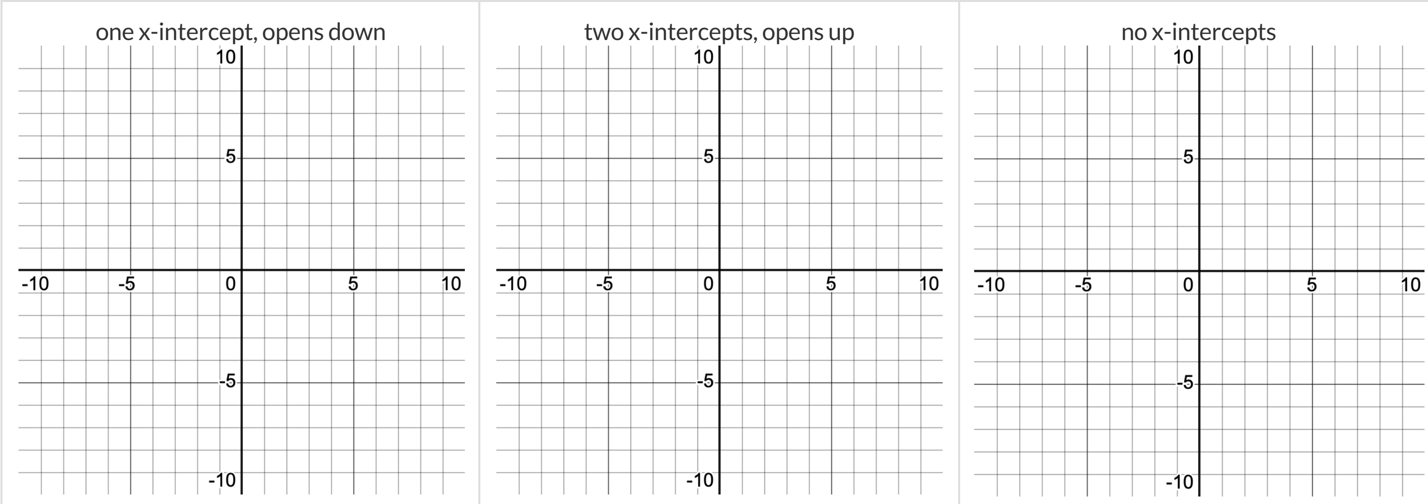
8

x	1	2	3	4	5	6	7
y	100	102	105	109	114	120	127

Linear
Quadratic
Neither

Parabolas

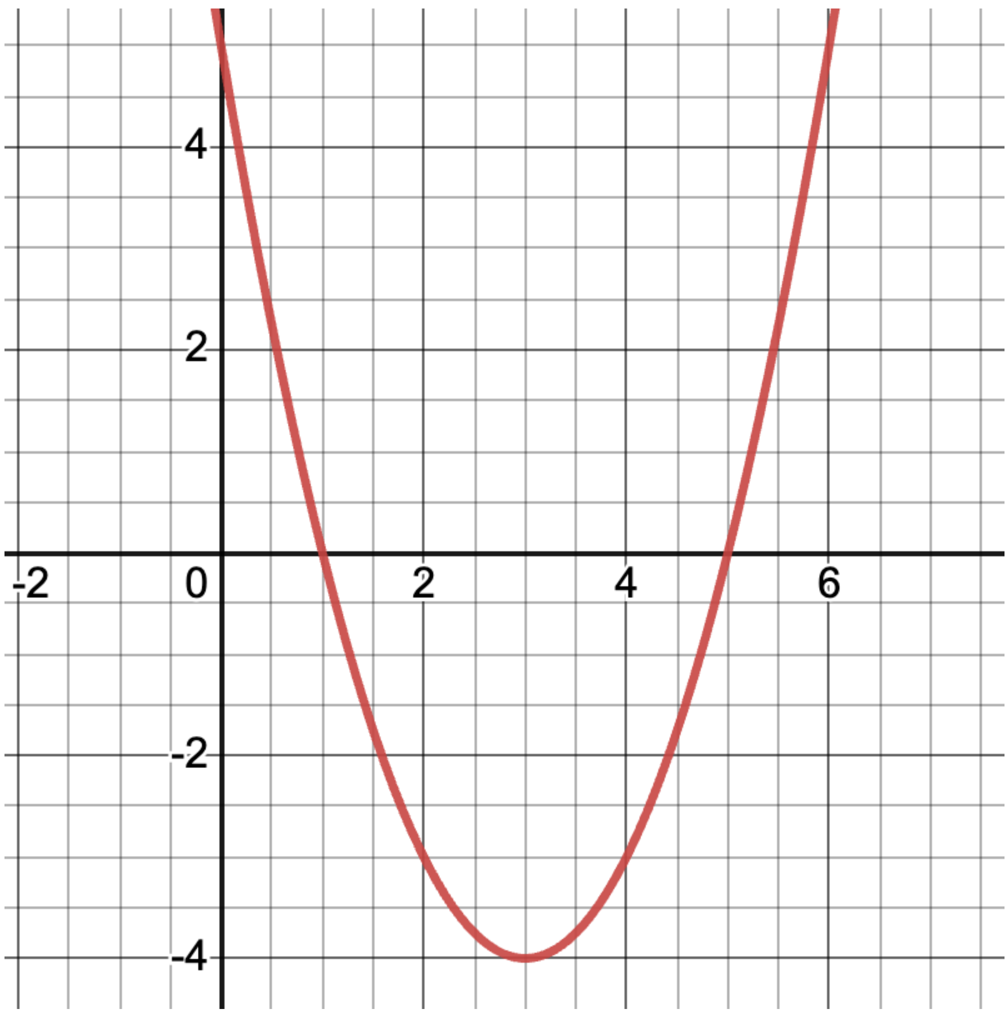
1) Sketch a ***parabola*** on each of the grids below that matches the description.



2) Label the ***vertex***, ***root(s)***, and ***y-intercept*** of the parabola below with:

A) their coordinates

B) the vocabulary word (above) that describes each



3) Draw a dotted line representing the ***axis of symmetry*** and label it with the equation that defines it.

Graphing Quadratic Models

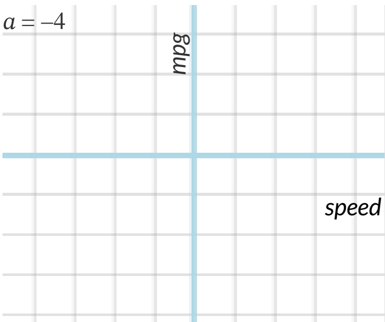
Open [Exploring Quadratic Functions (Desmos)](). The parabola you'll see is the graph of $f(x) = x^2$. Another, **identical** parabola is hiding behind it. This second parabola is written in Vertex Form: $g(x) = a(x - h)^2 + k$. Each coefficient starts at values to make $g(x)$ equivalent to $f(x)$.

1) Using the values of a, h, and k from Desmos, write the Vertex Form of $f(x) = x^2$: $f(x) =$ ____________________

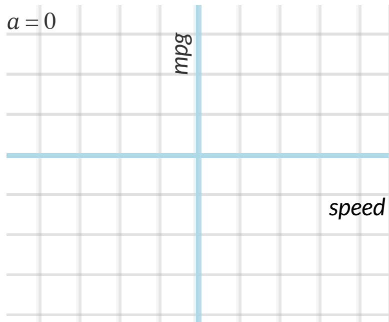
Magnitude a

2) Try changing the value of a to -4, 0, and 2, graphing each parabola in the squares below. Be sure to identify and **label the vertex and any roots with "V" and "R"!**

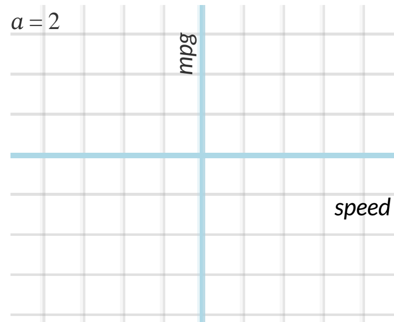
$a = -4$



$a = 0$



$a = 2$

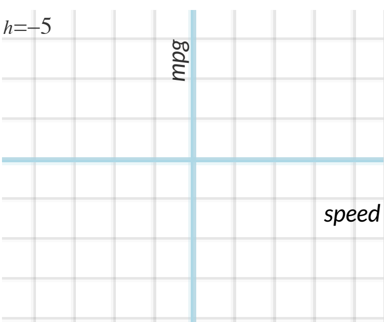


3) What does a tell us about a parabola? ____________________

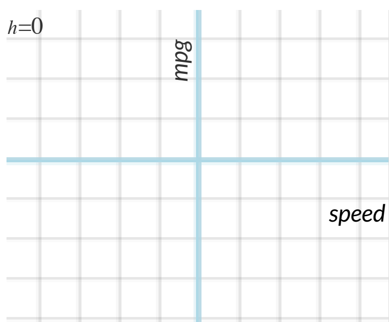
Horizontal Translation h

4) Set a back to 1. Change the value of h to -5, 0, and 5, graphing each parabola in the squares below. Be sure to identify and **label the vertex and any roots with "V" and "R"!**

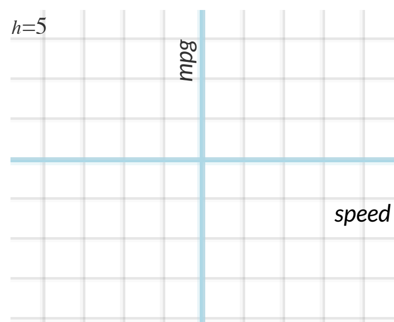
$h = -5$



$h = 0$



$h = 5$

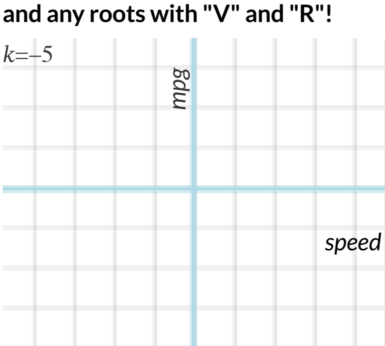


5) What does h tell us about a parabola? ____________________

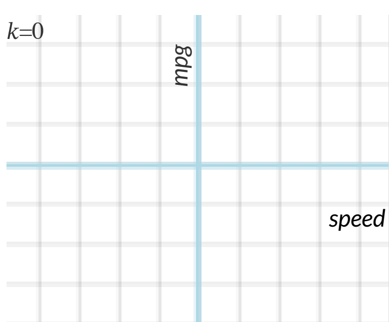
Vertical Translation k

6) Set h back to 0. Change the value of k to -5, 0, and 5, graphing each parabola in the squares below. Be sure to identify and **label the vertex and any roots with "V" and "R"!**

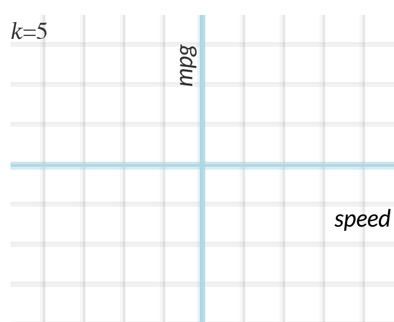
$k = -5$



$k = 0$



$k = 5$



7) What does k tell us about a parabola? ____________________

Modeling Fuel Efficiency v. Speed

Open your copy of the Fuel Efficiency Starter File and click "Run".
Before we try to model our fuel-efficiency data, we need to learn a new Pyret function!

1) Can you predict what the output of the following expressions will be? Enter them into the Interactions Area, and record the result.

`num-sqr(4)` ________________ `num-sqr(6 - 2)` ________________

2) What is the Contract for `num-sqr` ? ________________

3) What does `num-sqr` *do* ? ________________

Interpreting a Quadratic Model

In the Definitions Area of your Fuel Efficiency Starter File, you'll find the definition of a quadratic model `quad1` .

4) In `quad1` , the value of a is ________________, the value of h is ________________, and the value of k is ________________

5) Fit this model to your dataset, using `fit-model` . What R^2 value did you get? ________________

Hint: If you forgot the contract for `fit-model` *, look it up in the contracts pages!*

6) In your own words, describe what needs to change about this model to fit the data. ________________

____

____

Modeling Fuel Efficiency

Vertex Form: $y=a(x-h)^2+k$	• a: determines whether the parabola opens up or down and how steep the curve is • h: x-coordinate of the vertex • k: y-coordinate of the vertex (in quadratic models, this is also the vertical shift!)

7) We've determined that peak fuel efficiency is around 45 mph. What variable in the equation should we replace with 45? ________________

Update the definition of `quad1` , click "Run" and re-fit the model. What R^2 value did you get? ________________

8) What y-coordinate of the vertex would best match the shape of the curve? __________

Update the definition of `quad1` , click "Run" and re-fit the model. What R^2 value did you get? ________________

9) What value of a best matches the shape of the curve? __________

Update the definition of `quad1` , click "Run" and re-fit the model. What R^2 value did you get? ________________

10) See any small changes you'd like to make to the definition, trying to get R^2 as close to 1 as you can? Write your final definition below.

`fun f(x) :` ________________ `end` R^2: ________________

★ **What does this model actually mean? Try completing the sentence below:**

After experimenting, I came up with a quadratic model showing that *speed* explains ________________ (R-squared * 100) % of the variability in *gas mileage* for cars in this dataset. The vertex of the parabola drawn by this model is ________________ (x, y), which means that ________________

____

★ How does the fact that the value of a is negative impact this parabola? ________________

Looking up Rows and Columns

We can define names for values in Pyret, the same way we do in math:

```
name = "Flannery"
age = 16
logo = star(50, "solid", "red")
```

When **looking up a data Row** from a Table, programmers use the `row-n` function. This function takes a Table and a Number as its inputs. The numbers tell the computer which Row we want from the Table. *Note: Rows are numbered starting at zero!*
For example:

```
sasha   = row-n(animals-table, 0)  # define Sasha to be the first row
mittens = row-n(animals-table, 2)  # define Mittens to be the third row
```

When we define these rows, it's more useful to name them based on their *properties*, rather than their identifiers:

```
cat-row = row-n(animals-table,  0) # Sasha is a cat
dog-row = row-n(animals-table, 10) # Toggle is a dog
```

When **looking up a column** from a Row, programmers use square brackets and the name of the column they want.
For example:

```
# these two lines do the same thing! We can use the defined name to simplify our code
row-n(animals-table, 0)["age"] # look up Sasha's age (in row 0)
cat-row["species"]             # look up Sasha's age (using the defined name)
dog-row["age"]                 # look up Toggle's age (using the defined name)
```

Lookup Questions

The table below represents four pets at an animal shelter:

`pets-table`

name	sex	age	pounds
"Toggle"	"female"	3	48
"Fritz"	"male"	4	92
"Nori"	"female"	6	35.3
"Maple"	"female"	3	51.6

1) *Match* each Lookup Question (left) to the code that will give the answer (right).

"How much does Maple weigh?"	**1**	**A**	`row-n(pets-table, 3)`
"Which is the last row in the table?	**2**	**B**	`row-n(pets-table, 2)["name"]`
"What is Fritz's sex?"	**3**	**C**	`row-n(pets-table, 1)["sex"]`
"What's the third animal's name?"	**4**	**D**	`row-n(pets-table, 3)["age"]`
"How much does Nori weigh?"	**5**	**E**	`row-n(pets-table, 3)["pounds"]`
"How old is Maple?"	**6**	**F**	`row-n(pets-table, 0)`
"What is Toggle's sex?"	**7**	**G**	`row-n(pets-table, 2)["pounds"]`
"What is the first row in the table?"	**8**	**H**	`row-n(pets-table, 0)["sex"]`

2) Write the Pyret code that will produce each value on the right.

a.	row-n(*pets-table, 3)["name"]*	"Maple"
b.		"male"
c.		4
d.		48
e.		"Nori"

More Practice with Lookups

Consider the table below, and the four value definitions that follow:

`shapes-table`

name	corners	is-round
"triangle"	3	false
"square"	4	false
"rectangle"	4	false
"circle"	0	true

```
shapeA = row-n(shapes-table, 0)
shapeB = row-n(shapes-table, 1)
shapeC = row-n(shapes-table, 2)
shapeD = row-n(shapes-table, 3)
```

1) *Match* each Pyret expression (left) to the description of what it evaluates to (right).

`shapeD`	1	A	Evaluates to 4
`shapeA`	2	B	Evaluates to the last row in the table
`shapeB["corners"]`	3	C	Evaluates to `"square"`
`shapeC["is-round"]`	4	D	Evaluates to `true`
`shapeB["name"]`	5	E	Evaluates to `false`
`shapeA["corners"]`	6	F	Evaluates to 3
`shapeD["name"] == "circle"`	7	G	Evaluates to the first row in the table

2) Fill in the blanks (left) with the Pyret lookup code that will produce the value (right).

a.	______________________	"rectangle"
b.	______________________	"square"
c.	______________________	4
d.	______________________	0
e.	______________________	true

Defining Rows

We've already given you two row definitions: `cat-dog` and `dog-row`:

```
cat-row = row-n(animals-table,  0)  # Sasha is a cat
dog-row = row-n(animals-table, 10)  # Toggle is a dog
```

Remember: rows start at index zero! Use this to answer the questions below. (HINT: turn to The Animals Dataset and number the data rows first, then answer the questions below.)

1) The index of a row containing a lizard is ______________

2) The index of a row containing a rabbit is ______________

3) The index of a row containing a fixed animal is ______________

4) The index of a row containing a male animal ______________

5) The index of a row containing a female animal is ______________

6) The index of a row containing a hermaphroditic animal is ______________

7) The index of a row containing an unfixed animal is ______________

8) The index of a row containing a young animal (<2 years) is ______________

9) The index of a row containing an old animal (>10 years) is ______________

10) What code would you write to define `lizard-row`?

____

11) What code would you write to define `rabbit-row`?

____

12) What code would you write to define `fixed-row`?

____

13) What code would you write to define `male-row`?

____

14) What code would you write to define `female-row`?

____

15) What code would you write to define `hermaphrodite-row`?

____

16) What code would you write to define `young-row`?

____

17) What code would you write to define `old-row`?

____

Add this code to your Animals Starter File! You'll want these rows for later!

Exploring the Covid Dataset

*For this page, you'll need to have the [Covid Spread Starter File] open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

1) Click "Run", and evaluate `covid-table` in the Interactions Area.

2) Take a look at the Definitions Area and find the "notes on columns". What is the start date for the data in this table? ____________

3) In the Definitions Area we see rows defined for Connecticut (`CT1`), Massachusetts (`MA1`) and Maine (`ME1`).

What happens when you evaluate `MA1` in the Interactions Area? ____________

4) Evaluate `CT1`. What information do you learn? ____________

____

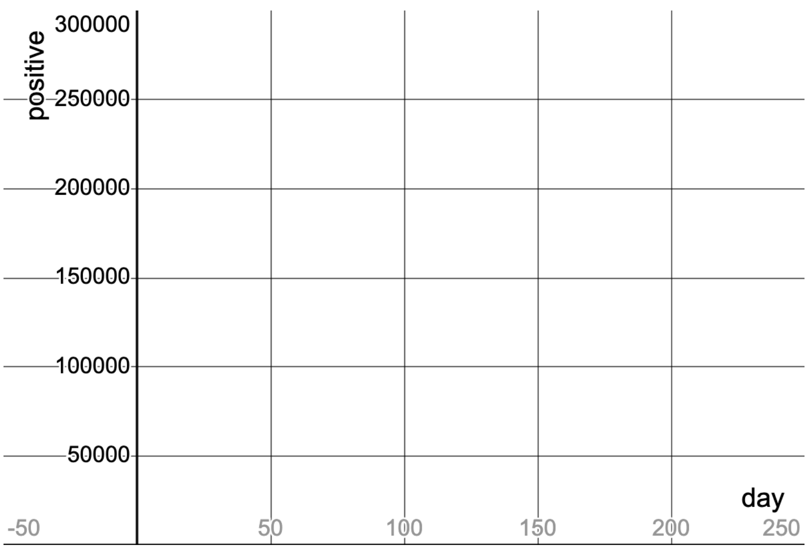
5) Define three new Rows called `VT1`, `NH1` and `RI1` for Vermont, New Hampshire and Rhode Island. Click "Run" and test them out.

a. How many people in Vermont tested positive on June 21st, 2020? ____________

b. How many people in New Hampshire tested positive on June 21st, 2020? ____________

c. How many people in Rhode Island tested positive on June 21st, 2020? ____________

6) In Pyret, make a scatter plot showing the relationship between `day` and `positive`, using `state` as your labels, then sketch the resulting scatter plot below.



7) In which state did the number of cases grow *fastest*?

____

8) In which state did the number of cases grow *slowest*?

____

9) Are these strong or weak relationship(s)?

____

10) What do you **Notice?** ____________

____

____

11) What do you **Wonder?** ____________

____

____

Filtering by State

*For this page, you'll need to have the [Covid Spread Starter File] open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you.*

1) Use `lr-plot` to obtain the best-possible linear model for the relationship between `day` and `positive` in the `covid-table`.

a. What do you notice about the line? ______________________________

____

b. What is the R^2 value? ____________ and what does it tell us about the model? ____________

____

2) Find the function called `is-MA` *in the Definitions Area under "Define some helper functions"*. Read the comments carefully!

a. What is the **Domain** of `is-MA`? ____________________ Its **Range**? ____________________

b. What do you *think* `is-MA(MA1)` will evalute to? __________. `is-MA(CT1)`? __________. `is-MA(ME1)`? __________

Try typing each of these helper functions into the Interactions Area on the right and confirm you were correct.

3) Find `MA-table` *in the Definitions Area under "Define some grouped and/or random samples"*.

a. What is that code doing? ______________________________

b. Type `MA-table` into the Interactions Area. What does it evaluate to? ______________________________

4) Use `lr-plot` to obtain the best-possible linear model for the relationship between `day` and `positive` in the `MA-table`.

a. What is the R^2 value? ____________

b. What does it tell us about the model? ______________________________

5) Using the code for `is-MA` and `MA-table` as a model, **define a new function** `is-VT` and **create a grouped sample** called `VT-table`. Use `lr-plot` to obtain the best-possible linear model for the relationship between `day` and `positive` in the `VT-table`.

a. What is the R^2 value? ____________

b. What does it tell us about the model? ______________________________

6) Why do these state-specific models fit so well, when model for all of New England fits so poorly?

____

____

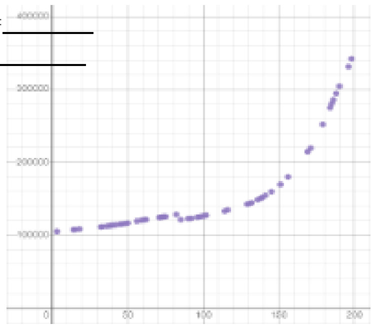
Linear Models for MA-table

Fitting the Model Visually $f(x) = mx + b$

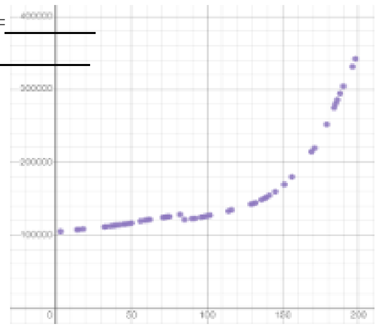
For this section, you'll need to have [Modeling Covid Spread (Desmos)]() open on your computer.

1) Try changing the value of m and b to find three promising linear models, graphing each one and labeling your values in the grids below.

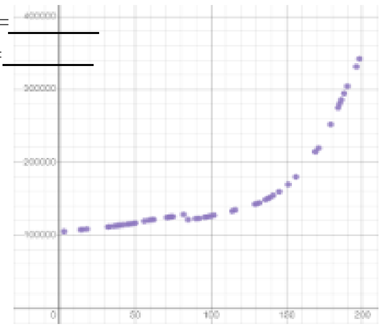
m=__________
b=__________



m=__________
b=__________



m=__________
b=__________



Fitting the Model Programmatically $f(x) = mx + b$

For this section, open your copy of the [Covid Spread Starter File]().

2) In the Definitions Area, define the three models you fit in Desmos, calling them `linear1`, `linear2` and `linear3` to.

3) Use `fit-model` to determine the R^2 value of each of your models for the `MA-table`.

R^2 for `linear1`: __________________ R^2 for `linear2`: __________________ R^2 for `linear3`: __________________

4) Use `lr-plot` to obtain the best-possible **linear model** for the MA Covid dataset.

- y = ____
- R^2 = ____

5) Look at the equation `lr-plot` generated. Do you see an `e`? What does it mean? ________________________

____

6) How does the model generated by `lr-plot` compare to the ones you fit visually in Desmos? __________________

____

Are Linear Models a Good Fit for This Data?

7) Would you feel good about making predictions based on these models? Why or why not? __________________

____

____

____

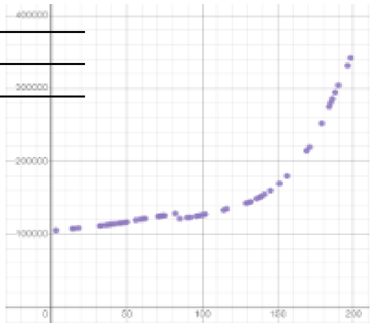
Quadratic Models for MA-table

Fitting the Model Visually $f(x) = a(x-h)^2 + k$

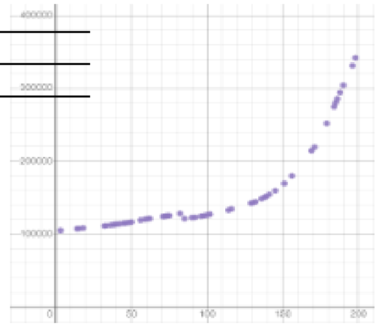
For this section, you'll need to have [Modeling Covid Spread (Desmos)] open on your computer.

1) Try changing the values of a, h and k to find three promising quadratic models, graphing each one and labeling your values in the grids below.

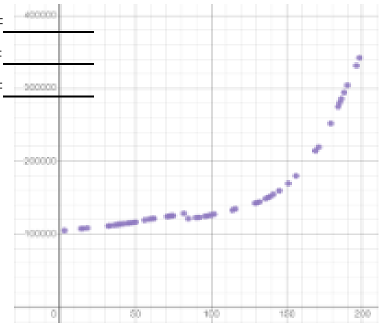
a= ________
h= ________
k= ________



a= ________
h= ________
k= ________



a= ________
h= ________
k= ________



Fitting the Model Programmatically $f(x) = a(x-h)^2 + k$

For this section, open your copy of the [Covid Spread Starter File].

2) In the space below, define `quadratic1` to be the first model you fit in Desmos.

```
fun quadratic1(x): ( ______________ * (num-sqr( x - ______________ ))) + ______________ end
                           a                              h                    k
```

3) In the Definitions Area, define `quadratic1`, `quadratic2` and `quadratic3` to describe the three models you fit in Desmos.

4) Use `fit-model` to determine the R^2 value of each of your models for the `MA-table`.

R^2 for `quadratic1`: ______________ R^2 for `quadratic2`: ______________ R^2 for `quadratic3`: ______________

Are Quadratic Models a Good Fit for This Data?

5) Would you feel good about making predictions based on these models? Why or why not? ____________________

____

____

____

Graphing Exponential Models

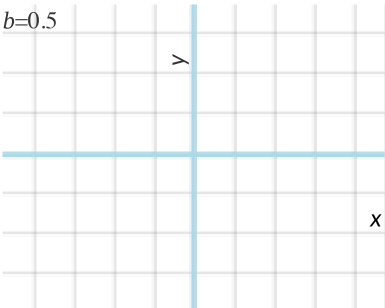
To complete this page, you'll need to open Modeling Covid Spread (Desmos). The curve you'll see is the graph of $h(x) = 2^x$.
Another, curve $f(x)$ is hiding behind it. For starters, the values of the coefficients of $f(x)$ have been set to make it equivalent to $h(x)$.

Base b

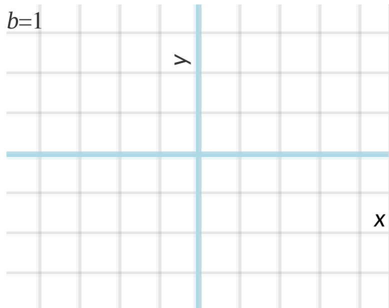
1) Set k back to 0, then try different values of b. **For what values is the function *undefined*** (the line disappears)? ________________

2) Keeping $a=1$ and $k=0$, change math{b} to 0.5, 1, and 2, graphing each curve below. For each curve, **label the coordinates at x=1, 2, and 3.**

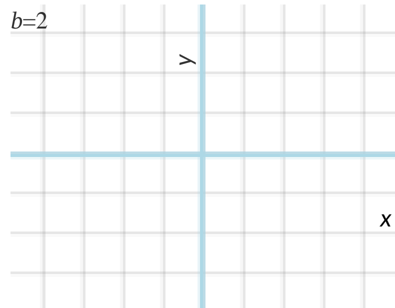
$b=0.5$



$b=1$



$b=2$



3) What does b tell us about an exponential function, when b is **greater than 1**? ________________

____

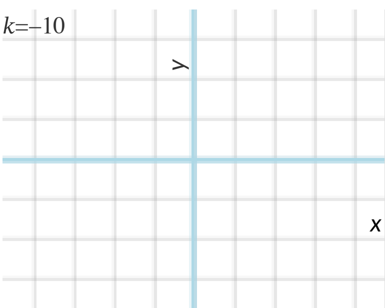
4) What does b tell us about an exponential function, when b is **less than 1**? ________________

____

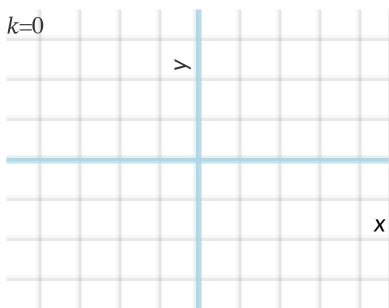
Vertical Shift...and Horizontal Asymptote k

5) Keeping $a=1$ and $k=0$, try changing the value of k to -10, 0, and 10, graphing each curve in the squares below. For each curve, find and label the y-value where the curve is "most horizontal", then **draw a horizontal line at that y-value**.

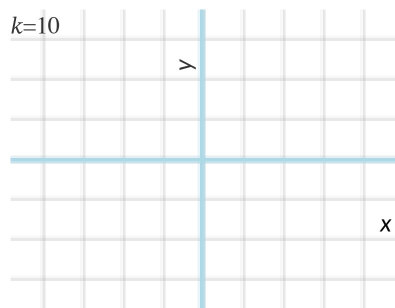
$k=-10$



$k=0$



$k=10$



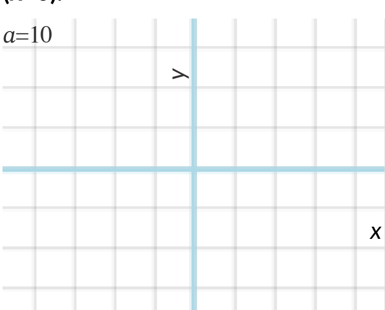
6) What does k tell us about an exponential function? ________________

____

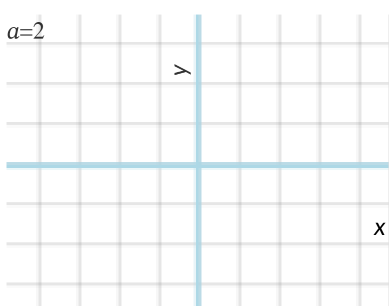
Initial Value a

7) Set $k=0$ and $b=2$. Change the value of a to 10, 2, and -5, graphing each curve in the squares below. **For each curve, label the y-intercept (x=0).**

$a=10$



$a=2$

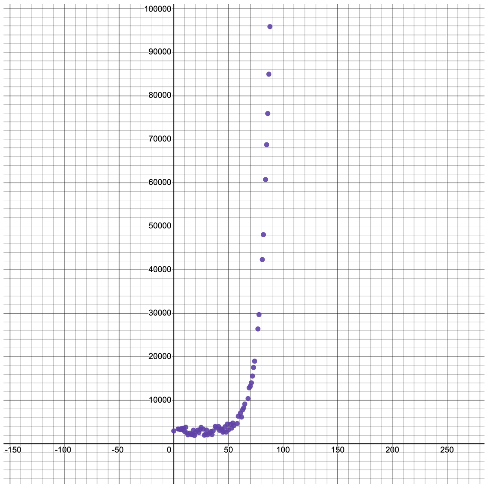


$a=-5$

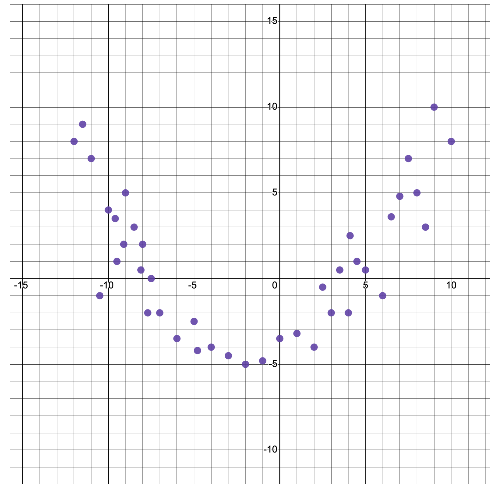
8) What does a tell us about an exponential function? ________________

What Kind of Model? (Graphs & Plots)

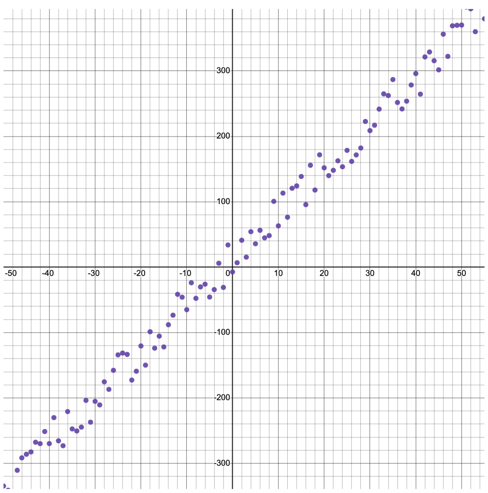
Decide whether each scatter plot appears to best be described by a linear, quadratic, or exponential model.



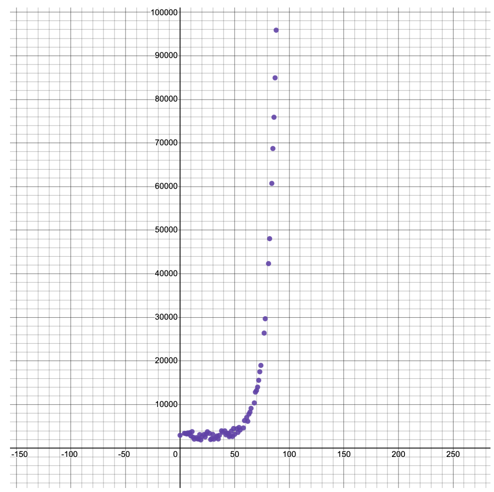
1) Linear Quadratic Exponential



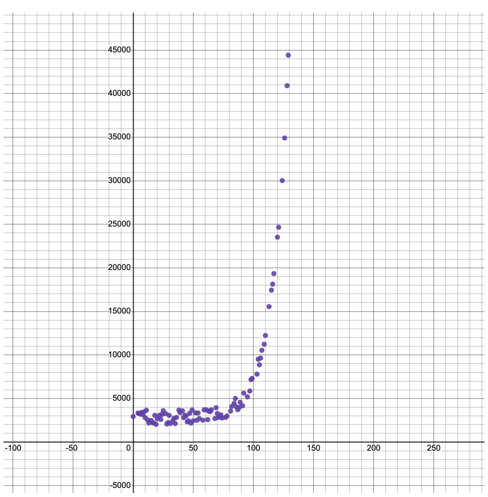
2) Linear Quadratic Exponential



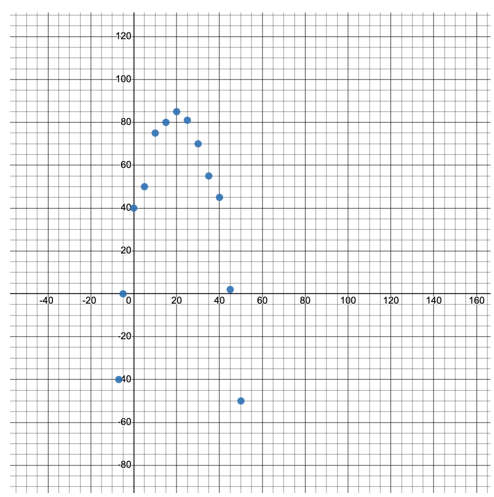
3) Linear Quadratic Exponential



4) Linear Quadratic Exponential



5) Linear Quadratic Exponential



6) Linear Quadratic Exponential

What Kind of Model? (Tables)

Decide whether each table is best described by a linear, quadratic, or exponential model.
If the model is exponential: Is it doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?

x	y
1	5
2	10
3	15
4	20
5	25
6	30
7	35

1) Linear Quadratic Exponential ________ factor

x	y
0	10
1	100
2	1000
3	10000
4	100000
5	1000000
6	10000000

2) Linear Quadratic Exponential ________ factor

x	y
70	-210
71	-169
72	-81
73	-34
74	15
75	66
76	119

3) Linear Quadratic Exponential ________ factor

x	y
-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

4) Linear Quadratic Exponential ________ factor

x	y
0	1
1	2
2	4
3	8
4	16
5	32
6	64

5) Linear Quadratic Exponential ________ factor

x	y
-5	1
-4	6
-3	36
-2	216
-1	1296
0	7776
1	466656

6) Linear Quadratic Exponential ________ factor

What Kind of Model? (Definitions)

Decide whether each representation describes a linear, quadratic, or exponential function.
If the function is exponential: Identify the growth factor and the initial value.

$$f(x) = 6x^2 - 5$$

1) Linear Quadratic Exponential

$$\text{miles}(\text{hours}) = \frac{22 \times \text{hours} + 14}{12 - 9}$$

2) Linear Quadratic Exponential

$$\text{cost}(w) = 1.2^w + 16$$

3) Linear Quadratic Exponential

$$t(g) = 42 - 2g^2$$

4) Linear Quadratic Exponential

$$\text{price}(d) = d^2 + 6d$$

5) Linear Quadratic Exponential

$$j(i) = \frac{1}{2}^i + 22$$

6) Linear Quadratic Exponential

$$f(x) = 20000 - 4.1^x$$

7) Linear Quadratic Exponential

$$g(x) = 3^{-4x}$$

8) Linear Quadratic Exponential

What Kind of Model? (Descriptions)

Decide whether each situation is best described by a linear, quadratic, or exponential function.
If the function is exponential: What is the growth factor. Is it doubling (factor of 2)? Tripling (factor of 3)? Factor of 5? 10?

1) The resale value of a car drops by a fixed percentage each year. A particular kind of car sells for $32,000, and its value drops by 12.5% each year

a. When the car is brand-new (x=0), how much is it worth? $32,000

b. How much is it worth after 1 year (x=1)? ________

c. After two years (x=2)? ________ After three years (x=3)? ________ Four (x=4)? ________

d. What is the **form** of this function (linear, quadratic, or exponential)? ________

e. **If it's exponential**, what is the initial value? ________ The base? ________ Is it growth or decay? ________

2) Sally is selling lemonade, for $1.25 a glass. She starts with $20 in cash, and hopes that by selling lemonade she will finally be able to get the power drill she's been wanting.

a. When Sally starts the day (x=0), how many dollars does she have? $0

b. How many dollars will she have after the first sale (x=1)? ________

c. After the sale (x=2)? ________ The third (x=3)? ________ The fourth (x=4)? ________

d. What is the **form** of this function (linear, quadratic, or exponential)? ________

e. **If it's exponential**, what is the initial value? ________ The base? ________ Is it growth or decay? ________

3) Mrs. Bidwell's club rules are that every student should high-five every other student. She starts out her year with only two students, but a new one joins the club every day.

a. How many high-fives happen at the start (x=0), with 2 students? 1

b. How many high-fives happen the next day (x=1), with 3 students? ________

c. With a fourth? (x=2)? ________ A fifth (x=3)? ________ A sixth (x=4)? ________

d. What is the **form** of this function (linear, quadratic, or exponential)? ________

e. **If it's exponential**, what is the initial value? ________ The base? ________ Is it growth or decay? ________

4) A meme goes viral on the internet, starting with one person posting an animation of a puppy doing a backflip into a pile of laundry. Every person that sees the meme falls in love with it, sharing it with 25 new friends.

a. When the person posts it (x=0), how many total times has it been shared? 1

b. How many times will it have been shared after those friends share it (x=1)? ________

c. When x=2? ________ When x=3? ________ When x=4? ________

d. What is the **form** of this function (linear, quadratic, or exponential)? ________

e. **If it's exponential**, what is the initial value? ________ The base? ________ Is it growth or decay? ________

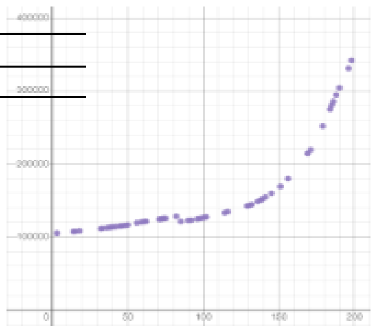
Exponential Models - MA Table

Fitting the Model Visually $f(x) = ab^x + k$

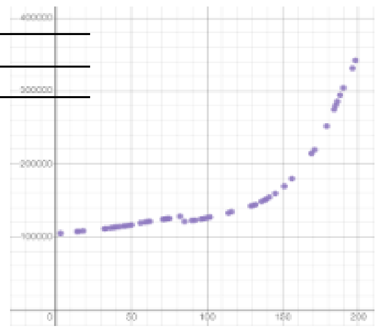
For this section, you'll need to have [Modeling Covid Spread (Desmos)] open on your computer.

1) Try changing the value of k, then a, then b to find three promising exponential models, graphing each one and labeling your values on the grids below.

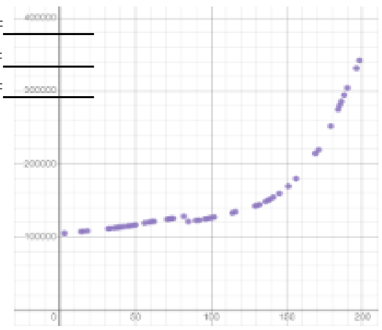
a=________
b=________
k=________



a=________
b=________
k=________



a=________
b=________
k=________



Fitting the Model Programmatically $f(x) = ab^x + k$

For this section, open your copy of the [Covid Spread Starter File].

2) In the space below, define `exponential1` for one of the models you fit in Desmos.

```
fun exponential1(x): ( ____________ * num-expt( ____________ , (~1 * x))) + ____________
                              a                       b                            k
end
```

Two Notes on this function definition:

- `num-expt` *is the function that we use for exponents. It takes in 2 numbers: the base and the power, in this case b and x.*
- *At first it may appear that x is being multiplied by negative 1 (−1), when it is actually being multiplied by* **~1** *(literally the value "roughly 1"). This tells Pyret to round off the calculation, prioritizing* ***speed*** *over* ***precision*** *to get a result that is "roughly accurate". We've added this to the function definition so that you won't have to wait for several minutes for Pyret to run* `fit-model` *to get an answer for question 4.*

3) Type your definition into the Definitions Area.

4) Use `fit-model` to determine how closely `exponential1` fits the `MA-table`. R^2 = ____________________

5) Are exponential models a good fit for this data? Why or why not? ______________________________

____

★) Rewrite the model so it doesn't multiply by `~1` to make Pyret do these calculations with extreme precision. WARNING: be sure to save your work first, as there's a good chance this will lock up your browser and require force-quitting! Data scientists perform calculations to do things like send satellites to far-away planets, or analyze large populations of a billion or more. You know the pros of using `~1` involve speed. What are the potential downsides of using `~1` to speed up a calculation?

____

____

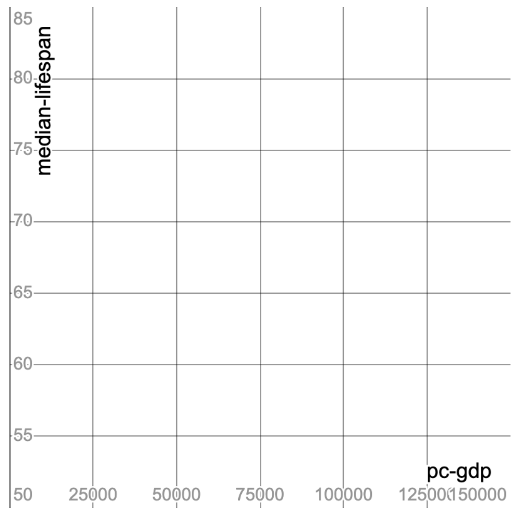
____

Exploring the Countries Dataset

For this page, you'll need the Countries of the World Starter File open on your computer. If you haven't already, select **Save a Copy** from the "File" menu to make a copy of the file that's just for you. The columns in this dataset are described below:

- **country** - name of the country
- **gdp** - total gross domestic product of the country
- **population** - number of people in the country
- **pc-gdp** - "per-capita gdp": the average GDP *per-person*
- **has-univ-healthcare** - indicates if the country has universal healthcare
- **median-lifespan** - the median life expectancy of people in the country

1) Make a scatter plot showing the relationship between `pc-gdp` and `median-lifespan`. Sketch the shape of the plot below.



2) What do you **Notice?** ____________________

3) What do you **Wonder?** ____________________

4) Are there any countries that stand out? Why or why not?

5) Suppose a wealthy country is suffering heavy causalties in a war. Draw a star on the plot, showing where you might expect it to be.

6) Do you think you see a relationship? If so, describe it. Is it linear or nonlinear? Strong or weak?

Stop here! We'll continue after some discussion.

Fitting Models

For each question below: (1) explore in Fitting Wealth-v-Health (Desmos); (2) define and fit your model in Pyret (the starter file already contains sample functions for you to change, called `linear1`, `quadratic1`, and `exponential1`!). Then write the model and the R^2.

7) Find the best **linear model** you can, using the first slide in the Desmos activity or `lr-plot` in the Countries of the World Starter File.

$f(x) =$ ________ (slope (m)) x + ________ (y-intercept (b)) ________ (R^2)

8) Find the best **quadratic model** you can, using the second slide *(quadratic)* in the Desmos activity.

$f(x) =$ ________ (quadratic coefficient (a)) $(x -$ ________ (vertex (h)) $)^2 +$ ________ (vertical shift (k)) ________ (R^2)

9) Find the best **exponential model** you can, using the third slide *(exponential)* in the Desmos activity.

$f(x) =$ ________ (initial value (a)) (________ (growth factor (b)) $^x) +$ ________ (vertical shift (k)) ________ (R^2)

10) Are any of these models a good fit for this data? Why or why not?

What Kind of Model? (Descriptions)

Decide whether each situation describes a quadratic, exponential, or logarithmic function.

1) The Richter Scale is measures the energy released by an earthquake. A magnitude 4 earthquake is 100 times more powerful as a magnitude 3 earthquake, which is 10 times as powerful as a magnitude 2 earthquake.

Quadratic Exponential Logarithmic

2) A car accelerates at a constant rate of 5mph/s.

Quadratic Exponential Logarithmic

3) The population of a colony of bacteria can double every 20 minutes, as long as there is enough space and food.

Quadratic Exponential Logarithmic

4) Benjamin Franklin set aside $4,400 in a savings account for the city of Philadelphia, knowing that the account would gain interest each year. 200 years later, the account was worth $1.625 million dollars!

Quadratic Exponential Logarithmic

5) Moore's law says that the number of transistors in a microprocessor will double roughly every 1.5 years. How many years will it take for the number of transistors in today's processors to increase by 100x?

Quadratic Exponential Logarithmic

6) As the *width* of a yard increases, the *area* of the yard increases much faster.

Quadratic Exponential Logarithmic

7) What explanation would you give to someone else, to help them identify which tables show **exponential** growth and which show **logarithmic** growth? ______________________________

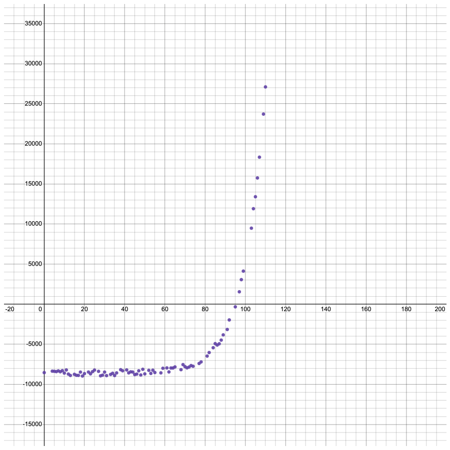
____

____

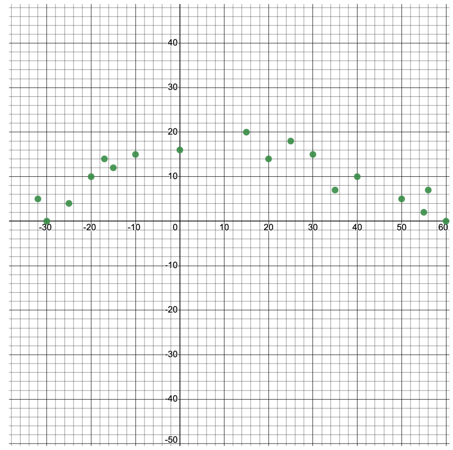
____

What Kind of Model? (Graphs & Plots)

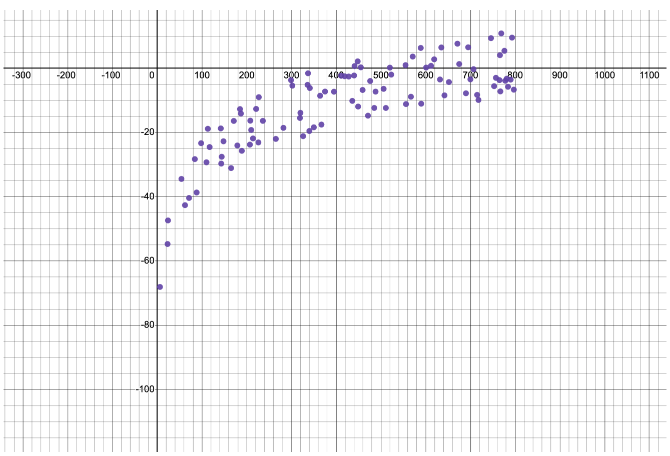
Decide whether each representation is best described by a quadratic, exponential, or logarithmic function.



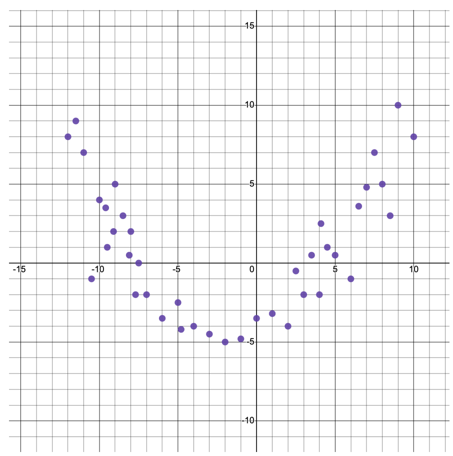
1) Quadratic Exponential Logarithmic



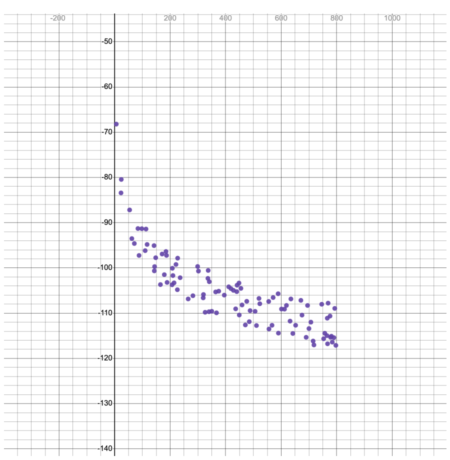
2) Quadratic Exponential Logarithmic



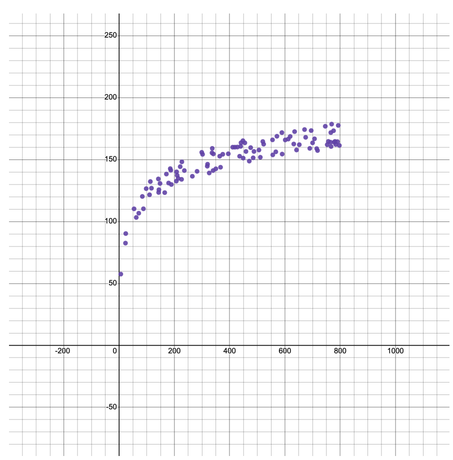
3) Quadratic Exponential Logarithmic



4) Quadratic Exponential Logarithmic



5) Quadratic Exponential Logarithmic



6) Quadratic Exponential Logarithmic

What Kind of Model? (Tables)

Decide whether each representation is best described by a quadratic, exponential, or logarithmic function.
If the function is logarithmic: How much does x need to increase (2x? 10x?) just to get a single increase in y?

x	y
1	0
10	1
100	2
1000	3
10000	4
100000	5
1000000	6

1) Quadratic Exponential Logarithmic

x	y
0	1
1	10
2	100
3	1000
4	10000
5	100000
6	1000000

2) Quadratic Exponential Logarithmic

x	y
70	-210
71	-169
72	-81
73	-34
74	15
75	66
76	119

3) Quadratic Exponential Logarithmic

x	y
2	1
4	2
8	3
16	4
32	5
64	6
128	7

4) Quadratic Exponential Logarithmic

x	y
-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

5) Quadratic Exponential Logarithmic

x	y
1	0
6	1
36	2
216	3
1296	4
7776	5
466656	6

6) Quadratic Exponential Logarithmic

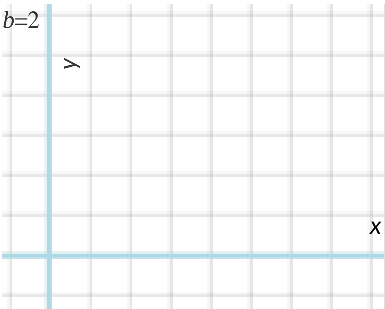
Graphing Logarithmic Models

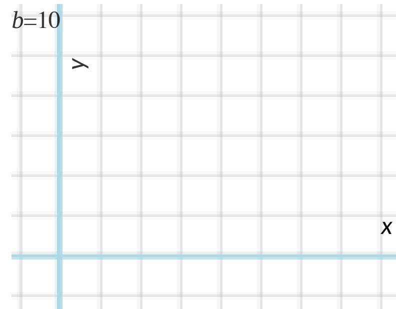
To complete this page, you'll need to open Exploring Logarithmic Functions (Desmos). The red curve is the graph of $h(x) = 1\ \log_2 x + 0$. It has a=1, b=2, and c=0. You can modify the curve $g(x)$ (behind h, shown in blue) by changing its a, b, and c.

Base *b*

Set *c* to zero and *a* to one.

1) Change the value of b to 3, 5, and 10, graphing each curve below. In each graph, **label the coordinate where *x*=1, and also where y=1, 2, and 3.**





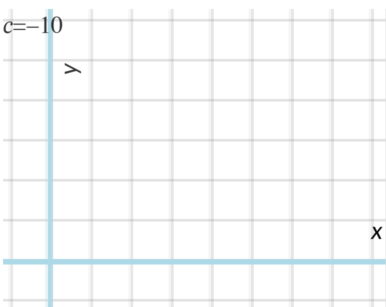
2) How does the value of b impact a logarithmic function? ______________________________

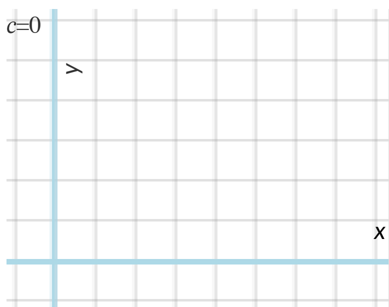
3) What connections can you draw between the value of b and exponents? ______________________________

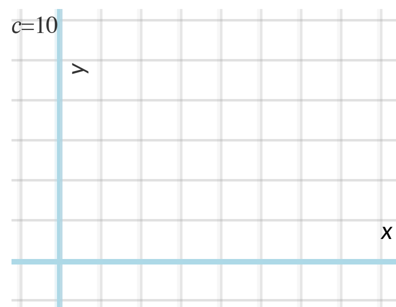
____

Vertical Shift *c*

4) Try changing the value of c to -10, 0, and 10, graphing each curve below. In each graph, **label the coordinate where *x*=1**.







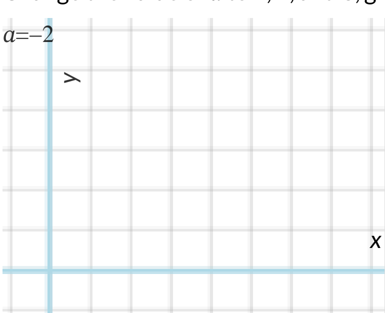
5) How does the value of c impact a logarithmic function? ______________________________

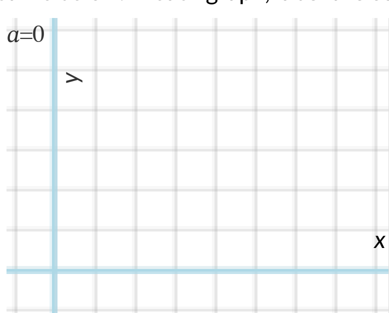
6) Why does $y = c$ when x=0? ______________________________

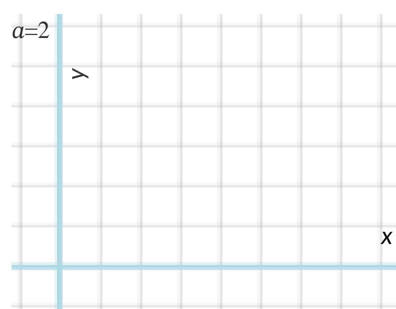
Logarithmic Coefficient *a*

7) *Set **c** to zero and **b** to ten, then zoom out so you can see as far as* $x = 1{,}000{,}000$

Change the value of a to 1, 2, and 3, graphing each curve below. In each graph, **label the coordinates where x=10, 100, and 1000.**







8) What is the value of x when $\log_{10}(x) = 6$? ____________ What about when $2\ \log_{10}(x) = 6$? __________ When $3\ \log_{10}(x) = 6$? __________

★) How are a and b related? ______________________________

Changing the Scale

For this page, you'll need to load Fitting Wealth-v-Health, Part 2 (Desmos).

Fitting a Logarithmic Model

Open the `Data Table` folder by clicking on the triangle (▶)

- x_1 is the per-capita income for each country, and y_1 is the median lifespan.
- Next to y_1 you'll see a dark circle with spots (⁘) inside. If the circle is dark, that means that those points are visible on our graph. Click the circle to "turn off" those dots, then click it again to turn them back on.
- Move the graph by clicking and dragging the background.
- Notice that a magnifying glass (🔍) appears to the bottom left of the table. Clicking on the magnifying glass resizes/rescales the graph to fit all the points in the table.

Look at the numbers along the x-axis, which increase as they go from left to right.

1) What would the next number be, if you were to add one at the far right: __________ *Describe* the pattern you used to find it: ____________

____

2) Move the sliders for a and c to create the best-fitting logarithmic model you can find, and write it below.

$f(x) =$ ____________________ $\log_{10}(x) +$ ____________________
log coefficient (a) vertical shift (c)

3) In Pyret, modify `f(x)` to define this model, and fit it using the `fit-model` function. What is your R^2? ____________________

Scaling the x-Axis

Open the "Graph Settings" window in Desmos by clicking on the wrench button (🔧) in the top-right corner of the graph.

- **Expand the "More Options" section** by clicking the triangle (▶).
- **Change the x-axis scale** from `Linear` to `Logarithmic`.
- **Zoom out** or **Zoom fit by click the magnifying glass beneath the table** 🔍 to put all of the points back into view.

Look at the numbers along the x-axis, which increase as they go from left to right.

4) What would the next number be, if you were to add one at the far right: __________ *Describe* the pattern you used to find it: ____________

5) What is the shape of the point cloud *now?* ______________________________
Is it linear? Quadratic? Exponential? Something else?

6) Adjust the sliders for a and c to create the best-fitting model you can find, and write it below.

$f(x) =$ ____________________ $\log_{10}(x) +$ ____________________
log coefficient (b) vertical shift (c)

7) In Pyret, modify `f(x)` to define this model, and fit it using the `fit-model` function. What is your R^2? ____________________

8) Why did transforming the **x-axis** make our data look linear? ______________________________

____

____

Transforming the Data

For this page, you'll need to load Fitting Wealth-v-Health, Part 2 (Desmos) open on your computer.

Transforming the Data

- **Find the `Wealth vs. Health` folder, which is open at the top of the expression list**
- This is the same table we've seen before, and the "points" circle (⁘) shows us that these dots are "on" and visible.
- Underneath the `Wealth vs. Health` folder, you'll see a **function** $g(x)$ and a **list** y_2 defined to be the same as y_1.
- **Open the `Log(Wealth) vs. Health` folder by clicking on the triangle (▶)**

1) Compare the two tables. What do you **Notice**? ______________________________

____

2) What do you **Wonder**? ______________________________

____

3) Why is the second column of both tables the same? ______________________________

4) How is the first column of this new table different from the original? ______________________________

____

Turn the points for the first table OFF, then turn the points for our new table ON. Our log transformation is so drastic that it looks like all the black datapoints are smashed against the y-axis!

5) **Rescale** the graph (🔍) to see the cloud. What is the shape of this point cloud? Is it linear? Quadratic? Exponential? ______________

6) Move the sliders for m and b to create the best-fitting linear model you can find, and write it below.

$f(x) =$ ______________ $x +$ ______________
slope (m) — y-intercept / vertical shift (b)

7) Why do you think transforming the **x-values** make our data look linear? ______________________________

____

____

Transforming Axes vs. Transforming Data

8) From your linear model above, copy your values for slope (m) and vertical shift (c):

______________ slope (m)   ______________ vertical shift (c)

9) Look back at the values you wrote for log coefficient (a) and vertical shift (c) on question 8 of Changing the Scale and copy them here:

______________ log coefficient (a)   ______________ vertical shift (c)

10) Are they similar? __________ Why or why not? ______________________________

____

____

Logarithmic Models

Open your copy of the Countries of the World Starter File and click "Run".

Part 1

1) Find the definition of `g(r)`. What does this function do? ______________________________

____

2) Find the Contract for `build-column` on the Contracts Page.

What is its Range? ______________________ What is its Domain? ______________________

3) At the end of the program, you'll find this code:

```
countries-transformed = build-column(countries-table, "log(pc-gdp)", g)
```

What do you think it does? ______________________________

____

4) Click "Run", and evaluate `countries-transformed` in the Interactions Area on the right to test it out!

a. What did you get back? ______________________________

b. What is different about this Table? ______________________________

c. Where did the column on the right come from? ______________________________

d. What does that line of code at the end of the program do? ______________________________

____

5) Use this new table to make an `lr-plot` comparing `log(pc-gdp)` and `median-lifespan` with `country` as the label.

6) Record the regression line here: $y =$ ______________ (slope) $x +$ ______________ (vertical shift) R^2: ______________

Part 2

7) Use the values produced by `lr-plot` to complete the model: $logarithmic(x) =$ ______________ (log coefficient) $\log_{10}(x) +$ ______________ (vertical shift)

8) Let's interpret this model:

A *10x* increase in `per-capita-GDP` is associated with `median-lifespan` increasing by ______________.

9) Rewrite your model as a pyret definition: `fun logarithmic(x):` ______________________________ `end`

10) Add the definition to your starter file and click "Run" to load it.

11) Use `fit-model` to calculate the value of R^2: ______________

12) Let's interpret this R^2 value:

Roughly ________ percent of the variation in ______________________ is explained by the variation in ______________________.

★ Are there other relationships you can think of, which might be logarithmic?

Contracts for Algebra 2

Contracts tell us how to use a function, by telling us three important things:

1. The **Name**
2. The **Domain** of the function - what kinds of inputs do we need to give the function, and how many?
3. The **Range** of the function - what kind of output will the function give us back?

For example: The contract `triangle :: (Number, String, String) -> Image` tells us that the name of the function is `triangle`, it needs three inputs (a Number and two Strings), and it produces an Image.

With these three pieces of information, we know that typing `triangle(20, "solid", "green")` will evaluate to an Image.

Name		Domain		Range
# bar-chart	::	(Table (table-name), String (column))	->	Image
bar-chart(animals-table, "species")				
# box-plot	::	(Table (table-name), String (column))	->	Image
box-plot(animals-table, "weeks")				
# count	::	(Table (table-name), String (column))	->	Number
count(animals-table, "species")				
# first-n-rows	::	(Table (table-name), Number (num-rows))	->	Table
first-n-rows(animals-table, 15)				
# fit-model	::	(Table (table-name), String (labels), String (xs), String (ys), (Num -> Num) (model-function))	->	Image
fit-model(animals-table, "name", "pounds","weeks", f)				
# histogram	::	(Table (table-name), String (labels), String (values), Number (bin-size))	->	Image
histogram(animals-table, "species", "weeks", 2)				
# line-graph	::	(Table (table-name), String (xs), String (ys))	->	Image
line-graph(animals-table, "name", "pounds","weeks")				
# lr-plot	::	(Table (table-name), String (labels), String (xs), String (ys))	->	Image
lr-plot(animals-table, "name", "pounds","weeks")				
# pie-chart	::	(Table (table-name), String (column))	->	Image
pie-chart(animals-table, "species")				
# r-value	::	(Table (table-name), String (xs), String (ys))	->	Number
r-value(animals-table, "name", "pounds","weeks")				
# row-n	::	(Table (table-name), Number (index))	->	Row
row-n(animals-table, 2)				

Name	Domain		Range
# scatter-plot	:: (Table [table-name], String [labels], String [xs], String [ys])	->	Image
`scatter-plot(animals-table, "name", "pounds","weeks")`			
# sort	:: (Table [table-name], String [column], Boolean [ascending])	->	Table
`sort(animals-table, "species", true)`			
# string-contains	:: (String [haystack], String [needle])	->	Boolean
`string-contains("hotdog", "dog")`			

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